# COMPLETE ARITHMETIC

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# COMPLETE ARITHMETIC

BY

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# PREFACE

A COMPLETE arithmetic should meet all the ordinary demands of the elementary school. It may omit that which is non-essential and all matter that properly belongs to text-books for secondary schools; but it should include a full treatment of all important topics taught in the elementary school, and a limited treatment of those of minor importance.

This book is intended for a complete arithmetic, to be used either with or without the author's Elementary Arithmetic. The work is divided into three parts. *Part One*, after giving a complete treatment of the fundamental operations, covers the work ordinarily found in the sixth year. *Part Two*, after reviewing the subjects of Bills and Accounts, Denominate Numbers, and Practical Measurements, covers the work of the seventh year. *Part Three* covers the work of the eighth year.

The aim of this book is threefold:

(1) To give the pupil skill in the art of computation.

(2) To make him a good mathematician.

(3) To give him a working knowledge of modern business methods.

The first necessarily suggests an abundance of graded work.

The second requires both inductive and deductive thought. The method, therefore, is inductive in the development of all mathematical principles, and deductive in their application. It requires also that all solutions shall be clear and

concise, and that the statements of all definitions, rules, and principles shall be as brief and comprehensive as possible.

The third aim demands a practical treatment of all subjects from the standpoint of actual business methods.

With these ends in view, attention is invited to the following:

1. The large number of graded problems under each subject and in the reviews.

2. The abundance of exercises for oral drill.

3. The study of problems and processes.

4. The treatment of Fractions, Practical Measurements, and the Comparative Studies in Percentage.

5. The problems arising out of business conditions.

6. The treatment of Promissory Notes, Banking, Commercial Discount, Exchange, and Stocks and Bonds according to the actual methods of modern business.

The author gratefully acknowledges his indebtedness to many prominent educators and friends for valuable aid, discriminating criticisms, and helpful suggestions.

SAMUEL HAMILTON.

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# COMPLETE ARITHMETIC

## PART I — SIXTH YEAR

## FUNDAMENTAL PROCESSES

### NOTATION AND NUMERATION

A **unit** is a single thing; as, one, one cent.

A **number** is a unit or a collection of units.

**Numbers** are used to tell *how many;* they are expressed by figures or letters. The figures we now use are of Hindu origin, but the Arabs were the first people to introduce them into Europe. They are

| NAUGHT | ONE | TWO | THREE | FOUR | FIVE | SIX | SEVEN | EIGHT | NINE |
|---|---|---|---|---|---|---|---|---|---|
| **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |

These ten figures are, therefore, called **Arabic numerals**.

The figure **0** is called **naught**, **zero**, or **cipher**, and has no value.

The **Arabic notation** is a method of expressing numbers by means of figures.

**Numeration** is a method of reading numbers expressed by means of figures or letters.

**Arithmetic** is the science of numbers and the art of computing by them.

## ARABIC SYSTEM OF NOTATION AND NUMERATION

Any number containing but one figure, simply stands for so many ones; thus, 9 stands for 9 units, or ones.

In any number containing two figures, the *first place* at the right is called **ones**; the *second place* is called **tens**; thus, in 25 there are 2 tens, or 20 ones, plus 5 ones, or 25 ones. 25 is read, "twenty-five."

**1.** Name the *ones* and *tens* in the following numbers and then state how many ones each number equals:

15 25 30 75 82 60 72 45 70 99 20 10 39 47 90

In any number containing three figures, the *third place* from the right is called **hundreds**; thus, in 325, the 3 stands for 300 ones. 325 is read, "three hundred twenty-five."

**2.** Name the *ones*, *tens*, and *hundreds* in the following and then state how many ones each number equals:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 125 | 329 | 879 | 801 | 600 | 650 | 803 | 132 | 400 | 904 |
| 109 | 705 | 105 | 550 | 900 | 901 | 502 | 999 | 570 | 809 |

In any number containing four figures, the *fourth place* from the right is called **thousands**. 4635 is read, "four thousand, six hundred, thirty-five."

**3.** Name the places in each number and then read:

2135 6005 6910 5604 5000 4025 2684 8709 7009 8900

For convenience in reading large numbers, in the Arabic system, the figures are generally separated by commas into groups of three figures each, called **periods**.

The first period, counting from the right, is *units;* the second, *thousands;* the third, *millions;* the fourth, *billions;* the fifth, *trillions;* etc.

The following table shows the arrangement of these periods, and the three **orders** of figures in each period:

| TRILLIONS' PERIOD | | | BILLIONS' PERIOD | | | MILLIONS' PERIOD | | | THOUSANDS' PERIOD | | | UNITS' PERIOD | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hundred-trillions | Ten-trillions | Trillions | Hundred-billions | Ten-billions | Billions | Hundred-millions | Ten-millions | Millions | Hundred-thousands | Ten-thousands | Thousands | Hundreds | Tens | Ones |
| 1 | 0 | 1, | 3 | 4 | 5, | 6 | 4 | 2, | 0 | 0 | 1, | 3 | 4 | 6 |

The number in the table is read, "101 trillion, 345 billion, 642 million, 1 thousand, 346."

The *ones* are not named in reading numbers.

Since *ten ones* make 1 *ten*, and *ten tens* make 1 *hundred*, etc., our system of writing numbers is called a **decimal system**. The word decimal comes from the word *decem*, meaning *ten*.

Name the places and periods, then read:

**4.** 129,475
**5.** 407,575
**6.** 600,905
**7.** 790,505
**8.** 1,700,425 pounds
**9.** 9,609,500 tons
**10.** 5,505,608 yards
**11.** 40,000,905 dollars
**12.** 400,000,000
**13.** 709,050,050
**14.** 8,000,000,000
**15.** 9,075,074,093

Copy, point off, then read:

**16.** 700102
**17.** 6067004
**18.** 8011100
**19.** 90314607
**20.** 910723586
**21.** 837421012
**22.** 987654321
**23.** 5050504
**24.** 12126012
**25.** 87649101
**26.** 104706950
**27.** 1202436414
**28.** 3183456109
**29.** 7891234560
**30.** 60414
**31.** 102365
**32.** 14763487
**33.** 243540038
**34.** 2452603467
**35.** 8703005020
**36.** 6010100100

### Writing Numbers

Write:

1. 25 million, 161 thousand, 104.
2. 2 million, 12 thousand, 8.
3. 36 million, 1 thousand, 109.
4. 300 billion, 304 million, 100 thousand, 40.
5. 16 billion, 9 million, 70 thousand, 700.
6. 26 million, 18 thousand, 9.
7. 7 billion, 46 million, 900 thousand, 90.
8. 74 billion, 7 million, 46 thousand, 809.
9. Two hundred six thousand, eight.
10. Twenty-five million, six hundred.
11. One-thousand thousand.

Write each of the numbers 12 to 23, first, when 10 is added, and second, when 10 is subtracted from each number.

| | | | |
|---|---|---|---|
| 12. 80,900 | 15. 60,804 | 18. 497,842 | 21. 50,001 |
| 13. 67,895 | 16. 50,000 | 19. 109,090 | 22. 70,080 |
| 14. 45,101 | 17. 70,800 | 20. 290,009 | 23. 59,009 |

24. One million, two hundred thousand.
25. Nine billion, six-hundred million, seven.
26. Six billion, six thousand, six hundred six.
27. Seven hundred nine thousand, two.
28. Sixty-nine million, eight thousand.
29. Nine hundred two million, forty-two thousand, sixty-two
30. Thirty-two thousand, thirty-two.
31. Six-hundred forty-four million, six hundred four.
32. Ten thousand, ten.
33. Fifteen hundred million.
34. Twenty-eight million, eight thousand, eight.

## ROMAN SYSTEM OF NOTATION AND NUMERATION

The seven letters used in Roman notation are :

| | | | | | | |
|---|---|---|---|---|---|---|
| I | V | X | L | C | D | M |
| 1 | 5 | 10 | 50 | 100 | 500 | 1000 |

The other numbers are represented by combinations thus:

I. *When a letter is followed by the same letter or by one of less value, the values of the letters are to be added.* Thus, XX represents 20; XI represents 11.

II. *When a letter is followed by one of greater value, the value of the smaller is to be subtracted from that of the greater.* Thus, IV represents 4; IX represents 9.

III. *When a letter is placed between two letters of greater value, the value of the smaller is to be subtracted from the sum of the other two.* Thus, XIV represents 14; XIX represents 19.

*A bar placed over a letter multiplies its value by* 1000. Thus, $\overline{\text{V}}$ represents 5000.

The following table further illustrates the system :

| | | | | |
|---|---|---|---|---|
| I, 1 | VIII, 8 | XVI, 16 | LXXX, 80 | DCC, 700 |
| II, 2 | IX, 9 | XX, 20 | XC, 90 | DCCC, 800 |
| III, 3 | X, 10 | XXX, 30 | C, 100 | CM, 900 |
| IV, 4 | XI, 11 | XL, 40 | CCC, 300 | M, 1000 |
| V, 5 | XII, 12 | L, 50 | CD, 400 | MCM, 1900 |
| VI, 6 | XIV, 14 | LX, 60 | D, 500 | $\overline{\text{V}}$, 5000 |
| VII, 7 | XV, 15 | LXX, 70 | DC, 600 | $\overline{\text{M}}$, 1,000,000 |

Read :

**1.** XLIII CDXLIX $\overline{\text{M}}$DII MCDXCII

**2.** XCIX MCMVIII MDLXXVI MDCCCLXI

**3.** Express in Roman notation: 41, 63, 84, 99, 107, 218, 572, 735, 996, 1907, 1564, 1616, 1000, 260,000.

## UNITED STATES MONEY

| | |
|---|---|
| **10 mills = 1 cent** | **10 dimes = 1 dollar** |
| **10 cents = 1 dime** | **10 dollars = 1 eagle** |

**1.** From the above table, tell why our money is a **decimal system** of money.

The dollar sign is \$; it is placed before the number of dollars. The sign for cent or cents is ¢; it is placed after the number of cents.

When dollars and parts of a dollar are written as one number, a period, called a **decimal point**, separates the dollars from the cents; thus, 8 dollars, 15 cents is written \$8.15.

Parts of a dollar may be written in three ways; thus, 15 cents may be written 15¢, \$0.15, or \$.15.

The first two places to the right of the decimal point are for cents.

**2.** Read in as many ways as you can:

| | | | | | | |
|---|---|---|---|---|---|---|
| 15¢ | \$8.32 | 5¢ | 105¢ | \$0.99 | \$.25 | \$0.50 |
| 900¢ | \$.01 | \$0.35 | \$0.05 | 100¢ | 1101¢ | \$9.90 |

The first place to the right of the point is for dimes, or **tenths** of a dollar; the second, for cents, or **hundredths** of a dollar; the third, for **mills**, or **thousandths** of a dollar. \$.025 is read, "two cents, five mills."

**3.** Read:

| | | | |
|---|---|---|---|
| \$.02 | \$8.06 | \$.901 | \$.515 |
| \$.022 | \$9.055 | \$.80 | \$.005 |
| \$.255 | \$1.005 | \$.801 | \$.50 |

**4.** Read, then write from dictation, using the dollar sign:

| | | |
|---|---|---|
| \$2.50 | 275 dollars, 5 cents | 100 cents, 8 mills |
| \$0.75 | 89 dollars, 2 mills | 525 cents, 5 mills |
| \$5.05 | 10 dollars, 1 cent | 875 cents, 3 mills |

## ADDITION

**1.** Count by 2's to 100; by 3's to 99; by 4's to 100.
**2.** Count by 6's to 102; by 7's to 105; by 8's to 104.
**3.** Count by 9's to 108; by 11's to 143; by 12's to 168.

**Addition** is the process of uniting two or more numbers to form one number.

The sign +, called *plus*, indicates *addition;* the sign =, called *equal* or *equals*, indicates *equality*.

Announce the sums at sight:

| **4.** | **5.** | **6.** | **7.** | **8.** | **9.** |
|---|---|---|---|---|---|
| 5 + 7 | 9 + 9 | 1 + 3 | 8 + 6 | 2 + 2 | 2 + 8 |
| 1 + 6 | 1 + 4 | 2 + 9 | 9 + 8 | 4 + 6 | 4 + 2 |
| 3 + 3 | 6 + 2 | 9 + 1 | 7 + 2 | 6 + 7 | 9 + 4 |
| 9 + 6 | 9 + 3 | 7 + 3 | 8 + 7 | 8 + 0 | 0 + 9 |
| 6 + 6 | 2 + 1 | 8 + 5 | 4 + 4 | 2 + 3 | 3 + 4 |
| 1 + 8 | 5 + 4 | 7 + 2 | 5 + 6 | 6 + 3 | 8 + 4 |
| 7 + 9 | 5 + 2 | 5 + 5 | 0 + 4 | 7 + 7 | 3 + 8 |
| 5 + 3 | 1 + 1 | 9 + 5 | 7 + 4 | 8 + 8 | 5 + 1 |

Announce the sums at sight, first in rows across, then in columns:

| **10.** | **11.** | **12.** | **13.** | **14.** | **15.** | **16.** | **17.** |
|---|---|---|---|---|---|---|---|
| 154 | 534 | 561 | 147 | 964 | 275 | 784 | 368 |
| 371 | 826 | 729 | 684 | 837 | 984 | 926 | 574 |
| 723 | 943 | 358 | 493 | 496 | 532 | 578 | 397 |

In like manner, announce the sums:

| **18.** | **19.** | **20.** | **21.** | **22.** | **23.** |
|---|---|---|---|---|---|
| 4356 | 9765 | 3742 | 8674 | 7248 | 5632 |
| 8547 | 6934 | 9746 | 7897 | 6754 | 9985 |
| 7961 | 9847 | 6957 | 8753 | 3976 | 7987 |
| 5943 | 4957 | 6582 | 7549 | 6937 | 9147 |

The **addends** in addition are the numbers to be added.

The **sum** or **amount** is the result of addition.

**Like numbers** are numbers that express the same units; as, 3 pounds, 5 pounds, or 3, 5.

**Unlike numbers** are numbers that express different units; as, 7 days, $3.

*Only like numbers can be added.*

**24.** Name and add the like numbers in the following: $5, 5 ft., 4 oz., $4, 3 oz., 3 gal., 6 gal., 6 ft.

**25.** Name the addends; then give the sum or amount:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 8 | 7 | 5 | 3 | 4 | 12 | 5 | 7 | 12 |
| 1 | 4 | 0 | 4 | 11 | 9 | 9 | 0 | 11 |
| 5 | 2 | 7 | 9 | 7 | 10 | 8 | 12 | 5 |

**26.** Add 45 and 23, thus: $45 + 23 = 45 + 20 + 3 = 68$.

Add in like manner:

**27.** 55 and 28 = ?
**28.** 29 and 47 = ?
**29.** 31 and 42 = ?
**30.** 57 and 32 = ?
**31.** 34 and 47 = ?
**32.** 69 and 28 = ?
**33.** 49 and 35 = ?
**34.** 44 and 57 = ?
**35.** 54 and 36 = ?
**36.** 72 and 19 = ?
**37.** 37 and 47 = ?
**38.** 46 and 53 = ?

Add the two columns, as in problem 26.

| **39.** | **40.** | **41.** | **42.** | **43.** | **44.** | **45.** | **46.** | **47.** |
|---|---|---|---|---|---|---|---|---|
| 5 | 7 | 3 | 9 | 11 | 10 | 15 | 11 | 26 |
| 10 | 15 | 10 | 8 | 12 | 12 | 10 | 7 | 37 |
| 11 | 20 | 15 | 16 | 14 | 7 | 4 | 4 | 12 |
| 27 | 4 | 11 | 4 | 10 | 14 | 9 | 12 | 29 |
| 4 | 9 | 9 | 7 | 5 | 9 | 7 | 9 | 17 |
| 12 | 12 | 7 | 3 | 7 | 7 | 10 | 6 | 13 |

NOTE. — Practice on similar work, until pupils can add rapidly and accurately.

## Seeing Groups

Add, observing the groups that make 10, 15, etc.:

| 1. | | 2. | 3. | 4. | 5. | 6. | 7. |
|---|---|---|---|---|---|---|---|
| 4 | | 7 | 8 | 7 | 8 | 3 | 2 |
| 6 | 10 | 8 | 3 | 4 | 9 | 9 | 6 |
| 7 | | 4 | 7 | 6 | 3 | 8 | 4 |
| 3 | 10 | 7 | 6 | 5 | 5 | 2 | 8 |
| 5 | | 8 | 4 | 7 | 4 | 7 | 4 |
| 9 | | 1 | 2 | 2 | 6 | 8 | 6 |
| 8 | | 0 | 1 | 6 | 7 | 5 | 5 |
| 2 | 10 | 9 | 7 | 5 | 9 | 4 | 7 |
| 5 | | 3 | 5 | 4 | 8 | 2 | 8 |
| 7 | 20 | 8 | 6 | 5 | 9 | 1 | 1 |
| 8 | | 9 | 4 | 6 | 6 | 3 | 9 |

## Written Work

**1.** Find the sum of 325, 436, 285.

| | |
|---|---|
| | 325 |
| Addends | 436 |
| | 285 |
| Sum | 1046 |

**Test by adding downwards.**

Write the numbers in their order, as indicated. In adding the first column, we think 5, 11, 16 ones; then write the **6** ones and carry the 1 ten to the tens' column. 9, 12, 14 tens; then write the **4** tens and carry the hundred to hundreds' column. 3, 7, **10** hundreds. The sum is 1046.

Practice until you can add these problems rapidly:

| **2.** | **3.** | **4.** | **5.** |
|---|---|---|---|
| $729.15 | 206793 | 534891 | 726432 |
| 804.90 | 429874 | 567321 | 874921 |
| 728.16 | 358079 | 623413 | 785341 |
| 574.34 | 427451 | 347621 | 634541 |
| 420.85 | 874297 | 983122 | 442211 |
| 345.78 | 298743 | 246893 | 532891 |

In adding two or more columns, business men frequently write the sum of each column separately on a slip of paper, and then add the separate sums as illustrated in problem 6. In case of interruption or error it is then not necessary to go over all the work.

| 6. | 7. | 8. | 9. | 10. | 11. |
|---|---|---|---|---|---|
| 279 | 224 | 746 | 581 | 910 | 375 |
| 874 | 921 | 317 | 841 | 725 | 421 |
| 398 | 234 | 567 | 789 | 234 | 291 |
| 569 | 321 | 104 | 902 | 803 | 123 |
| 768 | 572 | 571 | 846 | 931 | 146 |
| 343 | 937 | 847 | 641 | 261 | 143 |
| 41 | 921 | 803 | 731 | 283 | 197 |
| 39 | 321 | 830 | 450 | 847 | 190 |
| 28 | 731 | 476 | 903 | 275 | 913 |
| 3231 | | | | | |

Add these problems and test the work in 9 minutes:

| 12. | | 13. | 14. | 15. | 16. | 17. |
|---|---|---|---|---|---|---|
| 5608 | 12 | 6720 | 2954 | 3783 | 1751 | 5068 |
| 2094 | | 4604 | 3261 | 5427 | 7173 | 7504 |
| 8675 | 10 | 7259 | 5050 | 7891 | 5009 | 8795 |
| 6985 | | 8753 | 7406 | 5030 | 4287 | 6474 |
| 3745 | 20 | 6758 | 2834 | 6793 | 5783 | 2758 |
| 5268 | | 4326 | 6498 | 7406 | 7205 | 6471 |
| 7777 | | 6734 | 5065 | 5872 | 4987 | 2050 |
| 5989 | | 7583 | 8439 | 3458 | 6034 | 6579 |
| 7481 | 10 | 3586 | 9823 | 7295 | 2985 | 2068 |
| 5479 | | 2734 | 7984 | 8376 | 3046 | 6579 |
| 8705 | 9 | 4725 | 2030 | 2794 | 1154 | 2068 |
| 3074 | | 6050 | 5984 | 6384 | 3683 | 5432 |
| 5547 | 12 | 7438 | 3749 | 6589 | 4594 | 4280 |
| 6875 | | 5006 | 5308 | 7405 | 5181 | 5683 |

## Making Change.

**Business men make change by the adding method.** Thus, if a purchase is made for \$1.57, and \$2.00 is given in payment, the clerk will probably say: "One dollar fifty-seven cents, sixty, seventy, seventy-five, two dollars," laying down each time the piece of money that makes the sum named.

Acting as clerk when the following purchases and payments are made, give the exact language you might use, if the purchaser were actually present to receive the change:

| | Cost of Purchase | Amount Given | | Cost of Purchase | Amount Given |
|---|---|---|---|---|---|
| 1. | \$1.34 | \$2.00 | 16. | \$1.95 | \$2.00 |
| 2. | 2.95 | 5.00 | 17. | 7.23 | 10.00 |
| 3. | 2.11 | 3.00 | 18. | 3.67 | 5.00 |
| 4. | 3.17 | 4.00 | 19. | 2.85 | 4.00 |
| 5. | 1.15 | 1.50 | 20. | 5.02 | 6.00 |
| 6. | 4.12 | 4.50 | 21. | .91 | 5.00 |
| 7. | 2.24 | 3.00 | 22. | .79 | 5.00 |
| 8. | 3.54 | 3.75 | 23. | 6.01 | 10.00 |
| 9. | 1.12 | 1.50 | 24. | 7.11 | 8.00 |
| 10. | 4.02 | 5.00 | 25. | 3.95 | 5.00 |
| 11. | 2.79 | 4.00 | 26. | 2.01 | 10.00 |
| 12. | 2.36 | 5.00 | 27. | 6.79 | 7.00 |
| 13. | 3.09 | 4.00 | 28. | 7.04 | 8.00 |
| 14. | 2.71 | 3.00 | 29. | 8.31 | 10.00 |
| 15. | 3.06 | 3.50 | 30. | 2.98 | 4.00 |

### Written Work

1. Chicago is 468 miles, by rail, west of Pittsburg, and New York is 445 miles east of Pittsburg. What is the railroad distance from Chicago to New York, via Pittsburg?

2. A man purchased a farm for \$6500; built a barn on it at a cost of \$1980; a house for \$1825; and spent on improvements on the land, \$971. What was the cost of the farm and improvements?

3. The area in square miles of the main body of the United States is 3,088,519; the outlying possessions are Alaska, 590,884 square miles; Hawaii, 6449; Porto Rico, 3435; Philippines, 115,026; other outlying possessions, 761 square miles. Find the total possessions in square miles.

4. The average annual production of corn in the United States for a number of years was \$854,000,000; of hay, \$467,000,000; the production of cotton, \$406,000,000; of wheat, \$395,000,000; and of oats, \$254,000,000. Find the total average annual value of these five productions.

5. The values of the ten leading exports of the United States for the year 1906 were: cotton, \$413,137,936; meat and dairy products, \$208,586,501; iron and steel and machinery, \$172,555,588; copper, \$90,773,151; petroleum, \$85,738,866; wood, \$77,255,225; flour, \$58,399,727; corn, \$52,840,269; wheat, \$49,158,650; livestock, \$45,614,748. What was the value of all these products?

6. The values of the ten leading imports for the same year were: silk and silk manufactures, \$100,052,211; raw and manufactured fibers, \$99,635,731; hides, \$83,884,981; sugar, \$79,015,471; chemicals, \$78,647,978; coffee, \$72,252,465; cotton goods, \$68,911,371; wool including woolen goods, \$61,040,335; india rubber, \$58,664,651; jewelry, \$46,047,021. Find the value of all these imports.

## SUBTRACTION

**1.** Subtract by 2's from 97 to 1; by 3's from 75 to 9.

**2.** Subtract by 4's from 78 to 2; by 6's from 98 to 2.

**3.** Subtract by 7's from 101 to 3; by 8's from 106 to 2.

**Subtraction** is the process of finding the difference between two numbers, or of taking one number from another.

The **minuend** is the number from which we subtract.

The **subtrahend** is the number to be subtracted.

The **difference,** or **remainder,** is the result of subtraction.

The difference added to the subtrahend equals the minuend.

The sign –, called *minus*, indicates *subtraction.*

*Only like numbers can be subtracted.*

Give differences at sight:

| **4.** | **5.** | **6.** | **7.** | **8.** | **9.** |
|---|---|---|---|---|---|
| 3 – 2 | 12 – 7 | 11 – 9 | 10 – 4 | 11 – 2 | 14 – 3 |
| 4 – 1 | 9 – 4 | 7 – 4 | 19 – 3 | 16 – 8 | 13 – 9 |
| 5 – 4 | 9 – 8 | 9 – 5 | 17 – 9 | 15 – 6 | 16 – 7 |
| 7 – 3 | 11 – 4 | 17 – 8 | 11 – 3 | 14 – 5 | 15 – 7 |
| 8 – 4 | 9 – 7 | 12 – 6 | 12 – 9 | 12 – 8 | 13 – 8 |
| 7 – 2 | 5 – 1 | 11 – 5 | 16 – 9 | 14 – 7 | 18 – 9 |

**10.** Subtract each of the following numbers from 20; then, from 30, 40, etc.

4 6 11 17 13 14 7 8 12 10 15 14 16

**11.** Subtract each of the following numbers from 100.

40 70 60 25 45 75 44 64 84 37 57 68

**12.** Take 27 from 65, thus: $65 - 20 = 45$; $45 - 7 = 38$.

Subtract in like manner:

| **13.** | **14.** | **15.** | **16.** | **17.** | **18.** |
|---|---|---|---|---|---|
| 72 | 84 | 91 | 63 | 48 | 82 |
| <u>48</u> | <u>36</u> | <u>45</u> | <u>24</u> | <u>32</u> | <u>59</u> |

A clerk's sales book shows the following sales and amount given by customers. Give differences at sight:

| | Sales | Amount given | | Sales | Amount given |
|---|---|---|---|---|---|
| **19.** | $1.55 | $2.00 | **26.** | $ .49 | $ 2.00 |
| **20.** | 1.15 | 1.50 | **27.** | 1.23 | 5.00 |
| **21.** | 3.17 | 4.00 | **28.** | 1.06 | 2.00 |
| **22.** | 1.78 | 2.00 | **29.** | 2.14 | 3.00 |
| **23.** | 3.15 | 3.50 | **30.** | .99 | 2.00 |
| **24.** | 1.79 | 5.00 | **31.** | 1.29 | 10.00 |
| **25.** | 2.34 | 3.00 | **32.** | .87 | 5.00 |

## Written Work

**1.** From 632 take 374.

| | | |
|---|---|---|
| Minuend | $632 = 500 + 120 + 12$ | = 5 hundreds, 12 tens, 12 ones |
| Subtrahend | $374 = 300 + 70 + 4$ | = 3 hundreds, 7 tens, 4 ones |
| Difference | $258 = 200 + 50 + 8$ | = 2 hundreds, 5 tens, 8 ones |

Since 4 ones cannot be taken from 2 ones, take 1 ten = 10 ones, from 3 tens; 10 ones + 2 ones = 12 ones; 12 ones − 4 ones = **8** ones. Since 7 tens cannot be taken from the 2 tens remaining, take 1 hundred = 10 tens, from the 6 hundreds; 10 tens + 2 tens = 12 tens; 12 tens − 7 tens = **5** tens. 5 hundreds − 3 hundreds = **2** hundreds.

**Test.** — $374 + 258 = 632$. By adding the difference to the subtrahend, pupils can quickly discern whether the answer is correct.

Explain the steps in finding each remainder:

| **2.** | **3.** | **4.** | **5.** | **6.** |
|---|---|---|---|---|
| 800 | 9004 | 9080 | 7001 | 9040 |
| 594 | 7907 | 5987 | 4908 | 5879 |
| 206 | 1097 | 3093 | 2093 | 3161 |

Subtract:

| **7.** | **8.** | **9.** | **10.** | **11.** |
|---|---|---|---|---|
| 30984 | 54009 | 81704 | 41711 | 50000 |
| 24987 | 31047 | 54270 | 39111 | 42001 |

The following methods of subtraction are also convenient:

I. **By adding** (the method used in making change).

**1.** From 842 take 385.

| | |
|---|---|
| 842<br>385<br>457 | Think: What number added to 5 will make 12? (7.) Write down **7**; carry 1 to 8 tens in minuend. What number added to 9 (8 + 1) will make 14? (5.) Write down **5**; carry 1. What number added to 4 (3 + 1) will make 8? (4.) Write down **4**. |

II. **By subtracting from 10.**

**2.** From 653 take 378.

| | |
|---|---|
| 753<br>378<br>275 | Borrow 10, subtract 8, and add 3, thus: 10 − 8 in subtrahend = 2; 2 + 3 in minuend = **5**. 10 − 7 in subtrahend = 3; 3 + 4 (5 − 1) in minuend = **7**. 5 − 3 = **2**. The steps in the general method of subtraction are **borrow, add, subtract.** In this they are **borrow, subtract, add.** |

Write, subtract, and test four problems in 3 minutes:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **3.** | 85980<br>71409 | **9.** | \$670.01<br>340.97 | **15.** | 590680<br>289796 | **21.** | 6459871<br>2987598 |
| **4.** | 57004<br>20098 | **10.** | \$590.10<br>210.89 | **16.** | 998076<br>433011 | **22.** | 5798371<br>3099384 |
| **5.** | 39702<br>21308 | **11.** | \$953.01<br>391.54 | **17.** | 598801<br>303397 | **23.** | 8342901<br>5217809 |
| **6.** | 70001<br>39005 | **12.** | \$401.97<br>207.58 | **18.** | 743019<br>556601 | **24.** | 7654321<br>3456780 |
| **7.** | 98235<br>60104 | **13.** | \$701.49<br>511.10 | **19.** | 831001<br>397018 | **25.** | 6543903<br>4239001 |
| **8.** | 80021<br>51037 | **14.** | \$800.67<br>610.34 | **20.** | 458995<br>233450 | **26.** | 4932459<br>2013307 |

## PROBLEMS

**1.** The minuend is 6389, and the difference is 4360 Find the subtrahend.

**2.** From the sum of 3645 and 5796, subtract their difference.

**3.** In 1900 the population of New York State was 7,268,894; of Pennsylvania, 6,302,115. How much greater in population was New York than Pennsylvania?

**4.** In 1906 Iowa raised 373,275,000 bushels of corn; Missouri, 228,522,500 bushels. How much did the corn crop in Iowa exceed the crop in Missouri?

**5.** A and B together owe me \$7650; B owes me \$4675. After each pays me \$1600 on account, find the amount each one still owes me.

**6.** A retail merchant bought goods to the amount of \$1457. After selling stock from these goods to the amount of \$975, he found that the remainder of the goods unsold had cost him \$473. How much had he gained or lost?

**7.** Mr. Adams bought a farm for \$8670; he expended in improvements on barn and house, \$1790; on stock and farming utensils, \$2080. How much more did he pay for the farm than for improvements, live stock, and utensils?

**8.** The surface of the earth contains 196,907,000 square miles, of which 144,500,000 square miles are water. How much of the surface of the earth is land?

**9.** A father divided \$23,675 among his sons, giving to James \$6750 and to Henry \$5000 less than the part remaining after James was paid; to Frank he gave the remainder. How much did each receive?

## MULTIPLICATION

**1.** Count to 72 by 2's; by 3's; by 4's; by 6's; by 9's.

**2.** Count backwards from 48 by 2's; by 3's; by 4's; by 6's; by 8's.

**3.** Count forwards to 96 by 3's; by 4's; by 6's.

**4.** Count to 25 by 5's; to 60 by 4's; to 99 by 9's.

**5.** Build the multiplication tables by addition, thus:

$$\begin{array}{ccc} & & 2\\ & 2 & 2\\ 2 & 2 & 2\\ \underline{2} & \underline{2} & \underline{2} \end{array};\ \text{then write it in this form} \left\{\begin{array}{l} 2 \text{ times } 2 = 4\\ 3 \text{ times } 2 = 6\\ 4 \text{ times } 2 = 8 \end{array}\right.$$

**6.** Drill on these tables until pupils thoroughly know them:

| 1 | 2's | 3's | 4's | 5's | 6's | 7's | 8's | 9's | 10's | 11's | 12's |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

The first row of figures at the top stands for the *different tables.* By multiplying each of the numbers in *the first left-hand row* by each of the numbers in the *top row*, the tables can all be made. Thus, in the table of the twos, the products are directly below the number of the table, etc.

**7.** How many units are there in 4? $4 \times \$5$ means that we are to take \$5, *four* times to find the product; this may be found in two ways: $\$5 + \$5 + \$5 + \$5 = \$20$, or by multiplication, which is a short form of addition; thus, $4 \times \$5 = \$20$.

**Multiplication** is the process of taking one number as many times as there are units in another number.

The **multiplicand** is the number multiplied.

The **multiplier** is the number by which we multiply.

The **product** is the result of multiplication.

The sign $\times$ indicates multiplication; it is read, "times," when the multiplier *precedes* the sign, and, "multiplied by," when the multiplier *follows* the sign.

Read each statement and then give products.

| | | |
|---|---|---|
| **8.** $4 \times \$12$ | **10.** $12 \times 7$ yards | **12.** $11 \times 4$ pounds |
| **9.** $9 \times 6$ horses | **11.** $5 \times 8$ ft. | **13.** $7 \times 8$ bushels |

**14.** In the above statements, how many times are \$12 taken? 8 bushels? 7 yards?

**15.** Name the *multiplicands* in the above statements; the *multipliers*.

A **concrete number** is a number used with reference to a particular object; as, 5 days, 10 pounds, 8 inches.

An **abstract number** is a number used without reference to a particular object; as, 5, 8, 20.

Name the abstract and the concrete numbers in the following statements and then give products:

| | | |
|---|---|---|
| **16.** $12 \times 7$ days | **18.** $11 \times \$7$ | **20.** $6 \times 11$ |
| **17.** $15 \times 10$ | **19.** $9 \times 12$ ft. | **21.** $12 \times 5¢$ |

*The multiplier is always regarded as an abstract number. The multiplicand may be either abstract or concrete.*

**22.** In problems 16–21 are the products like the multiplicand or the multiplier?

*The product and the multiplicand are like numbers.*

### Oral and Written Analysis

**1.** How many eggs are there in 6 dozen?

| | |
|---|---|
| Since there are 12 eggs in one dozen, | 1 doz. = 12 eggs; |
| in 6 doz. there are $6 \times 12$ eggs = 72 eggs. | 6 doz. = $6 \times 12$ eggs = 72 eggs. |

**2.** How many trees are there in an orchard if there are 11 rows and 10 trees in each row?

**3.** James raised 7 bushels of potatoes on an average from each of 10 rows. How many bushels did he raise?

What is the cost of:

**4.** 10 quarts of cherries at 8¢ per quart?

**5.** 9 quarts of milk at 7¢ per quart?

**6.** 8 bushels of apples at $2 per bushel?

**7.** A twelve-pound cheese at 12¢ per pound?

**8.** 3 pecks of apples at 25¢ per peck?

**9.** How far does a boy ride on his automobile in 4 hours at the rate of 9 miles per hour?

**10.** How many miles are there in 4 streets, if the streets average 12 miles?

**11.** There are 32 quarts in a bushel. Find the number of quarts in 13 bushels.

**12.** How far does an automobile run in 4 hours, if it averages 14 miles per hour?

**13.** Find the cost of posting 18 letters at 2¢ each.

**14.** Find the cost of a 7-pound turkey at 13¢ per pound.

**15.** A lady purchased 2 dozen oranges at 40¢ per dozen. How much did they cost?

**16.** It takes John 15 minutes to walk to school. How many minutes will be required to walk to school 60 times?

**17.** Frank used 12 tablets, at 10¢ each, in a school term. How much did they cost?

## Written Work

**1.** Multiply 146 by 3.

| | |
|---|---|
| Multiplicand | 146 |
| Multiplier | 3 |
| Product | 438 |

**Test.** — $146 + 146 + 146 = 438$

$3 \times 6$ ones $= 18$ ones, or 1 ten and 8 ones. Write **8** in ones' place and carry the 1 ten. $3 \times 4$ tens $= 12$ tens; 12 tens + the 1 ten, carried from ones' place $= 13$ tens, or 1 hundred and 3 tens. Write **3** in tens' place and carry the 1 hundred. $3 \times 1$ hundred $= 3$ hundreds; 3 hundreds + the 1 hundred $=$ **4** hundreds. The product is 438.

Find products:

| | | | | | |
|---|---|---|---|---|---|
| **2.** 139 | **6.** 674 | **10.** 307 | **14.** 137 | **18.** 427 | **22.** 507 |
| 3 | 3 | 5 | 6 | 7 | 6 |
| **3.** 135 | **7.** 278 | **11.** 342 | **15.** 673 | **19.** 784 | **23.** 196 |
| 2 | 4 | 6 | 5 | 7 | 8 |
| **4.** 603 | **8.** 147 | **12.** 281 | **16.** 901 | **20.** 249 | **24.** 379 |
| 4 | 5 | 4 | 7 | 8 | 7 |
| **5.** 205 | **9.** 219 | **13.** 309 | **17.** 419 | **21.** 907 | **25.** 583 |
| 2 | 4 | 6 | 6 | 2 | 8 |

**26.** Multiply \$1.25 by 3.

| |
|---|
| \$1.25 |
| 3 |
| \$3.75 |

Place the decimal point in the product *directly under* the decimal point in the multiplicand.

| | | | | |
|---|---|---|---|---|
| **27.** \$2.53 | **30.** \$8.09 | **33.** \$3.27 | **36.** \$2.41 | **39.** \$3.19 |
| 9 | 9 | 8 | 10 | 9 |
| **28.** \$6.08 | **31.** \$2.25 | **34.** \$1.04 | **37.** \$3.74 | **40.** \$8.92 |
| 9 | 10 | 11 | 12 | 10 |
| **29.** \$9.09 | **32.** \$1.29 | **35.** \$3.05 | **38.** \$5.05 | **41.** \$9.08 |
| 9 | 11 | 10 | 12 | 12 |

## Multiplication by Larger Numbers

**Multiplying integers by 20, 100, 1000, etc.**

$10 \times 12 = 120$; $100 \times 2 = 200$; $1000 \times 2 = 2000$.

*Any integer may be multiplied by 10, 100, 1000, etc., by annexing to the integer as many naughts as there are naughts in the multiplier.*

Multiply each number by 10, by 100, by 1000, writing only the products: 16, 409, 290, 301, 205, 250, 175, 791.

### Written Work

**1.** Multiply 72 by 36.

| | | | |
|---|---|---|---|
| Multiplicand | 72 | | 72 |
| Multiplier | 36 | | 36 |
| 1st partial product | 432 | $= 6 \times 72$ | 432 |
| 2d partial product | 2160 | $= 30 \times 72$ | 216 |
| Entire product | 2592 | $= 36 \times 72$ | 2592 |

In practice, the 0 in the second partial product is omitted, and 2160 is written as 216 *tens*.

Find products:

| | | |
|---|---|---|
| **2.** $150 \times 40$ | **4.** $805 \times 16$ | **6.** $304 \times 71$ |
| **3.** $107 \times 35$ | **5.** $500 \times 70$ | **7.** $691 \times 74$ |

Multiply and test:

| | | |
|---|---|---|
| **8.** 6425 | | *a.* 245 |
| **9.** 1024 | | *b.* 344 |
| **10.** 8720 | | *c.* 564 |
| **11.** 9652 | | *d.* 746 |
| **12.** 8665 | by | *e.* 804 |
| **13.** 7894 | | *f.* 961 |
| **14.** 8465 | | *g.* 869 |
| **15.** 7695 | | *h.* 796 |
| **16.** 8425 | | *i.* 968 |
| **17.** 9476 | | *j.* 898 |

Form 100 problems by multiplying each multiplicand by each of the multipliers, thus:

8 *a.* $245 \times 6425 = ?$
8 *b.* $344 \times 6425 = ?$
15 *i.* $968 \times 7695 = ?$

Write, solve, and test each problem in $1\frac{1}{2}$ minutes.

**18** How much will 20,000 bricks cost at $7.75 per thousand?

Suggestion: 20,000 = 20 thousand = 20 M.

**19.** A ranchman sold 125 head of cattle at an average of $42.75 per head, and 625 sheep at $3.85 per head. Find the total amount of his sales.

**20.** The cost of drilling an oil well was 35¢ per foot for drilling and 65¢ for the tubing. If the well was drilled 1177 feet and tubed 700 feet, find the total cost.

**21.** The freight rate on corn in car-load lots from Omaha, Neb., to New York City is 20¢ per hundred pounds. Find the freight on a car of 42,000 lb.

**22.** A freight train of 32 cars is laden with corn. The cars contain an average of 700 bushels of 56 lb. each. Find the weight of the corn in pounds.

**23.** A commission merchant sold 1275 barrels of apples at the rate of $3.25 per barrel, charging $.325 per barrel for selling. How much was realized from the sale of the apples after the charges for selling were deducted?

**24.** A man owned a farm of 142 acres, worth $72.50 per acre; 5 city lots worth $1875 per lot; and a business house worth $6350. Find the value of his entire property.

**25.** Frank lives 87 rods from the schoolhouse. How many rods does he walk in going to school 140 days, if he returns home each day for his dinner?

**26.** Two trains leave a station at the same time. One travels west 38 miles per hour; the other travels east 45 miles per hour. How far apart are they in 9 hours?

**27.** A dealer bought a car load of coal, 42,000 lb., at $1.90 per ton of 2000 lb. If the freight was 70¢ a ton, and he retailed the coal at $3.25 a ton, find his profit.

## DIVISION

**1.** How many times is the number 6 contained in 24?

**Division** is the process of finding how many times one number is contained in another, or of separating a number into equal parts.

The **dividend** is the number to be divided.

The **divisor** is the number by which we divide.

The **quotient** is the result of division.

The **remainder** is the part of the dividend remaining when the quotient is not exact.

The sign ÷ indicates division, and is read, "divided by."

Give quotients:

| | | | | |
|---|---|---|---|---|
| $24 \div 6$ | $84 \div 7$ | $64 \div 8$ | $49 \div 7$ | $99 \div 11$ |
| $72 \div 9$ | $108 \div 9$ | $42 \div 6$ | $63 \div 9$ | $72 \div 12$ |
| $81 \div 9$ | $72 \div 8$ | $90 \div 9$ | $96 \div 8$ | $77 \div 11$ |

Division is indicated in *three* ways: $14 \div 2$; $2\overline{)14}$; and $\frac{14}{2}$.

How many times are:

**2.** 2 inches contained in (may be taken from) 12 inches?

**3.** 4 yards contained in (may be taken from) 12 yards?

*If both the dividend and divisor are concrete, they must be like numbers.*

Compare $18 \div 2$ with $\frac{1}{2}$ of 18; $15 \div 3$ with $\frac{1}{3}$ of 15.

How many:

**4.** Cents are $\frac{1}{8}$ of 48¢? 48¢ ÷ 8 = — cents.

**5.** Cents does one orange cost if 4 oranges cost 20¢?

*In separating a number into equal parts, the divisor is always an abstract number and the quotient is like the dividend.*

This kind of division is called **partition**.

In the following, point out the problems in partition:

**6.** \$120 ÷ 10 **7.** $\frac{1}{4}$ of 40¢ **8.** 24 ft. ÷ 2 ft.

## Remainder in Division

$34 \div 5 = 6$, and 4 remaining. $\$39 \div 5 = \$7$, and \$4 remaining.

Give quotients and remainders:

$\$26 \div 6$; $79 \div 8$; $\$48 \div \$9$; $37 \div 4$; $84¢ \div 8¢$; $49 \div 7$.

## Written Work

**1.** Divide 236 by 3.

Divisor 3)236 Dividend
78 Quotient
Remainder 2

**Test.** — $3 \times 78 = 234$;
$234 + 2 = 236$

23 tens ÷ 3 = 7 tens, and 2 tens (20 ones) remaining. Write the **7** in tens' place.

20 ones + 6 ones = 26 ones; 26 ones ÷ 3 = **8** ones, and **2** ones remaining.

Quotient 78; remainder 2.

We think: "3 in 23, **7** times, and 2 remaining; 3 in 26, **8** times, and **2** remaining."

Find quotients:

**2.** $344 \div 3$ **3.** $763$ ft. $\div 6$ ft. **4.** $466¢ \div 9$

Divide \$6.48 by 3.

3)\$6.48
\$2.16

Place the decimal point in the quotient *directly under* the decimal point in the dividend.

Divide and test:

| | | | | |
|---|---|---|---|---|
| **5.** | \$203.75 | by | *a.* | 2 |
| **6.** | \$678.34 | | *b.* | 3 |
| **7.** | \$209.07 | | *c.* | 4 |
| **8.** | \$390.08 | | *d.* | 5 |
| **9.** | \$720.93 | | *e.* | 6 |
| **10.** | \$379.38 | | *f.* | 7 |
| **11.** | \$297.34 | | *g.* | 8 |
| **12.** | \$427.84 | | *h.* | 9 |
| **13.** | \$918.07 | | *i.* | 10 |
| **14.** | \$847.12 | | *j.* | 11 |

Form 100 problems by dividing each dividend by each of the divisors, thus:

5 *a.* $\$203.75 \div 2 = ?$
5 *b.* $\$203.75 \div 3 = ?$
9 *e.* $\$720.93 \div 6 = ?$

Write, solve, and test two problems in 1 minute.

### Dividing by Larger Numbers

**1.** Divide 50, 90, 150, 600, 1000 by 10.

**2.** Name the quotients when 130, 170, 1200, 2000, 160, is each divided by 10.

**3.** Divide 500, 600, 1500, and 2500 by 100.

*Removing one naught from the right of a number divides the number by* 10; *removing two naughts, divides it by* 100; *removing three naughts divides it by* 1000; *etc.*

**4.** Divide 225 by 20, thus: $2|0\overline{)22|5}$ — 2 tens is contained in 22 tens 11 times, with 5 remaining. $5 \div 20 = \frac{5}{20}$.

$$\begin{array}{r} 2|0\overline{)22|5} \\ 11\ \frac{5}{20} \end{array}$$

**5.** Divide 2375 by 20; by 200.

Divide each number by 20; by 50; by 80; by 500.

| | | | |
|---|---|---|---|
| **6.** 37,845. | **8.** 90,200 | **10.** 409,805. | **12.** 390,075. |
| **7.** 50,240. | **9.** 74,079 | **11.** 790,086. | **13.** 985,000. |

## LONG DIVISION

**1.** Divide 4310 by 21.

$$\begin{array}{r} 205 \\ 21\overline{)4310} \\ \underline{42\phantom{0}} \\ 110 \\ \underline{105} \\ 5 \text{ remainder} \end{array}$$

**Test:** $21 \times 205 = 4305$
$4305 + 5 = 4310$

**Steps**

1. Divide 43 by 21. Write the quotient figure **2** over the figure 3 of the dividend.
2. Multiply 21 by 2.
3. Subtract 42 from 43.
4. Bring down the next figure. Is 21 contained an integral number of times in 11? Write **0** in the quotient.
5. Bring down the next figure and proceed as before. Write **5** in the quotient.

Divide and test:

| | | |
|---|---|---|
| **2.** $252 \div 21$ | **7.** $2214 \div 21$ | **12.** $1326 \div 51$ |
| **3.** $525 \div 21$ | **8.** $4601 \div 22$ | **13.** $1922 \div 62$ |
| **4.** $724 \div 22$ | **9.** $1271 \div 31$ | **14.** $2193 \div 51$ |
| **5.** $642 \div 31$ | **10.** $1344 \div 42$ | **15.** $7010 \div 91$ |
| **6.** $345 \div 31$ | **11.** $1024 \div 32$ | **16.** $6874 \div 81$ |

Divide and test:

| | | |
|---|---|---|
| **17.** 1364 by 22 | **25.** 6207 by 76 | **33.** 8538 by 94 |
| **18.** 1395 by 31 | **26.** 6572 by 68 | **34.** 7646 by 87 |
| **19.** 1728 by 42 | **27.** 7010 by 91 | **35.** 8544 by 79 |
| **20.** 2193 by 51 | **28.** 7284 by 92 | **36.** 9584 by 66 |
| **21.** 2583 by 63 | **29.** 6874 by 81 | **37.** 7001 by 84 |
| **22.** 3034 by 74 | **30.** 6986 by 83 | **38.** 8200 by 77 |
| **23.** 4345 by 65 | **31.** 7044 by 86 | **39.** 7909 by 96 |
| **24.** 5072 by 59 | **32.** 8406 by 92 | **40.** 8549 by 78 |

**41.**

```
      1009
395)398555
    395
      3555
      3555
```

Since 35 does not contain 395, the second figure in the quotient is 0.

**42.**

```
      $11.17
715)$7986.55
     715
      836
      715
      1215
       715
       5005
       5005
```

**43.**

```
      544
805)437920
    4025
     3542
     3220
      3220
      3220
```

Divide and test:

| | | |
|---|---|---|
| **44.** 6464341 | by | *a.* 268 |
| **45.** 7846760 | | *b.* 354 |
| **46.** 5864548 | | *c.* 676 |
| **47.** 8645341 | | *d.* 758 |
| **48.** 9624872 | | *e.* 865 |
| **49.** 7784100 | | *f.* 984 |
| **50.** 6810404 | | *g.* 789 |
| **51.** 7904025 | | *h.* 897 |
| **52.** 4867045 | | *i.* 509 |
| **53.** 3234567 | | *j.* 890 |

Form 100 problems by dividing each dividend by each of the divisors, thus:

44 *a.* $6464341 \div 268 = ?$
44 *b.* $6464341 \div 354 = ?$
49 *c.* $7784100 \div 676 = ?$

Write, solve, and test each problem in 2 minutes.

## Problems of Two or More Operations

**1.** If 48 barrels of flour cost \$324, how much will 275 barrels cost?

Cost of 48 bbl. = \$324.00
Cost of 1 bbl. = \$324.00 ÷ 48 = \$6.75
Cost of 275 bbl. = 275 × \$6.75 = \$1856.25

**Study of Problem**

1. What is given in this problem?

*a.* Number of barrels in each purchase, *b.* Cost of 48 bbl.

2. What is required? *a.* Cost of 1 bbl. *b.* Cost of 275 bbl.

3. How do you find what is required? *a.* Divide cost of first purchase by the number of barrels. *b.* Multiply the cost of 1 bbl. by the number of barrels purchased.

NOTE. — The purpose of these studies is threefold:

1. To train the pupil to see and understand the conditions of a problem.
2. To give that logical, analytic grasp of conditions that forms the basis of all mathematical power.
3. To direct the teacher in his efforts to attain these ends.

**2.** If 2675 bushels of wheat cost \$2728.50, how much are 196 bushels worth?

**3.** A water tank holds 8640 gallons. If it receives 728 gallons per hour by one pipe and discharges 512 gallons by another, in what time will it be filled?

**4.** Two steamers sail towards each other from opposite sides of the Pacific Ocean. If the distance across is 9872 miles, and one sails at the rate of 285 miles a day, and the other 332 miles per day, in how many days will they meet?

**5.** The daily pay of a railway conductor is \$3.45. If he works 310 days in a year, and spends on an average \$65 per month, how much has he left at the end of the year?

**6.** The receipts of a street railway for 365 days were \$119,685.23. Find the average daily profits if the total expenses were \$96,478.02.

**7.** A and B divide an estate of \$9875 between them. If A receives \$275 more than B, how much does each receive?

**8.** A man sold 128 acres of land at \$70 an acre, and 96 at \$90 an acre. He invested the money in town lots at \$550 each. How many did he buy?

**Study of Problem**

1. What is given in the problem?
2. What is required?
3. What is the first step in the solution? the second? the third? the fourth?

\$8960 value of 1st farm
\$8640 value of 2d farm
\$17600 value of both

$\$17600 \div \$550 = 32$, no. of lots bought.

**9.** If it costs 40 cents to ship a 10-gallon can of milk from Hickory to Pittsburg, how much does the railroad realize in 5 days, from a shipment of 135 cans per day?

**10.** A shipper pays 20 cents per barrel, per month, cold storage charges on apples, and 15 cents per firkin on butter. Find the charges for three months on 45 firkins of butter and 328 barrels of apples.

**11.** A locomotive in making a certain trip uses 18 tons of coal. If a trip is made in 2 days, how much coal will the engine consume in 190 days?

**12.** A dealer buys three boxes of oranges for \$3.50, \$2.75, and \$2.50, respectively. If he sells 10 dozen at 50 cents per dozen, 9 dozen at 40 cents per dozen, 12 dozen at 35 cents per dozen, and the remaining 5 dozen at 25 cents per dozen, find his gain.

**13.** An opera sale of tickets is as follows: 450 @ \$1.50; 380 @ \$1.00; 520 @ \$.75; 310 @ \$.50; and 240 @ \$.25. Find the total sale of the tickets, and the average cost of each ticket.

**14.** A steamboat consumes 23 tons of coal per day. Find the cost of the coal, at \$5.85 per ton, for a trip of 39 days.

## COMPARISON

**Comparison**, as here used, indicates the relation of two similar numbers, expressed by the *quotient* of the *first number divided by the second.*

**1.** Compare 10 and 5; 12 and 4; 16 and 8; 20 and 5; 24 and 6.

**2.** 20 is how many times 4? 30 is how many times 6? How does 40 compare with 4?

**3.** What is the quotient of 48 divided by 8? by 6? by 4?

**4.** Compare 200 and 50; 400 and 100; 500 and 250.

**5.** 125 is what part of 250? of 500? of 375? of 625?

**6.** Compare 48 feet and 2 yards: 75 feet and 5 yards.

NOTE. — Change yards to feet, or feet to yards; then compare.

**7.** 6 feet is what part of 3 yards? of 10 yards?

### Written Work

**1.** When 4 pounds of butter cost 80 cents, how much will 12 pounds cost?

NOTE. — 12 pounds = 3 × 4 pounds; hence, 12 pounds will cost 3 × 80 cents.

**2.** Find the cost of 10 barrels of apples, when 2 barrels cost $4.50.

**3.** At 3 pounds of coffee for $1, how much will 15 pounds cost?

**4.** How much will 30 yards of silk cost when 3 yards cost $3.75?

**5.** When 2 doz. oranges are selling for 60 cents, how much will 8 dozen cost?

**6.** Find the cost of 20 barrels of cement, when 5 barrels cost $6.25.

**7.** How much will 30 dozen eggs cost, when 3 dozen sell for $1?

## COMBINING PROCESSES

A **parenthesis** ( ) or a **vinculum** ——— groups together several numbers and shows that the operations within the groups are to be performed first; thus, $6-(3+2)=6-5=1$; $(5+3)\times 2=8\times 2=16$; $5+(3\times 2)=5+6=11$; $\overline{32\div 4}+3=8+3=11$; $\overline{4\times 2}-3=8-3=5$.

When no parenthesis or vinculum is used, the signs $\times$ and $\div$ indicate operations that are to be performed before those indicated by either $+$ or $-$; thus, $4+8\times 3=4+(8\times 3)$, or 28; $5+12\div 6=5+(12\div 6)$, or 7.

In an expression like $12\div 6\times 2$, mathematicians are not agreed as to which sign shall be used first. To avoid ambiguity, the parenthesis should be used in such expressions. Thus, $(12\div 6)\times 2=4$; but $12\div(6\times 2)=1$.

Find the value of:

1. $4\times 12-16\div 4$.
2. $7+8\times 7-26$.
3. $(14+8-6)\times 9$.
4. $(87-65+96)\times 24$.
5. $(240+98)\times(688-425)$.
6. $(56-18)\times\overline{11+4}-6\times 4$.
7. $(84-7\times 6+9\times 4-6)\div 9$.
8. $(56\div 7)\times 12+97-7\times 9$.
9. $6+10\times 5+8\div 2-4-2+8$.
10. $7\times\overline{5+4}+\overline{8\times 6}\div 2-3\times 4$.
11. $(6+2\times 3)\div 4+(3\times 6)\div 2+2\times(3+5-2)$.
12. $36-6\times 4+2\times 6+(40+5)\div 9+3\times 6$.
13. $\overline{10+20-5}\times 3+\overline{6\times 2}\div 3+5\times 6$.
14. $3\times(4+5-2)+4+5\times(\overline{4\times 5}\div 2)+5$.
15. $3\times(6+8)+7\times(8\div 2)-3\times(6\div 3)+15-7$.
16. $175-8\times(19-10)-\overline{25\div 5}+\overline{6\times 7}-9\div 3$.

## FACTORS AND DIVISORS

**1.** What two numbers will give 6 as a product? 8 as a product? 10 as a product?

**2.** What are 2 and 3 in relation to 6? 4 and 2 in relation to 8? 5 and 2 in relation to 10?

An **integer** or an **integral number** is a whole number.

The **factors** of a number are the integers whose product is the number; thus, 5 and 2 are factors of 10.

**3.** Name two factors that produce 24, 32, 40, 56, 49, 72, 96.

A factor of a number is an **exact divisor** of the number; that is, it is contained in the number an *integral number* of times.

**4.** Name the exact divisors of 54, 81, 48, 36, 66, 64, 63.

**5.** Observe the two equal factors that produce 4; 9; 16.

$2 \times 2 = 4$
$3 \times 3 = 9$
$4 \times 4 = 16$

**6.** Observe the three equal factors that produce 8; 27; 64.

$2 \times 2 \times 2 = 8$
$3 \times 3 \times 3 = 27$
$4 \times 4 \times 4 = 64$

Instead of repeating a factor, a small figure called an **exponent** may be written to the right and a little above the number to show how often it is used as a factor; thus, $3^3 = 3 \times 3 \times 3 = 27$; $2^4 = 2 \times 2 \times 2 \times 2 = 16$.

**7.** What number will divide 9 and 10? 21 and 25?

Numbers are **prime to each other** when they have no common factor; thus, 9 and 10 are prime to each other.

**Even numbers** are numbers that contain the factor 2.

**Odd numbers** are numbers that do not contain the factor 2.

**8.** What are the factors of 7? of 11? Observe that 7 and 11 have no exact divisors except *themselves* and *one*.

A **prime number** is one that has no exact divisor except itself and one; thus, 5, 2, and 3 are prime numbers.

**9.** Name all the prime numbers to 31.

**10.** What are the factors of 15? Observe that 15 can be divided by 3 and 5. It is *composed* of *other* factors than itself and one.

A **composite number** is one that has other exact divisors than itself and one; thus, 6 and 10 are composite numbers.

**11.** Name all the composite numbers to 50.

## TESTS OF DIVISIBILITY

**1.** Divide 12, 24, 26, 38, and 50 each by 2. What is the ones' figure in each of the dividends? Divide other numbers ending in 2, 4, 6, 8, or 0 by 2.

*A number is divisible by* 2, *if the ones' figure is* 2, 4, 6, 8, *or* 0.

**2.** Divide 15, 25, 40, 125, 150 each by 5. What is the ones' figure in each dividend? Divide other numbers ending in 5 or 0 by 5.

*A number is divisible by* 5, *if its ones' figure is* 5 *or* 0.

**3.** Divide 36, 69, 48, 72, 162, 369 each by 3. Notice that the sum of the *digits* (that is, of the *figures*) in each number is divisible by 3. Divide by 3, other numbers the sum of whose digits is divisible by 3.

*A number is divisible by* 3, *if the sum of its digits is divisible by* 3.

4. Divide 18, 27, 279, 819, 639 each by 9. Notice that the sum of the digits in each dividend is divisible by 9. Divide by 9, other numbers the sum of whose digits is divisible by 9.

*A number is divisible by 9, if the sum of its digits is divisible by 9.*

5. Select the numbers that are divisible by 2; by 3; by 5; by 9.

| | | | | | |
|---|---|---|---|---|---|
| 86 | 96 | 123 | 918 | 515 | 3672 |
| 94 | 72 | 321 | 819 | 450 | 1909 |

## FACTORING

1. Give the two factors that produce 15.

2. If one of them is given, how may the other be found?

*To separate a number into two factors, take any exact divisor for one factor and the quotient of the number by this factor for the other.*

**Factoring** is the process of separating a number into its factors.

A **prime factor** is a prime number used as a factor; thus, 3 and 5 are the prime factors of 15.

### Written Work

1. Find the prime factors of 126.

$$\begin{array}{r|r} 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Divide by the least prime factor; divide the quotient by the next smallest prime factor, etc., until the last quotient is a prime number. The divisors and the last quotient are the prime factors; thus, 2, 3, 3, and 7 are the prime factors of 126.

**Test**: $2 \times 3 \times 3 \times 7 = 126$

Or,

$2 \times 3^2 \times 7 = 126$

Find the prime factors of:

| | | | |
|---|---|---|---|
| **2.** 125 | **6.** 945 | **10.** 2431 | **14.** 25600 |
| **3.** 210 | **7.** 2934 | **11.** 7200 | **15.** 64640 |
| **4.** 225 | **8.** 4620 | **12.** 7700 | **16.** 97125 |
| **5.** 400 | **9.** 3822 | **13.** 6525 | **17.** 78000 |

## GREATEST COMMON DIVISOR

**1.** Name a number that will exactly divide both 16 and 24; 15 and 25; 14 and 27.

A **common divisor** of two or more numbers is a number that exactly divides each of them; thus, 4 is a common divisor of 16 and 24.

**2.** Is 4 the greatest number that will exactly divide 16 and 24? What is the greatest number that will exactly divide 16 and 24?

The **greatest common divisor** (g. c. d.) of two or more numbers is the greatest number that exactly divides each of them; thus, 9 is the g. c. d. of 27 and 36.

**3.** Name the g. c. d. of 24 and 36; of 32 and 40.

### Written Work

**1.** Find the greatest common divisor of 56, 98, 154.

$$\begin{array}{r|rrr} 2 & 56 & 98 & 154 \\ \hline 7 & 28 & 49 & 77 \\ \hline & 4 & 7 & 11 \end{array}$$

g. c. d $= 2 \times 7$, or 14.

*As the g.c.d. of two or more numbers is the product of all their common prime factors,* divide the numbers by their common prime factors. In the same way divide the quotients until they are prime to each other. The divisors *2 and 7 are all the common prime factors of the numbers.* Hence, the g.c.d. of 56, 98, and 154 is $2 \times 7$, or 14.

Find the g.c.d. of:

2. 42, 63, 189
3. 54, 216, 360
4. 48, 60, 96
5. 84, 252, 512
6. 21, 48, 78
7. 84, 56, 210
8. 22, 110, 132
9. 42, 84, 175
10. 17, 68, 85
11. 432, 720, 864

## LEAST COMMON MULTIPLE

1. Name a number that will exactly contain 6 and 9; 8 and 12; 7 and 9.

A **common multiple** of two or more numbers is a number that is exactly divisible by each of them; thus, 36 is a common multiple of 6 and 9.

2. Name the *least* number that is exactly divisible by 6 and 9; by 8 and 12.

The **least common multiple** (l.c.m.) of two or more numbers is the least number that is exactly divisible by each of them; thus, 18 is the l.c.m. of 6 and 9.

3. Name the l.c.m. of 6 and 8; of 9 and 12; of 8 and 12.

### Written Work

1. Find the l.c.m. of 18, 32, and 40.

$18 = 2 \times 3 \times 3$
$32 = 2 \times 2 \times 2 \times 2 \times 2$
$40 = 2 \times 2 \times 2 \times 5$
l.c.m. $= 2^5 \times 3^2 \times 5$, or 1440.

*The l. c. m. of two or more numbers is the product of all their prime factors, each factor being used as often as it occurs in any number.*

2 occurs 5 times as a factor in 32. It must, therefore, be used 5 times in the l. c. m. 3 occurs twice as a factor in 18; it must, therefore, be used twice in the l. c. m. 5 occurs once as a factor in 40; it must, therefore, be used once in the l. c. m. Hence, the l. c. m. of 18, 32, and 40 is $2^5 \times 3^2 \times 5 = 1440$.

**2.** Find the l. c. m. of 12, 36, 54, and 63.

$$\begin{array}{r|rrrr} 2 & \cancel{12} & 36 & 54 & 63 \\ \hline 3 & & 18 & 27 & 63 \\ \hline 3 & & 6 & 9 & 21 \\ \hline & & 2 & 3 & 7 \end{array}$$

l. c. m. $= 2^2 \times 3^3 \times 7 = 756$.

Since 12 is a divisor of 36 the l. c. m. of 36, 54, and 63 is also a multiple of 12. 12 may therefore be rejected from the work.

Divide any two of the numbers by a common prime factor. Then divide the quotients in like manner until the quotients are prime to each other. The product of the divisors and the last quotients is the l. c. m.

Find the l. c. m. of:

**3.** 24, 48, 72
**4.** 36, 70, 105
**5.** 32, 40, 48
**6.** 25, 35, 56
**7.** 30, 60, 105
**8.** 32, 48, 96
**9.** 45, 70, 90
**10.** 48, 64, 72
**11.** 144, 180, 240
**12.** 85, 51, 255
**13.** 120, 225, 540
**14.** 98, 42, 126
**15.** 180, 216, 120
**16.** 100, 110, 440

## CANCELLATION

$144 \div 36 = 4$. We may separate the dividend 144 into the factors 9 and 16, and the divisor 36 into the factors 9 and 4. We may, therefore, write $144 \div 36 = 4$ as follows:

$(\cancel{9} \times 16) \div (\cancel{9} \times 4) = 4.$

By striking out the common factor 9 in both dividend and divisor, the problem is: $(16 \div 4) = 4$.

*Striking out equal factors from both dividend and divisor does not change the quotient.*

When the product of a number of factors is to be divided by the product of another set of factors, the usual way is to write the dividend above a line and the divisor below, and strike out equal factors:

Thus, $\frac{56}{24}=\frac{\cancel{8}\times 7}{\cancel{8}\times 3}=\frac{7}{3}=2\frac{1}{3}.$

**Cancellation** is the process of shortening operations in division by striking out equal factors from both dividend and divisor.

### Written Work

**1.** Divide $3 \times 6 \times 8 \times 20$ by $11 \times 4 \times 20$.

$$\frac{3 \times 6 \times \overset{2}{\cancel{8}} \times \cancel{20}}{11 \times \cancel{4} \times \cancel{20}} = \frac{36}{11} = 3\tfrac{3}{11}.$$

Write the dividend above and the divisor below a line. First cancel the 20 from dividend and divisor. Then cancel the factor 4 from 8 in the dividend and from 4 in the divisor, leaving 2 in the dividend and 1 in the divisor. As there are no other factors common to dividend and divisor, you have $3 \times 6 \times 2$, or 36, divided by 11, or $\frac{36}{11}$, which equals $3\frac{3}{11}$.

NOTE. — When equal factors in the terms are canceled, the factor 1 always remains, but as it does not affect the product, it need not be written.

Divide:

**2.** $27 \times 56 \times 38 \times 50$ by $19 \times 35 \times 40$

**3.** $5 \times 51 \times 36 \times 63$ by $17 \times 9 \times 54 \times 10$

**4.** $25 \times 72 \times 64 \times 28$ by $40 \times 96 \times 21 \times 4$

**5.** $69 \times 56 \times 45 \times 27$ by $23 \times 45 \times 63 \times 9$

**6.** $72 \times 48 \times 84 \times 28$ by $24 \times 48 \times 42 \times 14$

**7.** $148 \times 64 \times 57 \times 12$ by $114 \times 32 \times 48$

**8.** By selling butter at 24¢ per pound a lady receives enough money to buy 48 pounds of coffee at 20¢ per pound. How many pounds of butter does she sell?

**9.** A man worked 16 days of 10 hours each at 20¢ per hour, and spent the money he received for corn at 40¢ per bushel. How many bushels of corn did he get?

# FRACTIONS

## FRACTIONAL UNITS

When we say 8, 6 ft., \$2, 6 rd., 7 mi., 5 in., what are the units of measure?

Observe that in each case the number and its unit of measure are of the same denomination; thus, 1 ft. is the unit of measure in 6 ft.

A **unit**, therefore, is any single quantity with which another quantity of the same kind is measured or compared; as, 1 is the unit of 10; 1 ft. is the unit of 8 ft.; 1 yd. of 2 yd.; 1 mi. of 12 mi.; 1 acre of 5 acres, etc.

A **fractional unit** is one of the equal parts into which an integral unit has been divided; as, $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{15}$, etc.

A **fraction** is one or more fractional units; as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{9}$, $\frac{5}{6}$, etc.

The **terms** of a fraction are the numerator and the denominator.

The **denominator** indicates the *size* of the fractional unit; it is written below the line, and shows into how many parts the integral unit has been divided. Thus, in the fraction $\frac{4}{5}$, 5 is the denominator, and shows that some unit has been divided into 5 parts.

The **numerator** indicates the *number* of fractional units; it is written above the line, and shows how many parts are taken. Thus, in the fraction $\frac{4}{5}$, 4 is the numerator, and shows that 4 parts have been taken.

**1.** What is the fractional unit in $\frac{3}{5}$? $\frac{1}{5}$? $\frac{5}{6}$? $\frac{3}{7}$?

**2.** Read the following fractional units in order of their size, beginning with the largest: $\frac{1}{16}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{12}$, $\frac{1}{9}$, $\frac{1}{7}$, and $\frac{1}{21}$.

**3.** The use of the numerator and the denominator in $\frac{9}{12}$ yd. may be explained thus, $\frac{9}{12}$ yd. $= 9 \times \frac{1}{12}$ yd.

As the *integral unit* is the basis by which we measure whole numbers, so the *fractional unit* is the basis by which we measure fractions of the same kind.

**4.** Name the unit of 4 ft.; 5 mi.; $\frac{5}{8}$; $\frac{3}{5}$.

**5.** Which is the larger, $\frac{7}{8}$ or 1? $\frac{3}{4}$ or 1? $\frac{5}{6}$ or 1? Explain how much larger in each case.

**6.** What is the difference in value between the fraction $\frac{8}{8}$ and an integral unit?

A **common fraction** is a fraction that has both terms expressed; as, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{1}{4}$.

A **proper fraction** is a fraction less in value than 1; as, $\frac{1}{2}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{8}{9}$, $\frac{1}{7}$, $\frac{3}{4}$, etc.

An **improper fraction** is a fraction equal to or greater in value than 1; as, $\frac{8}{8}$, $\frac{9}{8}$, $\frac{4}{3}$, $\frac{5}{5}$, $\frac{6}{5}$, $\frac{10}{9}$, etc.

A **mixed number** is a number expressed by a whole number and a fraction; as, $3\frac{1}{2}$, $12\frac{3}{4}$.

Change each of the following to integral units, or to mixed numbers. Thus, $\frac{9}{8} = 1\frac{1}{8}$.

| | | | |
|---|---|---|---|
| **7.** $\frac{10}{8}$ | **10.** $\frac{12}{4}$ | **13.** $\frac{21}{5}$ | **16.** $\frac{31}{6}$ |
| **8.** $\frac{5}{4}$ | **11.** $\frac{15}{4}$ | **14.** $\frac{59}{9}$ | **17.** $\frac{47}{7}$ |
| **9.** $\frac{6}{5}$ | **12.** $\frac{17}{8}$ | **15.** $\frac{27}{3}$ | **18.** $\frac{99}{8}$ |

## READING AND WRITING FRACTIONS

Read:

| | | |
|---|---|---|
| 1. $\frac{3}{4}$ | 6. $\frac{15}{40}$ | 11. $\$12\frac{3}{4}$ |
| 2. $\frac{8}{9}$ | 7. $\frac{4}{10}$ | 12. $\frac{3}{10}$ bu. |
| 3. $\frac{11}{12}$ | 8. $\frac{45}{75}$ | 13. $6\frac{3}{4}$ bbl. |
| 4. $\frac{1}{6}$ | 9. $\frac{38}{110}$ | 14. $125\frac{6}{31}$ |
| 5. $\frac{27}{38}$ | 10. $\frac{115}{248}$ | 15. 1 oz. = $\frac{1}{32000}$ of a ton. |

Write in figures:

1. One fourth.
2. Three fifths.
3. Six ninths.
4. Three fourths.
5. Seven tenths.
6. Eighteen twentieths.
7. Eight thousandths.
8. Sixty seventieths.
9. Nineteen forty-thirds.
10. Eighty-nine thousandths.
11. Eighty-nine three hundredths.
12. Twelve and three fourths.
13. Six and three fourths.
14. Five and one half.
15. Five ninths of three fifths.
16. One thousand ninety-fourths.
17. Nine hundred three thousandths.
18. Ten and three fourths.
19. Four hundred ninety and six thousand twenty-four ten-thousandths.

Write in words:

| | | |
|---|---|---|
| 20. $\frac{4}{5}$ | 24. $\frac{4}{14}$ | 28. $45\frac{9}{10}$ |
| 21. $\frac{5}{9}$ | 25. $\frac{35}{46}$ | 29. $2\frac{203}{222}$ |
| 22. $\frac{11}{15}$ | 26. $\frac{72}{85}$ | 30. $200\frac{3}{1000}$ |
| 23. $\frac{1}{4}$ | 27. $\frac{105}{118}$ | 31. $1001\frac{1001}{2000}$ |

## REDUCTION OF FRACTIONS

**Reduction** is the process of changing the form of a number without changing its value.

**1.** Divide both terms of the fraction $\frac{4}{8}$ by 2. How does $\frac{4}{8}$ compare in value with $\frac{1}{2}$?

**2.** Multiply both terms of the fraction $\frac{3}{4}$ by 2. How does $\frac{6}{8}$ compare in value with $\frac{3}{4}$? How may we obtain $\frac{6}{8}$ or $\frac{9}{12}$ from $\frac{3}{4}$?

*Multiplying or dividing both terms of a fraction by the same number does not change its value.*

**Changing a fraction to higher terms.**

**1.** Explain why a fraction is expressed in *smaller* fractional units when it is changed to *higher* terms.

**2.** Explain why changing a fraction to *higher* terms does not change the *value* of the fraction.

Change:

**3.** $\frac{1}{2}$ to 12ths

**4.** $\frac{3}{4}$ to 24ths

**5.** $\frac{5}{6}$ to 18ths

**6.** $\frac{3}{10}$ to 40ths

**7.** $\frac{7}{8}$ to 56ths

**8.** $\frac{8}{9}$ to 81sts

**9.** $\frac{11}{12}$ to 72ds

**10.** $\frac{7}{9}$ to 63ds

**11.** $\frac{3}{8}$ to 96ths

**12.** $\frac{6}{7} = \frac{?}{42} = \frac{?}{63}$

**13.** $\frac{9}{10} = \frac{?}{60} = \frac{?}{90}$

**14.** $\frac{4}{11} = \frac{?}{44} = \frac{?}{88}$

**15.** $\frac{7}{9} = \frac{?}{81} = \frac{?}{108}$

**16.** $\frac{5}{6} = \frac{?}{36} = \frac{?}{72}$

**17.** $\frac{7}{8} = \frac{?}{40} = \frac{?}{64}$

**18.** $\frac{5}{12} = \frac{?}{96} = \frac{?}{72}$

**19.** $\frac{6}{11} = \frac{?}{132} = \frac{?}{99}$

### Written Work

**1.** Change $\frac{5}{9}$ to 27ths.

$$27 \div 9 = 3$$

$$\frac{5}{9} = \frac{5 \times 3}{9 \times 3} = \frac{15}{27}$$

Since multiplying both terms of a fraction by the same number does not change its value, multiply both terms of the fraction by the quotient of $27 \div 9$, or 3.

Change:

**2.** $\frac{3}{4}$ to 20ths

**3.** $\frac{7}{8}$ to 56ths

**4.** $\frac{11}{12}$ to 96ths

**5.** $\frac{9}{13}$ to 78ths

**6.** $\frac{15}{22}$ to 132ds

**7.** $\frac{17}{18}$ to 72ds

**8.** $\frac{7}{15}$ to 135ths

**9.** $\frac{11}{23}$ to 276ths

**10.** $\frac{17}{25}$ to 275ths

**11.** $\frac{18}{31}$ to 372ds

**12.** $\frac{21}{38}$ to 494ths

**13.** $\frac{41}{45}$ to 765ths

**14.** $\frac{51}{83}$ to 415ths

**15.** $\frac{26}{35}$ to 315ths

**Changing a fraction to lower terms or to lowest terms.**

**1.** Explain why a fraction is expressed in *larger* fractional units when it is changed to *lower* terms. Explain also why changing a fraction to lower terms does not change its value.

Change:

**2.** $\frac{6}{12}$ to 4ths

**3.** $\frac{24}{36}$ to 6ths

**4.** $\frac{35}{40}$ to 8ths

**5.** $\frac{48}{80}$ to 10ths

**6.** $\frac{50}{60}$ to 12ths

**7.** $\frac{25}{40}$ to 8ths

**8.** $\frac{60}{72}$ to 12ths

**9.** $\frac{80}{96}$ to 6ths

**10.** $\frac{24}{54}$ to 9ths

**11.** $\frac{55}{80}$ to 16ths

**12.** $\frac{?}{5} = \frac{40}{50}$

**13.** $\frac{?}{12} = \frac{49}{84}$

**14.** $\frac{?}{21} = \frac{27}{63}$

**15.** $\frac{?}{9} = \frac{48}{108}$

**16.** $\frac{?}{12} = \frac{132}{144}$

**17.** $\frac{?}{8} = \frac{24}{64}$

**18.** $\frac{?}{8} = \frac{35}{56}$

**19.** $\frac{?}{9} = \frac{40}{72}$

## Written Work

A fraction is expressed in its *lowest terms* when the numerator and the denominator are *prime* to each other.

**1.** Change $\frac{42}{54}$ to lowest terms.

$$\frac{\cancel{42}}{\cancel{54}} = \frac{\cancel{21}}{\cancel{27}} = \frac{7}{9}$$

Or g.c.d. $= 6$

$$\frac{42 \div 6}{54 \div 6} = \frac{7}{9}$$

Since dividing both terms of a fraction by the same number does not change its value, we may reject by cancellation all the factors common to both terms, leaving the factors 7 and 9. Hence, $\frac{42}{54} = \frac{7}{9}$.

Or we may, in one step, divide both terms of the fraction by their g. c. d., 6.

**2.** Change $\frac{357}{483}$ to lowest terms.

$$\frac{357 \div 3}{483 \div 3} = \frac{119 \div 7}{161 \div 7} = \frac{17}{23} \quad \text{Or, g.c.d.} = 21 \quad \frac{357 \div 21}{483 \div 21} = \frac{17}{23}$$

*Cancel all the factors common to both numerator and denominator. Or, divide both numerator and denominator by their greatest common divisor.*

Change to lowest terms:

| | | | | |
|---|---|---|---|---|
| **3.** $\frac{18}{24}$ | **12.** $\frac{121}{132}$ | **21.** $\frac{125}{325}$ | **30.** $\frac{455}{530}$ | **39.** $\frac{414}{999}$ |
| **4.** $\frac{25}{55}$ | **13.** $\frac{54}{72}$ | **22.** $\frac{365}{605}$ | **31.** $\frac{750}{825}$ | **40.** $\frac{126}{189}$ |
| **5.** $\frac{42}{49}$ | **14.** $\frac{18}{28}$ | **23.** $\frac{480}{660}$ | **32.** $\frac{615}{945}$ | **41.** $\frac{435}{630}$ |
| **6.** $\frac{72}{81}$ | **15.** $\frac{42}{48}$ | **24.** $\frac{182}{196}$ | **33.** $\frac{462}{504}$ | **42.** $\frac{147}{196}$ |
| **7.** $\frac{21}{36}$ | **16.** $\frac{36}{42}$ | **25.** $\frac{264}{333}$ | **34.** $\frac{672}{936}$ | **43.** $\frac{216}{270}$ |
| **8.** $\frac{24}{28}$ | **17.** $\frac{58}{74}$ | **26.** $\frac{315}{345}$ | **35.** $\frac{756}{924}$ | **44.** $\frac{546}{588}$ |
| **9.** $\frac{35}{75}$ | **18.** $\frac{128}{176}$ | **27.** $\frac{200}{450}$ | **36.** $\frac{567}{621}$ | **45.** $\frac{396}{432}$ |
| **10.** $\frac{27}{72}$ | **19.** $\frac{94}{144}$ | **28.** $\frac{528}{624}$ | **37.** $\frac{294}{476}$ | **46.** $\frac{567}{783}$ |
| **11.** $\frac{84}{96}$ | **20.** $\frac{81}{96}$ | **29.** $\frac{288}{444}$ | **38.** $\frac{322}{504}$ | **47.** $\frac{837}{945}$ |

**Changing a mixed number to an improper fraction, or an improper fraction to a mixed number.**

Change to an improper fraction at sight:

| | | | |
|---|---|---|---|
| **1.** $1\frac{3}{4}$ | **4.** $3\frac{1}{7}$ | **7.** $6\frac{8}{10}$ | **10.** 12 to 3ds |
| **2.** $1\frac{7}{8}$ | **5.** $5\frac{2}{3}$ | **8.** $3\frac{2}{15}$ | **11.** 8 to 6ths |
| **3.** $2\frac{1}{3}$ | **6.** $2\frac{5}{9}$ | **9.** 8 to 12ths | **12.** 10 to 5ths |

## Written Work

**1.** Change $111\frac{3}{5}$ to an improper fraction.

$$1 = \frac{5}{5}$$
$$111 = 111 \times \frac{5}{5} = \frac{555}{5}$$
$$\frac{555}{5} + \frac{3}{5} = \frac{558}{5}$$

In 1 there are $\frac{5}{5}$; in 111 there are 111 times $\frac{5}{5}$, or $\frac{555}{5}$, which added to $\frac{3}{5}$ equal $\frac{558}{5}$. Hence, $111\frac{3}{5}$ equals $\frac{558}{5}$.

In small numbers the work may be done *mentally*, only the result being written.

Change to improper fractions:

| | | | |
|---|---|---|---|
| **2.** $12\frac{3}{4}$ | **8.** $75\frac{15}{28}$ | **14.** $268\frac{97}{120}$ | **20.** $391\frac{45}{56}$ |
| **3.** $15\frac{5}{6}$ | **9.** $95\frac{21}{25}$ | **15.** $324\frac{47}{78}$ | **21.** $16\frac{11}{90}$ |
| **4.** $22\frac{7}{8}$ | **10.** $103\frac{29}{32}$ | **16.** $502\frac{23}{105}$ | **22.** $901\frac{5}{32}$ |
| **5.** $30\frac{9}{10}$ | **11.** $118\frac{31}{45}$ | **17.** $109\frac{47}{144}$ | **23.** $100\frac{7}{15}$ |
| **6.** $48\frac{11}{12}$ | **12.** $175\frac{38}{56}$ | **18.** $600\frac{17}{30}$ | **24.** $390\frac{43}{50}$ |
| **7.** $56\frac{13}{14}$ | **13.** $215\frac{76}{105}$ | **19.** $305\frac{3}{7}$ | **25.** $231\frac{14}{75}$ |

**26.** Change $\frac{128}{7}$ to a mixed number.

$$\frac{128}{7} = 128 \div 7 = 18\frac{2}{7}$$

In 1 there are $\frac{7}{7}$, and in $\frac{128}{7}$ there are as many times 1, as 7 is contained times in 128, or $18\frac{2}{7}$. Hence, $\frac{128}{7}$ is equal to $18\frac{2}{7}$.

*Every fraction is an indicated division.*

Change to integers or mixed numbers:

| | | | |
|---|---|---|---|
| **27.** $\frac{31}{15}$ | **33.** $\frac{715}{32}$ | **39.** $\frac{745}{34}$ | **45.** $\frac{2368}{15}$ |
| **28.** $\frac{527}{11}$ | **34.** $\frac{770}{35}$ | **40.** $\frac{876}{38}$ | **46.** $\frac{3440}{75}$ |
| **29.** $\frac{956}{7}$ | **35.** $\frac{333}{18}$ | **41.** $\frac{576}{24}$ | **47.** $\frac{5286}{48}$ |
| **30.** $\frac{157}{16}$ | **36.** $\frac{625}{25}$ | **42.** $\frac{862}{84}$ | **48.** $\frac{4645}{50}$ |
| **31.** $\frac{261}{8}$ | **37.** $\frac{576}{24}$ | **43.** $\frac{534}{96}$ | **49.** $\frac{2200}{12}$ |
| **32.** $\frac{382}{25}$ | **38.** $\frac{240}{16}$ | **44.** $\frac{1245}{24}$ | **50.** $\frac{4032}{36}$ |

**Changing to least similar fractions.**

A **common denominator** of two or more fractions is a number that contains all the denominators of the fractions an integral number of times; thus, 24 is a common denominator of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

The **least common denominator** (l. c. d) of two or more fractions is the *least* number that contains all the denominators of the fractions an integral number of times; thus, 12 is the l. c. d. of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

The l. c. d is the least common multiple of the denominators.

**Similar fractions** are fractions that express the same unit value. They must therefore have a common denominator.

**By inspection:**

**1.** Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{9}$ to similar fractions having the least common denominator.

$$\frac{1 \times 9}{2 \times 9} = \frac{9}{18}$$

$$\frac{2 \times 6}{3 \times 6} = \frac{12}{18}$$

$$\frac{5 \times 2}{9 \times 2} = \frac{10}{18}$$

It is evident by inspection that 18 is the least common multiple of 2, 3, and 9. It is therefore the least common denominator of the given fractions. Changing the given fraction to 18ths, we find that $\frac{1}{2} = \frac{9}{18}$; $\frac{2}{3} = \frac{12}{18}$; and $\frac{5}{9} = \frac{10}{18}$. Hence, the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{9}$ may be changed to the similar fractions $\frac{9}{18}$, $\frac{12}{18}$, and $\frac{10}{18}$.

Change to least similar fractions:

| | | | |
|---|---|---|---|
| **2.** $\frac{1}{3}, \frac{1}{4}, \frac{5}{12}$ | **7.** $\frac{1}{5}, \frac{3}{10}, \frac{7}{40}$ | **12.** $\frac{2}{3}, \frac{1}{6}, \frac{19}{36}$ | **17.** $\frac{5}{6}, \frac{3}{7}, \frac{9}{14}$ |
| **3.** $\frac{2}{3}, \frac{1}{5}, \frac{2}{15}$ | **8.** $\frac{1}{3}, \frac{5}{12}, \frac{7}{24}$ | **13.** $\frac{1}{3}, \frac{1}{2}, \frac{1}{6}$ | **18.** $\frac{1}{3}, \frac{5}{16}, \frac{17}{24}$ |
| **4.** $\frac{5}{6}, \frac{2}{3}, \frac{7}{18}$ | **9.** $\frac{2}{5}, \frac{3}{8}, \frac{9}{40}$ | **14.** $\frac{3}{7}, \frac{11}{14}, \frac{19}{28}$ | **19.** $\frac{2}{5}, \frac{2}{3}, \frac{3}{20}$ |
| **5.** $\frac{1}{6}, \frac{3}{8}, \frac{17}{24}$ | **10.** $\frac{1}{9}, \frac{1}{6}, \frac{5}{18}$ | **15.** $\frac{3}{5}, \frac{5}{9}, \frac{7}{15}$ | **20.** $\frac{4}{7}, \frac{5}{8}, \frac{1}{56}$ |
| **6.** $\frac{1}{9}, \frac{4}{18}, \frac{5}{36}$ | **11.** $\frac{3}{5}, \frac{7}{10}, \frac{1}{20}$ | **16.** $\frac{4}{7}, \frac{3}{14}, \frac{1}{28}$ | **21.** $\frac{2}{3}, \frac{4}{5}, \frac{3}{4}$ |

**By factoring the denominators:**

**1.** Change $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{9}{10}$ to similar fractions having the least common denominator.

$$4 = 2 \times 2$$
$$6 = 2 \times 3$$
$$10 = 2 \times 5$$

In finding the l. c. m., use each factor as often as it occurs in any one number.

$$\text{l. c. m.} = 2 \times 2 \times 3 \times 5 = 60$$

$$\frac{3 \times 15}{4 \times 15} = \frac{45}{60}$$
$$\frac{5 \times 10}{6 \times 10} = \frac{50}{60}$$
$$\frac{9 \times 6}{10 \times 6} = \frac{54}{60}$$

The least common multiple of the denominators is 60, which is the least common denominator of the given fractions. Changing the given fractions to 60ths, we find that $\frac{3}{4} = \frac{45}{60}$; $\frac{5}{6} = \frac{50}{60}$; and $\frac{9}{10} = \frac{54}{60}$.

Change to least similar fractions:

**2.** $\frac{1}{4}, \frac{2}{5}, \frac{3}{8}, \frac{1}{2}$

**3.** $\frac{2}{3}, \frac{5}{16}, \frac{7}{12}, \frac{1}{4}$

**4.** $\frac{7}{10}, \frac{3}{5}, \frac{8}{15}, \frac{9}{20}$

**5.** $\frac{2}{3}, \frac{4}{5}, \frac{7}{8}, \frac{5}{6}$

**6.** $\frac{5}{9}, \frac{12}{25}, \frac{17}{18}$

**7.** $\frac{2}{7}, \frac{5}{9}, \frac{8}{11}$

**8.** $\frac{2}{5}, \frac{19}{20}, \frac{5}{12}, \frac{17}{30}$

**9.** $\frac{1}{7}, \frac{1}{2}, \frac{9}{28}, \frac{11}{18}$

**10.** $\frac{1}{4}, \frac{9}{10}, \frac{8}{35}, \frac{3}{16}$

**11.** $\frac{7}{18}, \frac{9}{15}, 1\frac{7}{9}, \frac{13}{30}$

**12.** $\frac{1}{18}, \frac{5}{21}, \frac{9}{28}, \frac{31}{54}$

**13.** $3\frac{1}{2}, \frac{11}{18}, 1\frac{5}{6}, \frac{22}{24}$

**14.** $\frac{7}{10}, \frac{5}{12}, \frac{9}{20}, \frac{29}{30}$

**15.** $\frac{3}{14}, \frac{9}{16}, 3\frac{3}{8}, 2\frac{17}{21}$

**16.** $\frac{6}{21}, \frac{17}{35}, \frac{3}{70}, \frac{51}{120}$

**17.** $\frac{5}{6}, \frac{8}{9}, \frac{5}{21}, \frac{37}{63}$

**18.** $\frac{8}{9}, \frac{14}{25}, 3\frac{7}{15}$

**19.** $\frac{9}{22}, \frac{16}{33}, \frac{19}{66}, \frac{67}{132}$

## ADDITION AND SUBTRACTION

### Size and kind of fractional units.

**1.** Why is it not possible to add 4 ft. to 5 oz.?

**2.** What is the sum of $\frac{1}{5}$ and $\frac{3}{5}$? of $\frac{4}{3}$ and $\frac{2}{3}$?

**3.** What is the size of the fractional unit of $\frac{4}{5}$ and $\frac{2}{5}$?

**4.** What is the kind of fractional unit in each?

**5.** Are the fractional units of $\frac{4}{5}$ and $\frac{2}{5}$ alike in *size* and *kind?*

**6.** What is the *size* of the fractional unit in $\$\frac{4}{5}$ and $\frac{2}{5}$ ft.? What is the *kind* of unit in each?

**7.** Why can we add or subtract $\$\frac{4}{5}$ and $\$\frac{2}{5}$, or $\frac{4}{5}$ ft. and $\frac{2}{5}$ ft.?

**8.** What kind of fractions can be added or subtracted?

*Before fractions can be added or subtracted they must be expressed in* **similar fractional units.**

Fractions having the same kind of units or having *related units* can be added; thus, $\frac{2}{3}$ and $\frac{1}{4}$, or $\frac{2}{3}$ yd. and $\frac{1}{4}$ ft. ($= \frac{1}{12}$ yd.) can be changed to similar fractions and then added. But fractions whose units are *unrelated,* as $\frac{2}{3}$ yd. and $\frac{1}{4}$ oz., cannot be added, because they cannot be changed to similar fractions.

**9.** $5\frac{1}{2}$ ft. and 2 yd. = —— ft. Why must you change yards to feet before adding?

**10.** Add the fractions in the following list that are similar:

| | | | | | |
|---|---|---|---|---|---|
| $\frac{1}{4}$ da. | $\frac{3}{8}$ hr. | $\$\frac{1}{5}$ | $\frac{3}{4}$ da. | $\frac{9}{10}$ min. | $\frac{7}{8}$ hr. |
| $\frac{4}{5}$ rd. | $\frac{2}{3}$ ft. | $\$\frac{3}{5}$ | $\frac{7}{10}$ min. | $\frac{1}{3}$ ft. | $\frac{1}{5}$ rd. |

Add quickly:

| | | | |
|---|---|---|---|
| **11.** $\frac{1}{2}+\frac{1}{4}$ | **15.** $\frac{3}{5}+\frac{1}{2}$ | **19.** $\frac{1}{12}+\frac{3}{4}$ | **23.** $\frac{5}{6}+\frac{1}{3}$ |
| **12.** $\frac{2}{3}+\frac{1}{6}$ | **16.** $\frac{4}{3}+\frac{1}{2}$ | **20.** $\frac{2}{3}+\frac{7}{9}$ | **24.** $\frac{2}{3}+\frac{8}{6}$ |
| **13.** $\frac{3}{4}+\frac{1}{8}$ | **17.** $\frac{2}{3}+\frac{1}{9}$ | **21.** $\frac{4}{5}+\frac{1}{2}$ | **25.** $\frac{3}{4}+\frac{5}{8}$ |
| **14.** $\frac{1}{4}+\frac{2}{3}$ | **18.** $\frac{5}{4}+\frac{3}{2}$ | **22.** $\frac{11}{12}+\frac{2}{3}$ | **26.** $\frac{7}{8}+\frac{1}{3}$ |

## Written Work

**1.** Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{8}{9}$.

$$\begin{array}{c|l} \multicolumn{2}{l}{36 = \text{l. c. d.}} \\ \frac{2}{3} & 24 \\ \frac{3}{4} & 27 \\ \frac{8}{9} & 32 \\ \hline & \frac{83}{36} = 2\frac{11}{36} \end{array}$$

The least common denominator is 36. Changing the given fractions to 36ths, we find that $\frac{2}{3}=\frac{24}{36}$; $\frac{3}{4}=\frac{27}{36}$; $\frac{8}{9}=\frac{32}{36}$. The sum of these fractions $=\frac{83}{36}=2\frac{11}{36}$.

1. What is the *first* step in the work?
2. What is the *second* step?
3. What is the *third* step?

Add:

**2.** $\frac{5}{6}, \frac{7}{8}, \frac{4}{5}$

**3.** $\frac{3}{7}, \frac{9}{14}, \frac{1}{2}$

**4.** $\frac{5}{7}, \frac{11}{25}, \frac{3}{4}$

**5.** $\frac{2}{3}, \frac{1}{4}, \frac{1}{2}$

**6.** $\frac{5}{8}, \frac{7}{12}, \frac{5}{6}$

**7.** $\frac{3}{4}, \frac{2}{5}, \frac{9}{10}$

**8.** $\frac{2}{7}, \frac{9}{14}, \frac{3}{4}$

**9.** $\frac{2}{5}, \frac{3}{7}, \frac{7}{9}$

**10.** $\frac{2}{3}, \frac{3}{5}, \frac{1}{4}, \frac{5}{6}$

**11.** $\frac{1}{6}, \frac{2}{3}, \frac{7}{12}, \frac{14}{15}$

**12.** $\frac{3}{8}, \frac{11}{12}, \frac{9}{32}, \frac{3}{4}$

**13.** $\frac{7}{8}, \frac{5}{12}, \frac{3}{16}, \frac{11}{24}$

**14.** $\frac{5}{14}, \frac{2}{7}, \frac{11}{35}, \frac{25}{49}$

**15.** $\frac{1}{4}, \frac{3}{5}, \frac{9}{10}, \frac{12}{25}$

**16.** $\frac{1}{4}, \frac{3}{8}, \frac{1}{5}, \frac{7}{12}$

**17.** Find the sum of $3\frac{1}{4}$, $2\frac{5}{9}$, and $7\frac{4}{10}$.

180 = l. c. d.

| | |
|---|---|
| $3\frac{1}{4}$ | 45 |
| $2\frac{5}{9}$ | 100 |
| $7\frac{4}{10}$ | 72 |

$$12 + \frac{217}{180} = 12 + 1\frac{37}{180} = 13\frac{37}{180}$$

Since the numbers are *mixed* numbers, the integers and fractions are added separately, and their sums are united. The sum of $\frac{1}{4}$, $\frac{5}{9}$, and $\frac{4}{10}$ is $\frac{217}{180}$, or $1\frac{37}{180}$. The sum of 3, 2, and 7 is 12. The sum of 12 and $1\frac{37}{180}$ is $13\frac{37}{180}$.

Add:

**18.** $3\frac{5}{6}, 7\frac{4}{8}, 9\frac{3}{16}$

**19.** $18\frac{9}{14}, 3\frac{3}{7}, 9\frac{9}{28}$

**20.** $8\frac{7}{8}, 10\frac{17}{24}, 15\frac{9}{16}$

**21.** $7\frac{3}{4}, 12\frac{8}{15}, 24\frac{5}{12}$

**22.** $16\frac{5}{9}, 30\frac{7}{8}, 45\frac{7}{24}$

**23.** $50\frac{3}{10}, 48\frac{7}{15}, 16\frac{19}{35}$

**24.** $20\frac{3}{7}, 12\frac{16}{21}, 5\frac{23}{28}$

**25.** $14\frac{5}{12}, 32\frac{10}{21}, 23\frac{29}{30}$

**26.** $2\frac{2}{3}, 7\frac{7}{8}, 11\frac{3}{10}, 14\frac{5}{14}$

**27.** $90\frac{7}{18}, 60\frac{9}{20}, 73\frac{7}{30}$

**28.** $3\frac{1}{2}, 4\frac{3}{7}, 6\frac{5}{9}, 8\frac{1}{10}$

**29.** $84\frac{5}{12}, 36\frac{5}{9}, 33\frac{3}{7}, 39\frac{2}{3}$

**30.** A bicycler rode $8\frac{1}{2}$ miles the first hour, $7\frac{3}{8}$ miles the second hour, $6\frac{9}{16}$ miles the third hour, and $8\frac{1}{4}$ miles the fourth hour. How many miles did he ride in the four hours?

**31.** Find the distance around a field $80\frac{3}{4}$ rods long and $60\frac{7}{8}$ rods wide.

**32.** Find the sum of the improper fractions $\frac{8}{5}$, $\frac{29}{25}$, $\frac{41}{15}$, $\frac{17}{10}$.

**33.** What number is that from which if $32\frac{7}{16}$ is taken, the remainder is $23\frac{99}{112}$?

### Subtracting fractions.

What kind of whole numbers can be added? what kind of fractions?

In adding like fractions we find the *sum* of the numerators; in subtracting like fractions we find the *difference* of the numerators.

### Written Work

**1.** From $\frac{7}{8}$ take $\frac{3}{18}$.

$72 = $ l. c. d.

| | |
|---|---|
| $\frac{7}{8}$ | 63 |
| $\frac{3}{18}$ | 12 |

$\frac{51}{72} = \frac{17}{24}$

Since fractions must be made similar before they can be subtracted, $\frac{7}{8}$ and $\frac{3}{18}$ are changed to 72ds. The difference between $\frac{63}{72}$ and $\frac{12}{72}$ is $\frac{51}{72}$, or $\frac{17}{24}$.

Observe the *three* steps in subtraction of fractions:

1. Change the fractions, if necessary, to like fractions.
2. Take the difference of their numerators.
3. Change the difference to its simplest form.

Find differences:

| | | |
|---|---|---|
| **2.** $\frac{7}{9} - \frac{3}{4}$ | **6.** $\frac{55}{96} - \frac{11}{24}$ | **10.** $\frac{11}{12} - \frac{10}{13}$ |
| **3.** $\frac{9}{10} - \frac{11}{15}$ | **7.** $\frac{13}{15} - \frac{17}{21}$ | **11.** $\frac{23}{36} - \frac{13}{25}$ |
| **4.** $\frac{7}{12} - \frac{7}{18}$ | **8.** $\frac{23}{24} - \frac{14}{15}$ | **12.** $\frac{28}{63} - \frac{12}{35}$ |
| **5.** $\frac{25}{27} - \frac{11}{12}$ | **9.** $\frac{93}{108} - \frac{13}{36}$ | **13.** $\frac{25}{56} - \frac{16}{42}$ |

**14.** A boy wishes to buy a pair of skates costing $\$\frac{9}{10}$, but he has only $\$\frac{3}{4}$. What part of a dollar is lacking?

**15.** From $\frac{4}{5}$ take $\frac{3}{4}$.

**16.** What fraction added to $\frac{5}{9}$ will give $\frac{43}{45}$?

**17.** From $\frac{3}{4}$ of an acre of land, subtract $\frac{5}{8}$ of an acre.

**18.** What part of a teacher's salary remained after he had spent $\frac{1}{3}$, $\frac{3}{16}$, and $\frac{1}{6}$ of it?

**19.** The minuend is $\frac{16}{21}$, and the remainder $\frac{2}{9}$. What is the subtrahend?

**20.** If a boy spends $\frac{1}{2}$ of his money one day, $\frac{1}{4}$ of it the next day, and has $\$1\frac{1}{2}$ left, how much money had he at first?

**Subtracting mixed numbers.**

## Written Work

**1.** From $7\frac{1}{4}$ take $3\frac{5}{9}$.

$$\begin{array}{rl|l} & \multicolumn{2}{l}{36 = \text{l. c. d.}} \\ \hline 7\frac{1}{4} = & 6\frac{5}{4} & 45 \\ & 3\frac{5}{9} & 20 \\ \cline{2-3} & 3 & \frac{25}{36} \end{array}$$

$7\frac{1}{4} = 6 + \frac{4}{4} + \frac{1}{4} = 6\frac{5}{4}$. The integers and fractions are subtracted separately. The least common denominator is 36. Changing the fractions to 36ths we find that $\frac{5}{4} = \frac{45}{36}$, and $\frac{5}{9} = \frac{20}{36}$. $\frac{45}{36} - \frac{20}{36} = \frac{25}{36}$. $6 - 3 = 3$, which added to $\frac{25}{36} = 3\frac{25}{36}$. Hence, the difference between $7\frac{1}{4}$ and $3\frac{5}{9} = 3\frac{25}{36}$.

*In subtracting mixed numbers subtract the integers and fractions separately.*

Find differences :

| | | |
|---|---|---|
| **2.** $7\frac{3}{4} - 3\frac{5}{12}$ | **8.** $63\frac{11}{18} - 24\frac{5}{34}$ | **14.** $60\frac{7}{16} - 35\frac{7}{12}$ |
| **3.** $9\frac{8}{9} - 2\frac{4}{7}$ | **9.** $71\frac{7}{22} - 19\frac{8}{25}$ | **15.** $71\frac{23}{39} - 54\frac{49}{78}$ |
| **4.** $18\frac{17}{21} - 8\frac{4}{9}$ | **10.** $73\frac{15}{40} - 18\frac{17}{36}$ | **16.** $39\frac{10}{27} - 18\frac{11}{15}$ |
| **5.** $30\frac{1}{4} - 20\frac{19}{30}$ | **11.** $92\frac{27}{35} - 29\frac{11}{18}$ | **17.** $82\frac{17}{24} - 45\frac{19}{32}$ |
| **6.** $45\frac{1}{6} - 24\frac{5}{9}$ | **12.** $82\frac{29}{32} - 29\frac{37}{64}$ | **18.** $29\frac{1}{48} - 11\frac{29}{72}$ |
| **7.** $50\frac{5}{13} - 11\frac{5}{6}$ | **13.** $95\frac{14}{27} - 47\frac{11}{32}$ | **19.** $20\frac{31}{35} - 15\frac{54}{70}$ |

**20.** If I pay a grocery bill of $\$22\frac{1}{2}$, a water bill of $\$3\frac{1}{4}$, and a gas bill of $\$5\frac{3}{4}$, how much shall I have left from 2 twenty-dollar bills?

**21.** The sum of three numbers is 150. The least number is $15\frac{1}{2}$, and it is $63\frac{7}{8}$ less than the greatest. Find the other number.

**22.** What fraction added to the sum of $\frac{1}{6}$, $\frac{2}{9}$, and $\frac{5}{16}$ will make $\frac{3}{4}$?

**23.** If 5 is added to each term of the fraction $\frac{3}{4}$, is the value of the fraction increased or diminished, and how much?

**24.** Two boys undertake to save \$50 apiece. When one of them lacks $\$8\frac{9}{10}$ of having \$50, both together have $\$84\frac{3}{4}$. How much has each?

**25.** A traveling man's grips, when starting out, weighed as follows: $12\frac{7}{8}$ pounds and $19\frac{3}{5}$ pounds. Find the weight of both, and the difference in their weight.

**26.** James lives $1\frac{3}{4}$ miles east of the schoolhouse, and Harry $1\frac{3}{16}$ miles west of the schoolhouse. Find the sum of the distances walked by both each day and the distance James walks farther than Harry.

**27.** Eight women do different parts in making a finished garment. The cost of the different parts is $1\frac{1}{2}$¢, $2\frac{3}{4}$¢, $4\frac{1}{5}$¢, $3\frac{3}{5}$¢, $4\frac{1}{3}$¢, $4\frac{1}{12}$¢, 1¢, and $\frac{7}{10}$¢. Find the cost of making one garment.

**28.** Four automobiles finish a race in the following time: $8\frac{4}{15}$ hours, $8\frac{3}{4}$ hours, $7\frac{4}{5}$ hours, and $9\frac{3}{20}$ hours. Find the difference between the time of the winner and each of the others.

**29.** The average cost per mile of a fleet of freight boats on the Great Lakes was: coal $13\frac{4}{7}$¢, crew $12\frac{3}{4}$¢, repairs $17\frac{3}{4}$¢, supplies 7¢. The average cost per mile of a freight train hauling the same freight was: coal $74\frac{3}{8}$¢, crew $37\frac{1}{2}$¢, repairs $19\frac{5}{12}$¢, supplies $4\frac{3}{4}$¢. Find the saving per mile by water.

**30.** A drayman hauls freight by the ton from the depot to a village. Find the amount hauled in 8 loads weighing respectively: $1\frac{7}{8}$ tons, $2\frac{1}{5}$ tons, $1\frac{3}{4}$ tons, $2\frac{5}{12}$ tons, $2\frac{3}{4}$ tons, $1\frac{5}{8}$ tons, $2\frac{1}{2}$ tons, $1\frac{9}{10}$ tons.

## MULTIPLICATION OF FRACTIONS

**To multiply a fraction by an integer.**

1. $1 + 1 + 1 + 1 =$ how many whole units?

2. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$ how many fractional units?

3. How many, then, are $4 \times 1$? $4 \times \frac{1}{3}$?

4. Does it make any difference in the process of multiplication in problem 3 whether the multiplicand is a whole number or a fraction?

5. What term of the fraction $\frac{1}{3}$ is multiplied by 4?

6. $\frac{2}{3}$ may be multiplied by 4, thus: $4 \times \frac{2}{3} = \frac{4 \times 2}{3}$, or $\frac{8}{3} = 2\frac{2}{3}$.

Give products:

| | | | |
|---|---|---|---|
| 7. $6 \times \frac{3}{4}$ | 11. $5 \times \frac{3}{7}$ | 15. $5 \times \frac{8}{11}$ | 19. $16 \times \frac{3}{8}$ |
| 8. $10 \times \frac{2}{5}$ | 12. $12 \times \frac{5}{7}$ | 16. $15 \times \frac{3}{4}$ | 20. $14 \times \frac{6}{7}$ |
| 9. $9 \times \frac{3}{7}$ | 13. $7 \times \frac{3}{8}$ | 17. $12 \times \frac{5}{6}$ | 21. $13 \times \frac{3}{9}$ |
| 10. $12 \times \frac{7}{8}$ | 14. $7 \times \frac{4}{9}$ | 18. $8 \times \frac{3}{4}$ | 22. $11 \times \frac{8}{11}$ |

23. In multiplying the above fractions by a whole number, did we increase the *size* or the *number* of the fractional units?

24. How, then, may any fraction be multiplied by an integer without increasing the *size* of its fractional units?

25. $2 \times \frac{2}{8}$ of a square $=$ how many eighths of the square? How many fourths of the same square? Draw figures to illustrate.

26. How does $\frac{4}{8}$ of a square compare in size with $\frac{2}{4}$ of the same square? Draw figure to illustrate.

**27.** $3 \times \frac{2}{9}$ of a square = how many ninths of the square? how many thirds of the same square? Draw figures to illustrate.

**28.** How does $\frac{6}{9}$ of a square compare with $\frac{2}{3}$ of the same square?

**29.** How much larger is the fractional unit in fourths than in eighths? in thirds than in ninths?

**30.** How can we increase the size of the fractional unit in $\frac{2}{8}$ without decreasing the number of fractional units? in $\frac{3}{9}$?

**31.** How, then, may any fraction be multiplied by an integer without increasing the *number* of its fractional units?

Then, $3 \times \dfrac{2}{9} = \dfrac{2}{9 \div 3}$, or $\dfrac{2}{3}$.

In what *two* ways, then, may we multiply a fraction by an integer?

*Multiplying the numerator or dividing the denominator of a fraction by a number multiplies the value of the fraction by that number.*

Give products:

| | | |
|---|---|---|
| **32.** $8 \times \frac{3}{4}$ | **40.** $15 \times \frac{2}{5}$ | **48.** $25 \times \frac{9}{75}$ |
| **33.** $9 \times \frac{7}{18}$ | **41.** $11 \times \frac{7}{77}$ | **49.** $15 \times \frac{3}{45}$ |
| **34.** $11 \times \frac{6}{7}$ | **42.** $9 \times \frac{5}{11}$ | **50.** $18 \times \frac{3}{54}$ |
| **35.** $6 \times \frac{9}{12}$ | **43.** $16 \times \frac{7}{32}$ | **51.** $12 \times \frac{9}{108}$ |
| **36.** $10 \times \frac{7}{10}$ | **44.** $7 \times \frac{11}{63}$ | **52.** $36 \times \frac{15}{72}$ |
| **37.** $9 \times \frac{8}{10}$ | **45.** $3 \times \frac{9}{24}$ | **53.** $42 \times \frac{13}{84}$ |
| **38.** $12 \times \frac{3}{8}$ | **46.** $4 \times \frac{13}{28}$ | **54.** $39 \times \frac{15}{78}$ |
| **39.** $18 \times \frac{5}{36}$ | **47.** $5 \times \frac{11}{7}$ | **55.** $64 \times \frac{3}{32}$ |

## Written Work

1. Multiply $\frac{7}{32}$ by 5.

$5 \times \frac{7}{32} = \frac{35}{32} = 1\frac{3}{32}$

Since multiplying the numerator of a fraction multiplies the fraction, 5 times $\frac{7}{32} = \frac{35}{32} = 1\frac{3}{32}$.

When possible, use cancellation.

Multiply :

| | | |
|---|---|---|
| 2. $\frac{8}{21}$ by 7 | 11. $\frac{27}{36}$ by 72 | 20. $\frac{7}{49}$ by 96 |
| 3. $\frac{5}{8}$ by 10 | 12. $\frac{11}{27}$ by 108 | 21. $\frac{5}{72}$ by 54 |
| 4. $\frac{27}{60}$ by 15 | 13. $\frac{3}{19}$ by 57 | 22. $\frac{9}{136}$ by 28 |
| 5. $\frac{6}{7}$ by 8 | 14. $\frac{21}{24}$ by 144 | 23. $\frac{3}{35}$ by 105 |
| 6. $\frac{17}{81}$ by 27 | 15. $\frac{9}{15}$ by 135 | 24. $\frac{5}{64}$ by 96 |
| 7. $\frac{11}{15}$ by 35 | 16. $\frac{7}{36}$ by 21 | 25. $\frac{10}{77}$ by 121 |
| 8. $\frac{17}{56}$ by 28 | 17. $\frac{9}{54}$ by 16 | 26. $\frac{15}{26}$ by 48 |
| 9. $\frac{35}{48}$ by 144 | 18. $\frac{5}{108}$ by 72 | 27. $\frac{9}{54}$ by 69 |
| 10. $\frac{18}{35}$ by 70 | 19. $\frac{11}{126}$ by 24 | 28. $\frac{17}{42}$ by 126. |

**Finding fractional parts of an integer.**

$8 \times \frac{3}{4}$ means that $\frac{3}{4}$ is to be taken as an addend as many times as there are units in the multiplier. Thus, $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$.

$\frac{3}{4}$ of 8 means that 8 is to be divided into 4 equal parts and 3 of these parts are to be taken.

What is the *first* step in finding the fractional parts of the whole number? the *second* step?

While the process of finding fractional parts of a whole number is classed as multiplication, the multiplicand at no time is *taken as an addend*, but is *partitioned*, that is, divided into equal parts, and a certain number of these parts is taken.

The sign $\times$ is read "of" when the number preceding it is a simple fraction ; as, $\frac{2}{3} \times \$6$ is read "$\frac{2}{3}$ of $6."

$\frac{3}{4} \times 12$ mo. means, therefore, $3 \times (\frac{1}{4}$ of 12 mo.) or $3 \times 3$ mo. $= 9$ mo.

$$\text{Or, } \frac{3}{4} \times 12 \text{ mo.} = \frac{3 \times \overset{3}{\cancel{12}}}{\cancel{4}} \text{ mo.} = 9 \text{ mo.}$$

Find :

1. $\frac{7}{8}$ of \$80
2. $\frac{5}{6} \times 42$
3. $\frac{7}{9} \times 63$
4. $\frac{3}{16} \times 48$
5. $\frac{7}{8} \times 20$
6. $\frac{5}{12}$ of 60 hr.
7. $\frac{7}{12}$ of 30 da.
8. $\frac{4}{5}$ of 47
9. $\frac{5}{7} \times 100$
10. $\frac{8}{9} \times 95$
11. $\frac{11}{12} \times 124$
12. $\frac{33}{40} \times 175$
13. $\frac{27}{50} \times 450$
14. $\frac{31}{36} \times 720$
15. $\frac{19}{27} \times 108$

### Written Work

1. If the Welsh mills turned out in a certain year 576000 tons of tin plate and the American mills $\frac{23}{24}$ as much, find the output of the American mills for that year.

2. A certain post office in one year handled 53678996 pieces of mail, of which $\frac{3}{7}$ were letters. Find the number of letters handled at that post office.

3. The records of a blast furnace show 1179360 tons of iron made during the year. If $\frac{5}{9}$ of this iron is sold as No. 1 iron, find the number of tons of No. 1 sold.

**Multiplying a whole number by a mixed number.**

### Written Work

1. Find the cost of $48\frac{3}{4}$ gallons of molasses @ \$.88.

$$\begin{array}{rl} \$.88 & \\ \underline{48\frac{3}{4}} & = 40 + 8 + \frac{3}{4} \\ .66 & = \frac{3}{4} \text{ of } .88 \\ 7.04 & = 8 \times .88 \\ \underline{35.20} & \underline{= 40 \times .88} \\ \$42.90 & = 48\frac{3}{4} \times .88 \end{array} \qquad\qquad \begin{array}{r} \$.88 \\ \underline{48\frac{3}{4}} \\ 66 \\ 704 \\ \underline{352} \\ \$42.90 \end{array}$$

Find the cost of:

**2.** $27\frac{3}{8}$ tons of hay at \$12 a ton.

**3.** $31\frac{3}{4}$ yards of cloth at \$.24 a yard.

**4.** $10\frac{1}{4}$ ounces of gold at \$18.75 an ounce.

**5.** $196\frac{9}{10}$ ounces of silver at \$.46 an ounce.

**6.** $97\frac{3}{4}$ bushels of apples at \$.70 a bushel.

**7.** $68\frac{5}{8}$ bushels of berries at \$2.65 a bushel.

**8.** $147\frac{1}{2}$ bushels of potatoes at \$.85 a bushel.

**9.** $84\frac{1}{4}$ pounds of prunes at \$.12 a pound.

**10.** $257\frac{3}{4}$ feet of curbing at \$.38 a foot.

**11.** A street-car conductor collects on an average \$3.60 per hour. How much does he collect in $11\frac{1}{2}$ hours?

**12.** In 4 days a carpenter worked $7\frac{1}{2}$ hours, 8 hours, $7\frac{1}{2}$ hours, and $6\frac{1}{2}$ hours. How much did the work cost at 45¢ per hour?

**13.** A ditch costs \$.27 a rod. Find the cost of $227\frac{1}{2}$ rods.

**14.** The "Pennsylvania Special" averages 58 miles per hour. How far does it travel in $16\frac{2}{3}$ hours?

**15.** A roller in a steel mill rolls $118\frac{3}{4}$ tons, $221\frac{1}{2}$ tons, $193\frac{1}{4}$ tons, and $180\frac{7}{8}$ tons in four days. If he is paid \$.12 per ton, how much should his pay envelope contain for the four days' work?

**16.** A car load of corn contains $821\frac{3}{5}$ bushels. How much is it worth at \$.57 a bushel?

**17.** Three men in a day cut respectively $2\frac{1}{8}$ cords, $1\frac{7}{8}$ cords, and $2\frac{3}{4}$ cords of wood. How much does each receive for his work at \$1.25 per cord?

**18.** At \$26 per ton, how much will the rails for 18 miles of railroad cost, if it takes $158\frac{2}{5}$ tons per mile?

**19.** Two men work respectively $23\frac{1}{4}$ days and $27\frac{3}{4}$ days in a month. How much more does one earn than the other, if each receives \$2.25 per day?

**Multiplying a mixed number by an integer.**

In the problem $9 \times 3\frac{2}{3}$, which number is the multiplier? the multiplicand? The multiplier usually *precedes* the sign $\times$.

Give products:

1. $9 \times 9\frac{1}{9}$
2. $6 \times \frac{2}{3}$
3. $8 \times 5\frac{1}{3}$
4. $7 \times 8\frac{3}{4}$
5. $9 \times 8\frac{7}{9}$
6. $10 \times 10\frac{3}{10}$
7. $15 \times 3\frac{1}{5}$
8. $14 \times 3\frac{3}{7}$
9. $11 \times 5\frac{2}{3}$
10. $6 \times 8\frac{8}{9}$
11. $18 \times 3\frac{2}{3}$
12. $20 \times 4\frac{4}{5}$
13. $16 \times 3\frac{3}{4}$
14. $9 \times 11\frac{1}{9}$
15. $11 \times 7\frac{3}{7}$
16. $25 \times 4\frac{3}{5}$
17. $13 \times 3\frac{1}{3}$
18. $12 \times 15\frac{5}{6}$
19. $15 \times 7\frac{2}{3}$
20. $8 \times 9\frac{7}{8}$
21. $9 \times 5\frac{5}{18}$

## Written Work

1. Multiply $14\frac{3}{8}$ by 9.

$$\begin{array}{l} 14\frac{3}{8} \\ \underline{\quad 9} \\ \quad 3\frac{3}{8} = 9 \times \frac{3}{8} \\ \underline{126 \quad = 9 \times 14} \\ 129\frac{3}{8} = 9 \times 14\frac{3}{8} \end{array}$$

$9 \times \frac{3}{8} = \frac{27}{8} = 3\frac{3}{8}$.
$9 \times 14 = 126$. $3\frac{3}{8} + 126 = 129\frac{3}{8}$.

Multiply:

2. $12\frac{2}{7}$ by 28
3. $11\frac{7}{9}$ by 45
4. $14\frac{5}{12}$ by 60
5. $25\frac{17}{25}$ by 75
6. $85\frac{39}{51}$ by 68
7. $64\frac{27}{42}$ by 56
8. $44\frac{17}{21}$ by 14
9. $78\frac{4}{7}$ by 63
10. $89\frac{4}{15}$ by 105
11. $715\frac{3}{5}$ by 45
12. $101\frac{4}{9}$ by 63
13. $205\frac{3}{5}$ by 75
14. $47\frac{3}{7}$ by 96
15. $609\frac{3}{7}$ by 21
16. $105\frac{4}{7}$ by 49
17. $290\frac{3}{10}$ by 45
18. $213\frac{3}{4}$ by 78
19. $735\frac{2}{3}$ by 21

Find products:

| | | |
|---|---|---|
| **20.** $213 \times 609\frac{3}{7}$ | **25.** $596 \times 56\frac{3}{4}$ | **30.** $379 \times 49\frac{7}{10}$ |
| **21.** $612 \times 48\frac{3}{5}$ | **26.** $972 \times 32\frac{5}{9}$ | **31.** $49 \times 465\frac{25}{49}$ |
| **22.** $842 \times 95\frac{7}{8}$ | **27.** $96 \times 325\frac{2}{9}$ | **32.** $786 \times 49\frac{7}{8}$ |
| **23.** $728 \times 34\frac{3}{10}$ | **28.** $856 \times 98\frac{2}{3}$ | **33.** $9872 \times 36\frac{7}{12}$ |
| **24.** $96 \times 207\frac{3}{5}$ | **29.** $54 \times 657\frac{4}{9}$ | **34.** $4398 \times 94\frac{3}{5}$ |

**35.** There are $16\frac{1}{2}$ ft. in a rod. How many feet are there in 12 rods?

**36.** Find the cost of 35 tons of railroad iron at $\$38\frac{3}{4}$ a ton.

**37.** At $\$1\frac{2}{5}$ each, how much will 40 "General Histories" cost?

**38.** If I sell 7 apples for 10 cents, how much shall I receive for 7 dozen apples?

**39.** When oranges are sold at 3 for 10 cents, how much will 3 crates carrying 180 each cost?

**40.** Stephenson's locomotive weighed $4\frac{3}{4}$ tons. Find the weight of a modern freight mogul weighing 40 times as much.

**41.** A mail boy averaged $13\frac{5}{9}$¢ per hour for 117 hours worked during the month. Find his earnings for the month.

**42.** A residence is lighted by 21 incandescent lights. The cost of each light per day is $1\frac{3}{7}$¢. Find the total electric light bill for a year of 365 days.

**43.** The average shipment of ore from a mine per week of 6 days is 347 tons. If $\$12\frac{7}{10}$ is the average profit realized from each ton of ore, find the net profit for 40 weeks.

**44.** A mail carrier's deliveries average $23\frac{7}{8}$ pounds. How many pounds of mail does he deliver in 2 trips each day for 312 days?

**45.** In a certain shoe factory a pair of shoes is finished every $3\frac{3}{8}$ minutes. Find the number of days of 10 hours each required to make 4600 pairs.

**46.** An establishment consumed in one year 8450 pounds of twine at $9\frac{1}{2}$¢ per pound; $36\frac{1}{2}$ dozen bottles of ink at \$1.90 per dozen; 6 gross pens at 90¢ per gross; 500 pads of paper at $\frac{3}{4}$¢ per pad. Find the total cost of the purchase.

**47.** A real estate agent sold 6 pieces of land containing respectively: $10\frac{3}{8}$ A., $12\frac{1}{2}$ A., $18\frac{3}{4}$ A., $26\frac{5}{16}$ A., $30\frac{2}{5}$ A., $3\frac{3}{4}$ A., at \$250 per acre. Find the amount of the sale.

**48.** A town has a population of 3600. If the average amount of water used by each person is $6\frac{3}{4}$ gallons per day, find the number of gallons of water used in 90 days.

**49.** If a motorman receives \$$\frac{1}{4}$ per hour, how much does he earn in 7 weeks of 6 days each, working 10 hours per day?

**Finding fractional parts of a fraction.**

**1.** What is $\frac{1}{3}$ of 9 hours? $\frac{2}{3}$ of 9 feet?

**2.** What is $\frac{1}{3}$ of 9 tenths? $\frac{2}{3}$ of 9 tenths?

**3.** $\frac{1}{3}$ of 9 tenths means that we are to take $\frac{1}{3}$ of 9 parts of a unit that has been divided into 10 equal parts.

**4.** $\frac{1}{3}$ of $\frac{9}{10}$ = how many tenths?

**5.** If $\frac{1}{3}$ of $\frac{9}{10} = \frac{3}{10}$, how much is $\frac{2}{3}$ of $\frac{9}{10}$?

Find mentally:

| | | |
|---|---|---|
| **6.** $\frac{1}{3}$ of $\frac{3}{4}$ | **11.** $\frac{7}{9}$ of $\frac{27}{35}$ | **16.** $\frac{2}{3}$ of $\frac{9}{16}$ |
| **7.** $\frac{2}{3}$ of $\frac{3}{10}$ | **12.** $\frac{11}{12}$ of $\frac{24}{33}$ | **17.** $\frac{5}{6}$ of $\frac{3}{4}$ |
| **8.** $\frac{5}{8}$ of $\frac{16}{21}$ | **13.** $\frac{9}{10}$ of $\frac{40}{43}$ | **18.** $\frac{11}{12}$ of $\frac{3}{4}$ |
| **9.** $\frac{3}{5}$ of $\frac{15}{16}$ | **14.** $\frac{4}{15}$ of $\frac{45}{53}$ | **19.** $\frac{9}{10}$ of $\frac{1}{6}$ |
| **10.** $\frac{6}{7}$ of $\frac{21}{25}$ | **15.** $\frac{3}{8}$ of $\frac{24}{35}$ | **20.** $\frac{5}{8}$ of $\frac{3}{10}$ |

**21.** What does the expression $\frac{9}{10}$ mean?

**22.** Can $\frac{9}{10}$ be separated into 3 equal parts; thus, $\frac{3}{10}$, $\frac{3}{10}$, $\frac{3}{10}$, without changing the size of the fractional unit?

**23.** To what fractional unit must we change $\frac{3}{5}$ before we can separate it into 4 equal parts; that is, take $\frac{1}{4}$ of $\frac{3}{5}$?

**24.** Is $\frac{12}{20}$ the same in value as $\frac{3}{5}$?

**25.** When $\frac{12}{20}$ is separated into 4 equal parts, how many fractional units are there in each part?

**26.** Then $\frac{1}{4}$ of $\frac{3}{5}$ = how many twentieths?

**27.** Observe that $\frac{1}{4}$ of $\frac{3}{5} = \frac{1 \times 3}{4 \times 5}$, or $\frac{3}{20}$.

**28.** What is $\frac{3}{4}$ of $\frac{3}{5}$? If $\frac{1}{4}$ of $\frac{3}{5} = \frac{3}{20}$, $\frac{3}{4}$ of $\frac{3}{5} = 3 \times \frac{3}{20}$, or $\frac{9}{20}$.

**29.** Observe that $\frac{3}{4}$ of $\frac{3}{5} = \frac{3 \times 3}{4 \times 5}$, or $\frac{9}{20}$.

A fractional part of a fraction equals *a fraction times a fraction.*

## Written Work

**1.** Find $\frac{7}{8} \times \frac{16}{21}$.

$$\frac{7}{8} \times \frac{16}{21} = \frac{7 \times 16}{8 \times 21} = \frac{112}{168} = \frac{2}{3}$$

To find $\frac{7}{8} \times \frac{16}{21}$ means to find $\frac{7}{8}$ of $\frac{16}{21}$. $\frac{1}{8}$ of $\frac{16}{21} = \frac{2}{21}$ and $\frac{7}{8}$ of $\frac{16}{21} = 7 \times \frac{2}{21} = \frac{14}{21}$, or $\frac{2}{3}$.

Or,

$$\frac{\cancel{7}}{\cancel{8}} \times \frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{21}}} = \frac{2}{3}$$

*A* **fractional part of a fraction** *is found by multiplying the numerators for the numerator of the product and the denominators for the denominator of the product.*

Indicate the operation, and cancel when possible.

A fractional part of a fraction is sometimes called a **compound fraction**. Thus, $\frac{1}{2}$ of $\frac{1}{6}$ is a compound fraction.

An integer may be expressed in fractional form, thus, —

$$8 = \frac{8}{1}. \qquad \frac{3}{4} \text{ of } 8 = \frac{3}{\cancel{4}} \text{ of } \frac{\overset{2}{\cancel{8}}}{1} = \frac{6}{1} = 6.$$

Find products:

| | | | |
|---|---|---|---|
| 2. $\frac{3}{8} \times \frac{16}{27}$ | 6. $\frac{25}{49} \times \frac{14}{45}$ | 10. $\frac{35}{58} \times \frac{87}{98}$ | 14. $\frac{36}{41} \times \frac{123}{148}$ |
| 3. $\frac{5}{19} \times \frac{38}{45}$ | 7. $\frac{21}{44} \times \frac{33}{56}$ | 11. $\frac{27}{62} \times \frac{155}{162}$ | 15. $\frac{38}{43} \times \frac{129}{130}$ |
| 4. $\frac{15}{28} \times \frac{14}{25}$ | 8. $\frac{18}{26} \times \frac{65}{72}$ | 12. $\frac{13}{68} \times \frac{34}{39}$ | 16. $\frac{343}{512} \times \frac{192}{245}$ |
| 5. $\frac{51}{64} \times \frac{8}{17}$ | 9. $\frac{11}{69} \times \frac{46}{121}$ | 13. $\frac{71}{74} \times \frac{37}{38}$ | 17. $\frac{111}{220} \times \frac{110}{444}$ |

18. Find $\frac{3}{4}$ of $12\frac{5}{8}$.

NOTE. — Change the mixed number to an improper fraction.

Find:

| | | |
|---|---|---|
| 19. $\frac{2}{3}$ of $7\frac{1}{2}$ | 23. $3\frac{3}{4} \times \frac{4}{5}$ | 27. $2\frac{3}{4} \times 7\frac{5}{8}$ |
| 20. $\frac{5}{6}$ of $12\frac{3}{4}$ | 24. $7\frac{3}{8} \times \frac{8}{9}$ | 28. $8\frac{1}{4} \times 5\frac{1}{2}$ |
| 21. $\frac{8}{9}$ of $13\frac{5}{7}$ | 25. $5\frac{5}{12} \times \frac{15}{16}$ | 29. $7\frac{8}{9} \times 3\frac{1}{4}$ |
| 22. $\frac{7}{12}$ of $10\frac{4}{5}$ | 26. $8\frac{3}{5} \times \frac{5}{8}$ | 30. $4\frac{7}{12} \times 3\frac{2}{3}$ |

Find the value of:

31. $4\frac{3}{8} \times 6\frac{6}{7}$

$$4\frac{3}{8} \times 6\frac{6}{7} = \frac{\overset{5}{\cancel{35}}}{\cancel{8}} \times \frac{\overset{6}{\cancel{48}}}{\cancel{7}} = \frac{30}{1}, \text{ or } 30.$$

| | |
|---|---|
| 32. $6\frac{3}{5} \times 2\frac{3}{11} \times 7\frac{8}{9}$ | 38. $35\frac{1}{16} \times 22\frac{1}{32} \times 12$ |
| 33. $8\frac{7}{10} \times 2\frac{7}{9} \times 2\frac{2}{3}$ | 39. $51\frac{3}{7} \times 19\frac{9}{15} \times 12\frac{1}{8}$ |
| 34. $10\frac{1}{2} \times 1\frac{1}{3} \times 7\frac{3}{4}$ | 40. $6\frac{3}{7} \times 5\frac{4}{5} \times \frac{7}{8} \times 2\frac{3}{10}$ |
| 35. $14\frac{1}{7} \times 17\frac{8}{9} \times 8\frac{2}{5}$ | 41. $20\frac{2}{9} \times 20\frac{4}{7} \times 20\frac{11}{16} \times 10$ |
| 36. $27\frac{15}{16} \times 42\frac{2}{3} \times 6\frac{1}{4}$ | 42. $7\frac{31}{32} \times 15 \times 2\frac{16}{55} \times 12\frac{9}{17}$ |
| 37. $2\frac{1}{2} \times 3\frac{3}{4} \times 4\frac{4}{5} \times 5\frac{5}{6}$ | 43. $172\frac{1}{2} \times 2\frac{7}{25} \times 3\frac{7}{9}$ |

**44.** The schedule of a train between two cities is $12\frac{3}{4}$ hours, and the train's speed is $35\frac{11}{30}$ miles per hour. Find the distance between the places.

**45.** Find the number of tons of sugar cane on $19\frac{5}{8}$ acres, if the average number of tons per acre is $11\frac{2}{3}$ tons.

**46.** An automobile's average rate of speed is $25\frac{3}{16}$ miles per hour. How far does it travel in $3\frac{3}{5}$ hours?

**47.** Estimate the value for one season of a Vermont sugar camp at $\$\frac{1}{10}$ per pound if $6\frac{3}{5}$ pounds are obtained on an average from each of 1275 maple trees.

## DIVISION OF FRACTIONS

### Dividing a fraction by an integer.

**1.** $15 \div 5$ means that 15 is to be separated into 5 equal parts of 3 units each; and one of the equal parts taken; thus, $15 \div 5 = 3$. Explain in the same way what $18 \div 9$ means.

**2.** $\frac{10}{15} \div 5$ means that $\frac{10}{15}$ is to be separated into 5 equal parts of $\frac{2}{15}$ each, and one of the equal parts taken; thus, $\frac{10}{15} \div 5 = \frac{2}{15}$. Which term of the fraction was divided by the integer?

*A fraction may be divided by an integer by dividing the numerator of the fraction by the integer.*

Divide:

| | | |
|---|---|---|
| **3.** $\frac{8}{9}$ by 4 | **5.** $\frac{15}{26} \div 5$ | **7.** $\frac{24}{25}$ by 8 |
| **4.** $\frac{12}{13}$ by 6 | **6.** $\frac{36}{37} \div 9$ | **8.** $\frac{42}{47}$ by 7 |

**9.** In $\frac{2}{3} \div 5$, can you separate $\frac{2}{3}$ into 5 equal parts in which the fractional unit is thirds?

**10.** Can you divide $\frac{2}{3}$ by 5 by dividing the numerator by the integer an integral number of times?

**11.** $\frac{2}{3} \div 5$ means that $\frac{1}{5}$ of $\frac{2}{3}$ is to be taken. Since $\frac{1}{5}$ of $\frac{2}{3} = \frac{2}{15}$, then, $\frac{2}{3} \div 5 = \frac{?}{15}$.

**12.** $\frac{2}{3} = \frac{?}{15}$. Can $\frac{10}{15}$ be separated into 5 equal parts of $\frac{2}{15}$ each? Then $\frac{2}{3} \div 5 = \frac{2}{15}$. Observe that multiplying the denominator of $\frac{2}{3}$ by 5 changes the fractional unit from thirds to fifteenths and takes one of the 5 equal parts into which $\frac{2}{3}$ has been changed.

*A fraction may be divided by an integer by multiplying the denominator by the integer.*

Solve the following problems by the more convenient method and give your reasons:

| | | |
|---|---|---|
| **13.** $\frac{35}{50} \div 7$ | **16.** $\frac{8}{15} \div 6$ | **19.** $\frac{6}{10} \div 25$ |
| **14.** $\frac{3}{4} \div 8$ | **17.** $\frac{96}{108} \div 9$ | **20.** $\frac{105}{125} \div 15$ |
| **15.** $\frac{25}{40} \div 10$ | **18.** $\frac{6}{100} \div 3$ | **21.** $\frac{35}{49} \div 7$ |

## Written Work

**1.** Divide $\frac{24}{25}$ by 8.

*a.* $\frac{24 \div 8}{25} = \frac{3}{25}$

In *a* the division is performed by dividing the *numerator* by 8.

*b.* $\frac{24}{25 \times 8} = \frac{24}{200} = \frac{3}{25}$

In *b* the division is performed by multiplying the *denominator* by 8, and changing the fraction to its lowest terms.

*c.* $\frac{\overset{3}{\cancel{24}}}{25 \times \cancel{8}} = \frac{3}{25}$

In *c* the division is indicated and the quotient is found by cancellation.

**A fraction may be divided by an integer** *either by dividing the numerator or by multiplying the denominator of the fraction by the integer.*

Find quotients:

| | | |
|---|---|---|
| **2.** $\frac{3}{4} \div 5$ | **11.** $\frac{21}{65} \div 8$ | **20.** $\frac{27}{28} \div 18$ |
| **3.** $\frac{7}{8} \div 9$ | **12.** $\frac{3}{31} \div 9$ | **21.** $\frac{72}{83} \div 30$ |
| **4.** $\frac{15}{16} \div 7$ | **13.** $\frac{8}{21} \div 24$ | **22.** $\frac{96}{113} \div 64$ |
| **5.** $\frac{21}{30} \div 18$ | **14.** $\frac{12}{13} \div 16$ | **23.** $\frac{57}{61} \div 38$ |
| **6.** $\frac{11}{13} \div 5$ | **15.** $\frac{18}{19} \div 27$ | **24.** $\frac{132}{163} \div 48$ |
| **7.** $\frac{80}{120} \div 25$ | **16.** $\frac{16}{17} \div 40$ | **25.** $\frac{117}{127} \div 52$ |
| **8.** $\frac{16}{23} \div 7$ | **17.** $\frac{36}{37} \div 12$ | **26.** $\frac{111}{112} \div 37$ |
| **9.** $\frac{24}{29} \div 8$ | **18.** $\frac{11}{18} \div 12$ | **27.** $\frac{256}{275} \div 80$ |
| **10.** $\frac{9}{17} \div 5$ | **19.** $\frac{35}{43} \div 42$ | **28.** $\frac{180}{197} \div 50$ |

**29.** Divide $3\frac{1}{8}$ by 5.

$$3\tfrac{1}{8} = \tfrac{25}{8}. \quad \frac{25}{8} \div 5 = \frac{\overset{5}{\cancel{25}}}{8 \times \cancel{5}} = \frac{5}{8}$$

NOTE.—Small mixed numbers are frequently reduced to improper fractions and then divided by the same principle as proper fractions.

| | | |
|---|---|---|
| **30.** $2\frac{4}{7} \div 6$ | **36.** $28\frac{4}{7} \div 25$ | **42.** $10\frac{2}{13} \div 24$ |
| **31.** $4\frac{1}{5} \div 7$ | **37.** $28\frac{1}{8} \div 18$ | **43.** $29\frac{2}{5} \div 42$ |
| **32.** $4\frac{8}{11} \div 13$ | **38.** $32\frac{8}{11} \div 21$ | **44.** $22\frac{3}{11} \div 58$ |
| **33.** $2\frac{22}{25} \div 48$ | **39.** $21\frac{7}{8} \div 25$ | **45.** $30\frac{2}{3} \div 23$ |
| **34.** $5\frac{13}{25} \div 23$ | **40.** $18\frac{7}{9} \div 26$ | **46.** $19\frac{1}{2} \div 26$ |
| **35.** $17\frac{7}{9} \div 8$ | **41.** $19\frac{1}{11} \div 45$ | **47.** $21\frac{7}{9} \div 56$ |

**48.** Divide $1286\frac{6}{7}$ by 9.

$$\begin{array}{r} 9\overline{)1286\frac{6}{7}} \\ \hline 142\frac{62}{63} \end{array}$$

NOTE.—The division may be performed by changing both numbers to sevenths, but it is a much shorter process to perform the work as indicated, changing the remaining mixed number, $8\frac{6}{7}$, to an improper fraction, $\frac{62}{7}$, and dividing it by 9. Thus, $\frac{1}{9}$ of $\frac{62}{7} = \frac{62}{63}$.

49. $25681\frac{2}{11} \div 7$
50. $9863\frac{7}{16} \div 6$
51. $6532\frac{11}{12} \div 8$
52. $6879\frac{9}{10} \div 5$
53. $36370\frac{5}{8} \div 11$
54. $8002\frac{7}{13} \div 12$
55. $4428\frac{13}{15} \div 12$
56. $10935\frac{3}{5} \div 4$
57. $9720\frac{2}{3} \div 11$
58. $7090\frac{9}{10} \div 9$

**Dividing fractions by first making them similar.**

**1.** What is the unit of measure in each of the following:

8; 6 yd.; 5 ft.; 4 in.; 8 rd.; 5 oz.; 2 lb.; 10 mi.?

**2.** What kind of units may be added or subtracted?

**3.** Since both the dividend and the divisor of concrete numbers must represent like units, to divide 24 yd. by 3 ft. we must either change yards to feet, thus, 72 ft. ÷ 3 ft.; or feet to yards, thus, 24 yd. ÷ 1 yd.

**4.** In $\frac{6}{8} \div \frac{2}{8}$ are the fractional units alike in *size* and *kind?* Then how often is $\frac{2}{8}$ contained in $\frac{6}{8}$?

**5.** Since $\frac{2}{8}$ is contained 3 times in $\frac{6}{8}$, how can we divide one fraction by another when the denominators are alike?

***A*** **fraction may be divided by another fraction** *by changing both fractions to a common denominator and dividing the numerators.*

**6.** Divide $\frac{3}{4}$ by $\frac{5}{6}$.

$$\frac{3}{4} = \frac{9}{12}; \quad \frac{5}{6} = \frac{10}{12}$$
$$\frac{9}{12} \div \frac{10}{12} = 9 \div 10 = \frac{9}{10}$$

Find quotients:

| | | |
|---|---|---|
| **7.** $\frac{5}{9} \div \frac{5}{6}$ | **11.** $\frac{7}{10} \div \frac{7}{8}$ | **15.** $\frac{23}{25} \div \frac{4}{5}$ |
| **8.** $\frac{5}{12} \div \frac{2}{3}$ | **12.** $\frac{5}{9} \div \frac{8}{11}$ | **16.** $\frac{11}{16} \div \frac{7}{8}$ |
| **9.** $\frac{8}{9} \div \frac{3}{5}$ | **13.** $\frac{5}{16} \div \frac{1}{3}$ | **17.** $\frac{5}{18} \div \frac{2}{9}$ |
| **10.** $\frac{13}{15} \div \frac{3}{4}$ | **14.** $\frac{5}{8} \div \frac{4}{7}$ | **18.** $\frac{18}{25} \div \frac{2}{5}$ |

**By inverting the terms of the divisor and multiplying.**

**1.** How many half inches are there in 1 in.? in 2 in.? 3 in.? 4 in. Then how many times is $\frac{1}{2}$ contained in 1? in 2? in 3? in 4?

**2.** If $1 \div \frac{1}{4} = 4$, what does $2 \div \frac{1}{4}$ equal? $3 \div \frac{1}{4}$? $12 \div \frac{1}{4}$?

**3.** What does $1 \div \frac{1}{5}$ equal? $3 \div \frac{1}{5}$? $6 \div \frac{1}{5}$?

*To divide any number by a fractional unit multiply the number by the denominator;* thus, $12 \div \frac{1}{4} = 12 \times \frac{4}{1} = \frac{48}{1}$, or 48.

| | | |
|---|---|---|
| **4.** $12 \div \frac{1}{4} = ?$ | **7.** $10 \div \frac{1}{12} = ?$ | **10.** $20 \div \frac{1}{9} = ?$ |
| **5.** $15 \div \frac{1}{2} = ?$ | **8.** $9 \div \frac{1}{12} = ?$ | **11.** $25 \div \frac{1}{4} = ?$ |
| **6.** $16 \div \frac{1}{3} = ?$ | **9.** $9 \div \frac{1}{15} = ?$ | **12.** $18 \div \frac{1}{3} = ?$ |

| | |
|---|---|
| **13.** $1 \div \frac{2}{4}$ = how many? | **17.** $1 \div \frac{5}{10}$ = how many? |
| **14.** $1 \div \frac{3}{6}$ = how many? | **18.** $1 \div \frac{4}{16}$ = how many? |
| **15.** $1 \div \frac{7}{14}$ = how many? | **19.** $1 \div \frac{2}{3}$ = how many? |
| **16.** $1 \div \frac{4}{8}$ = how many? | **20.** $1 \div \frac{3}{4}$ = how many? |

Observe that the number of times each of the above fractions is contained in 1 equals the number of times the numerator is contained in the denominator.

The number of times a fraction is contained in 1 is called the **reciprocal** of the fraction. Thus, $\frac{3}{4}$ is contained in 1, $\frac{4}{3}$ times. Hence, $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$.

1 divided by any fraction equals the fraction **inverted.**

Find quotients:

| | | | |
|---|---|---|---|
| **21.** $8 \div \frac{2}{3}$ | **23.** $4 \div \frac{3}{8}$ | **25.** $10 \div \frac{2}{5}$ | **27.** $25 \div \frac{5}{6}$ |
| **22.** $6 \div \frac{3}{4}$ | **24.** $15 \div \frac{3}{5}$ | **26.** $12 \div \frac{6}{7}$ | **28.** $14 \div \frac{2}{3}$ |

**29.** $\frac{2}{3} \div \frac{3}{4}$ = how many?

$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$ Since $1 \div \frac{3}{4} = \frac{4}{3}$, $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3}$ of $\frac{4}{3}$, or $\frac{8}{9}$.

*Any number is divided by a fraction by inverting the terms of the divisor and multiplying.*

**30.** Work $\frac{18}{25} \div \frac{3}{5}$ by both methods. What principle does the *second* method introduce that the *first* does not?

NOTE. — The process should be shortened by *cancellation* when possible.

### Written Work

Change the mixed numbers to improper fractions before dividing. Divide:

| | | | | | |
|---|---|---|---|---|---|
| **1.** | 18 by $\frac{6}{7}$ | **20.** | $\frac{13}{18} \div \frac{26}{27}$ | **39.** | $\frac{119}{138} \div \frac{7}{115}$ |
| **2.** | 12 by $\frac{4}{5}$ | **21.** | $\frac{5}{22} \div \frac{3}{8}$ | **40.** | $\frac{85}{87} \div \frac{51}{145}$ |
| **3.** | 25 by $\frac{8}{9}$ | **22.** | $\frac{27}{32} \div \frac{9}{16}$ | **41.** | $\frac{39}{64} \div \frac{13}{24}$ |
| **4.** | 32 by $\frac{12}{17}$ | **23.** | $\frac{25}{36} \div \frac{15}{16}$ | **42.** | $22\frac{3}{4} \div 2\frac{5}{8}$ |
| **5.** | 40 by $\frac{25}{36}$ | **24.** | $\frac{28}{39} \div \frac{42}{65}$ | **43.** | $45\frac{2}{3} \div 5\frac{12}{25}$ |
| **6.** | 48 by $\frac{15}{16}$ | **25.** | $\frac{65}{72} \div \frac{5}{8}$ | **44.** | $6\frac{3}{16} \div 8\frac{5}{8}$ |
| **7.** | $172 \div \frac{8}{9}$ | **26.** | $\frac{133}{144} \div \frac{95}{192}$ | **45.** | $128 \div 17\frac{9}{32}$ |
| **8.** | $235 \div \frac{4}{5}$ | **27.** | $\frac{99}{125} \div \frac{209}{250}$ | **46.** | 160 by $\frac{5}{8}$ of $\frac{3}{4}$ |
| **9.** | $770 \div \frac{11}{15}$ | **28.** | $\frac{81}{112} \div \frac{45}{72}$ | **47.** | $\frac{5}{8}$ of 640 by $6\frac{2}{13}$ |
| **10.** | $882 \div \frac{3}{7}$ | **29.** | $\frac{49}{108} \div \frac{91}{132}$ | **48.** | 198 by $12\frac{3}{8}$ |
| **11.** | $1035 \div \frac{5}{6}$ | **30.** | $\frac{243}{256} \div \frac{189}{320}$ | **49.** | $3\frac{3}{4} \div 4\frac{1}{6}$ |
| **12.** | $984 \div \frac{3}{8}$ | **31.** | $\frac{196}{225} \div \frac{56}{375}$ | **50.** | $10\frac{1}{8} \div 6\frac{3}{4}$ |
| **13.** | $\frac{3}{4} \div \frac{2}{3}$ | **32.** | $\frac{185}{224} \div \frac{111}{896}$ | **51.** | $10\frac{1}{2} \div 3\frac{1}{5}$ |
| **14.** | $\frac{2}{3} \div \frac{3}{4}$ | **33.** | $\frac{175}{282} \div \frac{125}{375}$ | **52.** | $(\frac{3}{4}$ of $\frac{8}{9}) \div (\frac{1}{2}$ of $\frac{4}{9})$ |
| **15.** | $\frac{7}{8} \div \frac{3}{4}$ | **34.** | $\frac{230}{231} \div \frac{2}{33}$ | **53.** | $(\frac{2}{7}$ of $\frac{7}{8}) \div (\frac{3}{4}$ of $\frac{8}{9})$ |
| **16.** | $\frac{9}{10} \div \frac{3}{5}$ | **35.** | $\frac{143}{170} \div \frac{65}{136}$ | **54.** | $(\frac{5}{9}$ of $\frac{12}{15}) \div (\frac{6}{27}$ of $\frac{33}{50}$ of $\frac{22}{27})$ |
| **17.** | $\frac{13}{14} \div \frac{6}{7}$ | **36.** | $\frac{97}{114} \div \frac{3}{76}$ | | |
| **18.** | $\frac{7}{12} \div \frac{2}{3}$ | **37.** | $\frac{83}{84} \div \frac{5}{56}$ | **55.** | $(\frac{22}{25}$ of $\frac{25}{42}) \div \frac{18}{25}$ |
| **19.** | $\frac{15}{16} \div \frac{11}{24}$ | **38.** | $\frac{55}{78} \div \frac{22}{195}$ | **56.** | $(\frac{11}{19}$ of $\frac{38}{44}) \div (\frac{7}{9}$ of $\frac{36}{21})$ |

## Miscellaneous

**1.** At $\$1\frac{1}{2}$ per day how long will it take a laborer to earn $\$67\frac{1}{2}$?

**2.** Divide $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{8}$ by $\frac{5}{6}$ of $\frac{4}{7}$ of $\frac{14}{25}$.

**3.** A man who owned $\frac{7}{11}$ of an estate sold $\frac{2}{5}$ of his share. What part of the estate did he then own?

**4.** A man who owned $\frac{3}{4}$ of a store sold $\frac{3}{8}$ of his share for $1406.25. What was the value of the store? What part had he left?

**5.** One man has $\$35\frac{1}{4}$; another has $\$62\frac{1}{2}$. If each gives to the other $\frac{2}{5}$ of what he has, how much more will the one then have than the other?

**6.** If $\frac{3}{4}$ of 10 bushels of oats cost $\$3\frac{3}{5}$, how much will $20\frac{5}{8}$ bushels cost?

**7.** What number must be multiplied by $\frac{8}{9}$ of $3\frac{3}{4}$ to give a product of $32\frac{4}{19}$?

**8.** A storm moves eastward at the rate of $18\frac{1}{3}$ miles per hour. In how many hours after the storm is first observed in Chicago should it be due in Pittsburg, 468 miles east?

**9.** Kerosene oil weighs $6\frac{2}{5}$ pounds to the gallon. Find the number of gallons in a tank car of oil weighing 39200 pounds.

**10.** A steam threshing machine averages $1\frac{3}{4}$ bushels of wheat per minute. How many minutes at the same rate will it take to thresh $358\frac{5}{8}$ bushels?

**11.** The average cost of the education per pupil in a certain city is $\$5\frac{1}{3}$. If the total cost is $16480, find the number of pupils attending school.

**12.** A contractor employs 6 men for $27\frac{1}{2}$ days and pays them $363. Find the average daily wages.

## COMPLEX FRACTIONS

**A complex fraction** is a fraction which has a fraction or a mixed number in either or both of its terms.

Thus, $\frac{\frac{1}{2}}{3}$, $\frac{4}{\frac{3}{8}}$, $\frac{\frac{2}{3}}{\frac{4}{5}}$, $\frac{3\frac{1}{2}}{4\frac{1}{4}}$ are complex fractions.

Such examples are simplified by the principles of division of fractions.

Thus, $\frac{\frac{1}{2}}{3} = \frac{1}{2} \div 3 = \frac{1}{6}$. They rarely occur except in advanced courses of study.

### Written Work

1. Find the quotient of $2\frac{1}{4}$ divided by $\frac{5}{6}$.

$$\frac{2\frac{1}{4}}{\frac{5}{6}} = \frac{9}{4} \div \frac{5}{6} = \frac{9}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{5} = \frac{27}{10} = 2\tfrac{7}{10}$$

Simplify:

2. $\dfrac{\frac{7}{9}}{7\frac{1}{3}}$

3. $\dfrac{\frac{8}{15}}{12}$

4. $\dfrac{\frac{2}{3} \text{ of } \frac{9}{15}}{\frac{5}{8} \text{ of } \frac{18}{25}}$

5. $\dfrac{\frac{4}{9} + 2\frac{1}{3}}{5\frac{5}{9}}$

6. $\dfrac{(\frac{5}{12} - \frac{1}{3}) \times (\frac{5}{6} + \frac{3}{4})}{\frac{1}{8} \times \frac{1}{2}}$

7. $\dfrac{\frac{3}{5} \text{ of } 2\frac{1}{5}}{1\frac{5}{7}}$

8. $\dfrac{\frac{1}{2}}{\frac{3}{5}} \times \dfrac{\frac{7}{8}}{\frac{5}{9}}$

9. $\dfrac{\frac{5}{8} - \frac{3}{5}}{\frac{3}{10} - \frac{1}{4}}$

10. $\dfrac{3\frac{1}{5} \times 4\frac{1}{3}}{2\frac{1}{2} \div 4\frac{1}{2}}$

11. $\dfrac{(\frac{3}{4} \text{ of } 8\frac{1}{2}) - \frac{1}{4}}{\frac{7}{8} - (\frac{1}{4} \times \frac{1}{5})}$

12. $\dfrac{\frac{3}{4} \times 6\frac{2}{3}}{3\frac{1}{3} \div \frac{3}{8}}$

13. $\frac{3}{4} \text{ of } \dfrac{4\frac{1}{2}}{6}$

14. $\frac{9}{10} \text{ of } \dfrac{4}{3\frac{1}{2}}$

15. $\dfrac{8 - (\frac{1}{4} \text{ of } \frac{2}{3})}{\frac{3}{4} \div (6\frac{1}{2} - 3\frac{1}{4})}$

16. $\dfrac{(2\frac{2}{5} \times 3\frac{3}{4}) \div 1\frac{3}{5}}{1\frac{1}{4} \div (\frac{5}{13} \times 10\frac{5}{6})}$

## FRACTIONAL RELATIONS

**Finding what part one number is of another.**

**1.** What part of 12 is 4? What part of 18 is 5? Express the answers also in the form of division. Thus, $\frac{4}{12} = 4 \div 12$; $\frac{5}{18} = 5 \div 18$.

**2.** What part of $\frac{5}{8}$ is $\frac{3}{8}$? of $\frac{4}{9}$ is $\frac{2}{9}$? How, then, can we find what part one number is of another?

*Divide the smaller number by the larger number.*

### Written Work

**1.** What part of 208 is 96?

$$96 \div 208 = \frac{96 \div 16}{208 \div 16} = \frac{6}{13}$$

We divide the smaller number 96 by 208, and reduce the resulting fraction $\frac{96}{208}$ to its lowest terms, $\frac{6}{13}$.

**2.** What part of $\frac{7}{8}$ is $\frac{3}{16}$?

$$\frac{3}{16} \div \frac{7}{8} = \frac{3}{\underset{2}{\cancel{16}}} \times \frac{\cancel{8}}{7} = \frac{3}{14}$$

We divide $\frac{3}{16}$ by $\frac{7}{8}$ by inverting the divisor of $\frac{7}{8}$ and multiplying. The result shows that $\frac{3}{16}$ is $\frac{3}{14}$ of $\frac{7}{8}$.

What part of

| | | |
|---|---|---|
| **3.** 90 is 16? | **6.** $\frac{8}{9}$ is $\frac{2}{3}$? | **9.** 4 is $3\frac{1}{5}$? |
| **4.** 120 is 50? | **7.** $\frac{7}{8}$ is $\frac{5}{32}$? | **10.** $13\frac{4}{5}$ is $1\frac{3}{5}$? |
| **5.** 200 is 18? | **8.** $\frac{11}{12}$ is $\frac{13}{48}$? | **11.** $12\frac{3}{8}$ is $1\frac{3}{8}$? |

**12.** 250 pupils belong to a school, and only 128 are present. What part of the whole number is present?

**13.** One year the population of a city was 220,000, and the next year 250,000. What fraction of the second year's population was the first year's?

**14.** The product of two numbers is 29,160, and one of the numbers is 27. What part of the other number is 27?

**Finding a number when a fractional part of it is given.**

**1.** If $\frac{3}{4}$ of a number is 30, what is the number?

If *three* fourths of the number is 30, *one* fourth of the number is one third of 30, or 10, and *four* fourths of the number, or the whole number, is $4 \times 10$, or 40. Hence, 30 is $\frac{3}{4}$ of 40.

## Written Work

**1.** 360 is $\frac{6}{17}$ of what number?

$\frac{1}{17}$ of the number $= \frac{1}{6}$ of 360, or 60
$\frac{17}{17}$ of the number $= 17 \times 60$, or 1020

Since 360 is $\frac{6}{17}$ of a number, $\frac{1}{17}$ of it is $\frac{1}{6}$ of 360, or 60, and $\frac{17}{17}$ of it is $\frac{17}{6}$ of 360, or 1020.

**2.** If $\frac{7}{8}$ is $\frac{3}{4}$ of a number, what is the number?

$\frac{1}{4}$ of the number $= \frac{1}{3}$ of $\frac{7}{8}$, or $\frac{7}{24}$,
$\frac{4}{4}$ of the number $= 4 \times \frac{7}{24} = \frac{28}{24}$, or $1\frac{1}{6}$

Since $\frac{3}{4}$ of the number is $\frac{7}{8}$, $\frac{1}{4}$ of it is $\frac{1}{3}$ of $\frac{7}{8}$, or $\frac{7}{24}$, and $\frac{4}{4}$ of the number is $4 \times \frac{7}{24}$, which equals $\frac{28}{24}$, or $1\frac{1}{6}$.

Find the number of which:

| | | | |
|---|---|---|---|
| **3.** 84 is $\frac{7}{8}$ | **5.** $3\frac{1}{2}$ is $\frac{7}{9}$ | **7.** $\frac{7}{9}$ is $\frac{7}{16}$ | **9.** $\frac{11}{12}$ is $\frac{22}{23}$ |
| **4.** 196 is $\frac{4}{11}$ | **6.** $12\frac{2}{3}$ is $\frac{19}{20}$ | **8.** $\frac{12}{13}$ is $\frac{6}{7}$ | **10.** $5\frac{3}{6}$ is $\frac{11}{14}$ |

**11.** If $\frac{9}{13}$ of a man's salary is \$900, what is his salary?

**12.** A man lost in speculation \$2700, which was $\frac{3}{16}$ of his entire fortune. What was his fortune?

**13.** If $\frac{11}{12}$ of the number of books in a library is 9922, how many books are there in the library?

**14.** There are 30,205 women in a certain town, which is $\frac{7}{15}$ of the number of men. How many men are there?

**15.** An author spent \$2100 for a piece of property, which was $\frac{7}{15}$ of what he was paid for a novel. How much did he receive for the novel?

## REVIEW OF FRACTIONS

**1.** $\frac{5}{6}$ of 30 is $\frac{5}{7}$ of what number?

**2.** If 6 is added to both terms of the fraction $\frac{3}{4}$, will the value be increased or diminished, and how much?

**3.** Change $\frac{160}{184}$ to its lowest terms.

**4.** The sum of two numbers is $43\frac{11}{12}$. One of the numbers is $18\frac{5}{6}$. What is the other?

**5.** If $5\frac{1}{2}$ tons coal cost \$28.27, find the cost of $12\frac{3}{5}$ tons.

**6.** If $2\frac{3}{4}$ acres of land cost \$110, how much will $12\frac{5}{8}$ acres cost at the same rate?

**7.** The dividend is 165, and the quotient is $6\frac{7}{8}$. What is the divisor?

**8.** Five tubs of butter contain, respectively, $27\frac{1}{2}$ lb., $30\frac{1}{4}$ lb., $24\frac{1}{16}$ lb., $32\frac{7}{8}$ lb., and $34\frac{3}{4}$ lb. How many pounds are there in the five tubs?

**9.** Multiply the sum of $\frac{5}{6}$ and $\frac{3}{4}$ by their difference.

**10.** If 10 men can build a wall in 35 days, how long will it take 25 men to do the work?

**11.** There are $272\frac{1}{4}$ square feet in 1 square rod. How many square rods are there in 43560 square feet?

**12.** If a man travels $2\frac{1}{2}$ miles in $\frac{4}{5}$ of an hour, at the same rate how far could he travel in $2\frac{1}{2}$ hours?

**13.** A man owning $\frac{2}{15}$ of a mine sold his interest for \$48700. Find the value of the mine at that rate.

**14.** Find the difference between $2\frac{1}{4} \times 3\frac{3}{5}$ and $2\frac{1}{4} \div 3\frac{3}{5}$.

**15.** A merchant owned $\frac{3}{4}$ of a store, and sold $\frac{3}{5}$ of his share for \$5760. Find the value of the whole store at that rate.

**16.** How much will 18000 stamped envelopes cost at $\$21\frac{1}{4}$ per thousand?

**17.** When oysters yield $1\frac{1}{4}$ gallons to the bushel, how many bushels will be required to fill a 10-gallon tub?

**18.** One buyer offered $\frac{6}{5}$ of the cost of a property, another $\frac{9}{8}$ of the cost. The difference in their offers was \$186. Find the cost of the property.

**19.** Find the cost of 18 yards of cloth if $3\frac{3}{4}$ yards cost \$9.

**20.** A clothier paid \$180 for 12 suits of clothing, and sold them at $\$19\frac{3}{4}$ a suit. How much did he gain?

**21.** E, who owns $\frac{2}{3}$ of a factory, sells $\frac{5}{6}$ of his share for \$3560. What is the value of the factory?

**22.** What is the distance from Pittsburg to Philadelphia if $\frac{3}{4}$ of the distance is $265\frac{1}{2}$ miles?

**23.** The product of two fractions is $\frac{5}{8}$; one is $\frac{11}{15}$, what is the other?

**24.** If C's wages are $\$3\frac{1}{2}$ a day, and his daily expenses $\$1\frac{3}{4}$, how many days must he labor to save \$28?

**25.** A traveler walked $25\frac{1}{2}$ miles the first day, $15\frac{5}{6}$ miles the second day, $19\frac{3}{4}$ miles the third day, $20\frac{3}{5}$ miles the fourth day, and $22\frac{7}{8}$ miles the fifth day. How far did he travel in the 5 days, and what was the average rate per day?

**26.** A field is $40\frac{2}{3}$ rods long, which is $\frac{4}{3}$ of its width. What is its width and what is the distance around the field?

**27.** Find the cost of $10\frac{1}{2}$ cords of wood at \$3 a cord, and $8\frac{3}{4}$ cords at \$4 a cord.

**28.** If $12\frac{1}{2}$ bushels of apples cost \$5, how much will $15\frac{3}{4}$ bushels cost?

**29.** A farmer raised 225 bushels of potatoes. He sold $\frac{2}{5}$ of them to one merchant, and $\frac{1}{5}$ of the remainder to another. Find the number of bushels he had left.

**30.** Reduce $\frac{15}{16}$ to a fraction whose denominator is 320.

**31.** Find the value of a mill if $\frac{3}{8}$ of $\frac{4}{5}$ of it is worth \$3750.

**32.** If $\frac{1}{2}$ the trees in an orchard are peach trees, $\frac{1}{4}$ apple trees, $\frac{1}{8}$ cherry trees, and the remaining 21, plum trees, how many trees are there in the orchard?

**33.** What number is it whose $\frac{4}{5}$ exceeds its $\frac{2}{3}$ by 40?

**34.** Divide into 5 equal parts the product of the sum and difference of $1\frac{1}{4}$ and $1\frac{1}{2}$.

**35.** A real estate dealer sold some lots for \$12360, gaining $\frac{1}{5}$ of the cost. If he had sold them for \$10500, would he have gained or lost, and how much?

**36.** A piece of land was sold at \$90 an acre, which was a gain of $\frac{2}{7}$ of the cost. How much did the land cost per acre?

**37.** My board and room cost \$32 per month. What does each cost if $\frac{2}{3}$ of the cost of the room equals $\frac{2}{5}$ of the cost of the board?

**38.** A father left \$30000 to his two children, giving the daughter $\frac{7}{8}$ as much as the son. What was the share of each?

**39.** The owner of $\frac{2}{3}$ of a mill sold $\frac{2}{5}$ of his share for \$4800. How much at this rate would a man who owns $\frac{1}{3}$ of the mill get for $\frac{3}{20}$ of his share?

**40.** One fourth of a certain number minus $3\frac{1}{8}$ equals $\frac{9}{40}$. What is the number?

**41.** A carpenter, working $9\frac{1}{2}$ hours a day, built a shed in 16 days. How many hours a day, at the same rate, must he work to build it in 18 days?

**42.** A merchant sold 10 dozen hats of a certain kind at \$2.75 each, and a number of dozens of another kind at \$2 each, receiving \$474 for all. How many dozen did he sell at \$2 each?

**43.** A man engaging in trade lost $\frac{3}{8}$ of the money invested, then gained \$495, after which he had \$2551. How much was his capital at first?

**44.** Simplify $\dfrac{3\frac{1}{8}}{\frac{3}{5}\text{ of }\frac{5}{9}} + (\frac{5}{16} \div \frac{5}{8})$.

**45.** A hardware merchant bought a bill of hardware at auction for $\frac{11}{12}$ of its value, and retailed it for $\frac{9}{8}$ of its value. If his gain was \$48.75, how much did he pay for it?

**46.** A boy bought lemons at the rate of 4 for 5 cents, and sold them at the rate of 3 for 5 cents. If he made \$6 in 2 weeks of 6 days each, what were his daily average sales?

**47.** A path leading to the top of a hill has a rise of 9 inches in 90 feet. What is the elevation of the hill in feet, if the path is $\frac{3}{4}$ of a mile long?

**48.** A teacher taught $8\frac{1}{2}$ months, and after spending $\frac{2}{5}$ of his salary for board had left \$204. How much did he earn per month?

**49.** Divide $\frac{3}{4}$ of 9 by $\frac{4}{5}$ of $8\frac{3}{8}$.

**50.** Two men, A and B, each bought farms, A's farm costing $2\frac{5}{8}$ times as much as B's. Find the cost of each, if both cost \$58,000.

**51.** Divide $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of $3\frac{5}{9}$ by $\frac{3}{4}$ of $\frac{5}{12}$ of $8\frac{8}{27}$.

**52.** A ship is worth \$120000, and the owner of $\frac{3}{4}$ of it sells $\frac{1}{4}$ of his share. Find the value of the part he has remaining.

**53.** Find the value of $(18\frac{3}{4} \div \frac{5}{6}) - (20\frac{1}{4} \div 1\frac{1}{8})$.

**54.** I purchased 160 acres of land at \$60 an acre, and sold $\frac{5}{8}$ of it at \$70 an acre. Find the number of acres I had left, and my gain on the number of acres sold.

**55.** A real estate agent bought land for \$7200, and sold it so as to gain $\frac{1}{10}$ of the cost. If the gain was \$6 per acre, how many acres did he buy?

**56.** A man paid $\frac{2}{5}$ of his indebtedness the first year, $\frac{2}{5}$ of the remainder the second year, $\frac{2}{5}$ of what then remained the third year, when he found that he still owed \$1296. Find the amount he owed at first.

**57.** If a man can do a piece of work in $6\frac{2}{3}$ days by working 9 hours a day, how long will it take him to do it, working 8 hours a day?

**58.** Mr. Williams has $\frac{1}{3}$ of his money in government bonds, $\frac{3}{5}$ in the bank, and \$520 in cash. How much is he worth?

**59.** A real estate agent bought 2 houses; his income from the one was \$480, which was $\frac{3}{4}$ of the income from the other. How much was his income from both houses?

**60.** Four brothers agreed to pay the mortgage on their father's farm. The first payment made was $\frac{1}{4}$ of the mortgage, the second $\frac{1}{2}$ of the remainder, and the third $\frac{3}{5}$ as much as the first and second. If the difference between the first and third payments was \$300, how much was the mortgage?

**61.** A merchant bought 250 barrels flour at \$4.80 per barrel. He sold 10 barrels which were damaged for $\frac{3}{4}$ of the cost. On the sale of 150 barrels he gained 15 cents per barrel. If his total gain was \$28.50, at what price did he sell the remaining number of barrels?

**62.** A merchant buys 36 pairs men's shoes, at $\$2\frac{1}{4}$ per pair; 24 pairs women's shoes, at $\$1\frac{3}{4}$ per pair; and 30 pairs slippers, at $\$\frac{3}{4}$ per pair. What is the amount of his bill?

**63.** If the men's shoes in example 62 are sold for \$3 per pair, the women's shoes for $\$2\frac{1}{2}$ per pair, and the slippers for $\$1\frac{1}{4}$ per pair, how much does the merchant make?

**64.** A cubic foot of water weighs $62\frac{1}{2}$ pounds. A barrel contains about $4\frac{1}{5}$ cubic feet. Find the weight of a barrel of water.

**65.** A student, in a half year at college, spent his money as follows: tuition, \$50; books, \$$9\frac{3}{4}$; 18 weeks' boarding, at \$$2\frac{3}{4}$ per week; incidental expenses, \$$41\frac{4}{5}$. What were his expenses for the half year?

**66.** $\frac{9}{17}$ of a barrel of oil, containing 51 gallons, was sold at 13 cents per gallon; the remainder of the barrel at 14 cents a gallon. If the oil cost $11\frac{7}{8}$ cents per gallon, what was the gain?

**67.** A merchant bought a stock of goods for \$6000. He sold $\frac{1}{2}$ of it at a gain of $\frac{1}{3}$ of the cost, $\frac{1}{4}$ of it at a gain of $\frac{1}{4}$ of the cost, and the remainder at a loss of $\frac{1}{5}$ of the cost. How much did he gain or lose?

**68.** A hall is $21\frac{2}{3}$ feet long and $10\frac{1}{4}$ feet wide. At $8\frac{1}{2}$ cents per foot, how much will it cost to put a molding around this hall?

**69.** Find the perimeters of each of five rooms, the dimensions being as follows: $22\frac{1}{3}$ ft. × 16 ft., $14\frac{3}{4}$ ft. × $15\frac{1}{2}$ ft., $12\frac{2}{3}$ ft. × 17 ft., $18\frac{1}{4}$ ft. × $15\frac{2}{3}$ ft., $30\frac{1}{2}$ ft. × 24 ft.

**70.** From a certain number $18\frac{7}{9} + 27\frac{5}{12}$ was subtracted, leaving a remainder of $9\frac{5}{18}$. What was the number?

**71.** From New York to Chicago by the Baltimore and Ohio railroad the distance is 1052 miles. From Chicago to El Paso by the Santa Fé railroad it is 1630 miles, and from El Paso to Mexico City by the Mexican Central railroad it is $1224\frac{1}{10}$ miles. What is the distance from New York to Mexico City, and how long would such a journey take, traveling $31\frac{3}{5}$ miles per hour?

**72.** A freight train runs from Kansas City to St. Louis, 288 miles, traveling 16 miles per hour. How long does it take it to make the trip? How long would it take a passenger train, running $\frac{9}{4}$ as fast, to make the same trip?

## PROBLEMS FOR ANALYSIS

Pupils should be taught:

(1) To express themselves accurately and rapidly; (2) to do this work without the aid of pencil and paper; (3) to give clear analytic statements in the solution of a problem.

1. The coal for the school buildings in a certain town cost \$94.50. How many tons were purchased at \$3 per ton?

2. A shoe dealer buys shoes at \$21 per dozen pairs and retails them at \$2.50 a pair. How much does he gain on each pair?

3. A laborer earns \$18 in 12 days. At that rate how much can he earn in 80 days?

4. What is $\frac{3}{4}$ of $\frac{5}{9}$ of 24?

5. A lady buys 10 yards of ribbon and uses $6\frac{2}{3}$ yards. What part of the ribbon has she left?

6. If eggs are bought at the rate of 4 dozen for \$1.08 and sold at 30 cents per dozen, what will be the gain on 3 dozen?

7. A man buys a farm of 102 acres and divides it into lots of 6 to the acre. How many lots are there if $\frac{1}{6}$ of the farm is laid out in streets?

8. How many sheep, at \$5 per head, may be purchased with the money from the sale of 10 head of cattle at \$42 per head?

9. A contractor buys 8000 bricks at \$12.50 per thousand. Find the amount of his bill.

**10.** If I pay 6 cents for the use of \$1 for one year, how much should I pay for the use of \$150 for 2 years?

**11.** If I buy goods at \$1.37 per yard and sell them at \$1.50 per yard, how much do I gain on 12 yards?

**12.** A boy earns 75 cents per day and pays $\frac{3}{5}$ of his wages for board. At that rate how much can he save in 26 days?

**13.** My father pays \$252 per year rent. How much is that per month?

**14.** A lady purchased 2 handkerchiefs at 35 cents each; 6 yards of ribbon at 12 cents per yard; 6 yards of cloth at \$1.12 per yard. How much change should she receive from a ten-dollar bill?

**15.** A man buys a farm for \$2000 and pays $\frac{7}{8}$ of the cost, giving his note for the balance. For how much does he give his note?

**16.** A farmer buys fertilizer at \$28 per ton and retails it at \$1.90 a hundred pounds. How much does he gain on 35 tons?

**17.** At 16 cents per pound, how many pounds of steak does a woman get if the amount of the purchase is 83 cents?

**18.** A huckster bought $6\frac{1}{2}$ pounds of butter at 16 cents per pound, and $6\frac{1}{4}$ dozen eggs at 18 cents per dozen. How much did he pay for both?

**19.** A huckster buys chickens at $8\frac{1}{3}$ cents per pound and sells them at $12\frac{1}{2}$ cents per pound. How many pounds must he purchase and sell in order to gain \$25?

SOLUTION. — On each pound he gains $4\frac{1}{6}$ cents.
To gain \$1 he must sell 24 pounds.
To gain \$25 he must sell 25 times 24 pounds, or 600 pounds.

**20.** A street car conductor earns $23\frac{1}{2}$ cents per hour. How much does he earn in 10 hours?

**21.** If the conductor averages 10 hours per day, how much will he earn in 30 days?

**22.** If a box of 40 dozen oranges is purchased for \$5 and retailed at 40 cents a dozen, what is the entire gain?

**23.** What fraction of a gallon is a pint? a gill? a quart?

**24.** A man sold $\frac{2}{3}$ of his farm for \$1800. At the same rate, how much would he receive for the whole farm?

SOLUTION. — $\frac{2}{3}$ of the amount received for the farm = \$1800.
$\frac{1}{3}$ of the amount received for the farm = $\frac{1}{2}$ of \$1800, or \$900.
$\frac{3}{3}$, or the amount received for the farm = 3 times \$900, or \$2700.

**25.** In a certain school there are 40 pupils in the grammar grade, which are $\frac{8}{9}$ of the number in the intermediate grade. How many pupils are there in both grades?

**26.** A man sold a piano for \$360, which was $\frac{4}{5}$ of what it cost him. How much did it cost him?

**27.** There are 112 cubic feet in $\frac{7}{8}$ of a cord of wood. How many cubic feet are there in 1 cord?

**28.** A farmer sold $\frac{3}{5}$ of his farm for \$1521. At that rate, what was the value of $\frac{2}{3}$ of the farm?

**29.** There are 198 cubic inches in $\frac{6}{7}$ of a gallon. How many cubic inches are there in 2 gallons?

**30.** If $\frac{4}{5}$ of a ton of hay costs \$16, how much will $2\frac{1}{2}$ tons cost?

**31.** A lady paid 84 cents for $\frac{3}{4}$ of a yard of silk. At the same rate, how much would she pay for $\frac{1}{2}$ yard?

**32.** If $\frac{5}{7}$ of an acre produces 160 bushels of potatoes, how many bushels will an acre produce?

**33.** A man saves \$220, which is $\frac{2}{7}$ of his yearly salary. What is his yearly salary?

**34.** If it takes 4 men 9 days to do a piece of work, how long will it take 6 men, at the same rate, to do the work?

**35.** If $\frac{3}{4}$ of a farm is valued at \$1240, what is the value of $\frac{1}{5}$ of the farm?

**36.** A lady spends $\frac{3}{5}$ of her income for board, and $\frac{1}{3}$ of the remainder for clothes and travel. If she saves \$160 per year, what is her income?

**37.** A farmer has 600 bushels of wheat, which is $\frac{2}{5}$ of $\frac{3}{4}$ of what his neighbor has. How many bushels has his neighbor?

**38.** A house was sold for \$1800, which was $\frac{3}{5}$ of its cost. What was the loss?

**39.** If 20 men can dig a ditch in 25 days, how long will it take 5 men?

**40.** A farmer sold 36 head of cattle, which was 6 more than $\frac{3}{7}$ of all he owned. How many had he remaining?

**41.** If a man works $\frac{3}{4}$ of a day and receives \$1.50, how much should he receive for $\frac{4}{5}$ of a day?

**42.** I purchased an overcoat for \$45, and found I had $\frac{4}{9}$ of my money left. How much had I at first?

**43.** James and John have together \$40, and John has seven times as much money as James. How much does each have?

SOLUTION. — Once James's money = James's money.
7 times James's money = John's money.
8 times James's money = amount of both, or \$40.
Once James's money = \$5.
7 times James's money = \$35, or John's money.

**44.** There are two numbers whose sum is 40; one number is $\frac{1}{7}$ of the other. What are the numbers?

**45.** A man invests $\frac{1}{4}$ of his money in a mill, $\frac{3}{5}$ in a farm, and the remainder, which is \$900, he deposits in a bank. How much is he worth?

**46.** A town has 3000 people. The number of pupils in the school is $\frac{1}{4}$ of the remaining population. How many pupils are there in the school?

**47.** In a farm of 500 acres the woodland is $\frac{2}{3}$ of the cleared land. How many acres are there of each?

**48.** A miller has 360 bushels of wheat. $\frac{2}{3}$ of the number of bushels of wheat equals $\frac{3}{4}$ of the number of bushels of corn. How much is the corn worth at 50 cents per bushel?

**49.** If $\frac{2}{3}$ of a clerk's salary for the year is \$800, how much is $\frac{3}{5}$ of his salary?

**50.** $\frac{3}{7}$ of 21 is $\frac{3}{10}$ of what number?

**51.** $\frac{9}{5}$ of 20 is $\frac{4}{13}$ of what number?

**52.** $\frac{2}{3}$ of $\frac{3}{4}$ of a number is 16. What is the number?

**53.** $\frac{1}{2}$ of C's farm equals $\frac{1}{3}$ of D's, and C has 100 acres. How many acres has D?

**54.** There are two numbers: $\frac{2}{3}$ of the first is $\frac{3}{4}$ of the second. The first number is 18. What is the second?

**55.** A and B agree to do a piece of work for \$80. If A works 7 days and B 9 days, how much should each receive?

Suggestion. — Together they work 16 da. Hence, A should receive $\frac{7}{16}$ of \$80, and B $\frac{9}{16}$ of \$80.

**56.** Sarah earns \$10 a week, and $\frac{4}{5}$ of what Sarah earns is $\frac{2}{3}$ of what Edna earns. How much does Edna earn?

**57.** $\frac{5}{6}$ of $\frac{3}{10}$ of 160 is $\frac{8}{11}$ of what number?

**58.** If $\frac{3}{4}$ of $\frac{5}{6}$ of a farm costs \$2500, how much does the farm cost?

**59.** $\frac{3}{4}$ of my money is invested in coal land, $\frac{1}{2}$ of the remainder in building lots, and the remainder, which amounts to \$800, is in the bank. How much have I invested in coal lands?

**60.** A man sold a horse for \$120, which was $\frac{1}{3}$ less than the horse cost him. Find the cost.

**61.** If 24 is $\frac{6}{7}$ of a number, 21 is what part of that number?

**62.** I spend $\frac{1}{2}$ of my monthly salary for board and room, $\frac{1}{3}$ of the remainder for clothing, and save the remainder, which is \$20. What is my salary?

**63.** If $\frac{3}{7}$ of a piece of work can be done in 9 days, how long will it take to complete the work after $\frac{3}{7}$ has been done?

**64.** By selling land at \$120 an acre, I gain $\frac{1}{4}$ of the cost. Find the cost.

**65.** If $\frac{3}{4}$ of the value of a farm is \$900, what is the value of $\frac{7}{8}$ of the farm?

**66.** If $4\frac{1}{2}$ tons of coal cost \$27, how much will 3 tons cost?

**67.** What is the difference between $\frac{1}{3}$ of $\frac{1}{2}$ and $\frac{2}{3}$ of 1?

**68.** If I can do a piece of work in 7 days, what part can I do in 1 day? in 3 days? in $\frac{2}{3}$ of a day? in $1\frac{3}{4}$ days?

**69.** A fruit grower sold 200 bushels of apples, which was $\frac{4}{5}$ of his crop. How much did he realize from the sale of his crop at \$1.20 per bushel?

**70.** An heir gets $\frac{8}{9}$ of an estate and invests $\frac{5}{8}$ of his share, and still has \$1600. What is the value of the estate?

**71.** A tank holds 120 gal. and is $\frac{3}{5}$ full. $\frac{2}{3}$ of the quantity is drawn off. How many gallons will it take to fill the tank?

**72.** One man bids $\frac{7}{12}$ of the cost of an article, another man bids $\frac{7}{8}$ of the cost of the article. The difference between their bids is 70 cents. Find the cost of the article.

## DECIMAL FRACTIONS

**1.** What is the largest common fractional unit?

**2.** Show that an integral unit may be divided into any number of fractional units.

**3.** Name the different fractional units from $\frac{1}{2}$ to $\frac{1}{20}$ in order of their size.

**4.** What fractional unit divides the integral unit into 10 equal parts? into 100 equal parts? into 1000 equal parts?

The divisions of an integral unit into 10ths, 100ths, 1000ths, etc., are called **decimal divisions.**

There are three ways by which decimal divisions may be expressed:

(1) by words, as nine tenths;
(2) by common fractions, as $\frac{9}{10}$, $\frac{9}{100}$;
(3) by decimals, as .9, .75.

A **decimal fraction** is any number of 10ths, 100ths, 1000ths, etc. of an integral unit. When expressed after a decimal point and without a written denominator, it is usually called a **decimal.**

A **decimal point** is a period placed after ones' place and before tenths' place.

**5.** What is the *largest* decimal unit? the *second* largest? the *third* largest?

*In any decimal system* $10 \times 1$ *unit in any place* $= 1$ *unit of the next higher place.*

**6.** Show that United States money is a *decimal system.*

## NOTATION AND NUMERATION OF DECIMALS

**1.** Since the first decimal division of an integral unit is tenths, what is the first place to the right of the decimal point?

**2.** What is the second place called? the third place?

**3.** Five tenths is written .5; five hundredths is written .05; five thousandths is written .005, etc. Write 7 tenths, 6 hundredths, 8 thousandths.

**4.** Express decimally $\frac{5}{10}$, $\frac{75}{100}$, $\frac{6}{1000}$, $\frac{54}{1000}$, $\frac{6}{100}$, $\frac{72}{1000}$.

*Every decimal contains as many decimal places as there are naughts in the denominator of the equivalent fraction.*

Table of places and names of *integral* and *fractional* units:

| Ten-millions | Millions | Hundred-thousands | Ten-thousands | Thousands | Hundreds | Tens | Ones | Decimal point | Tenths | Hundredths | Thousandths | Ten-thousandths | Hundred-thousandths | Millionths | Ten-millionths |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 5 | 7 | 8 | 4 | 5 | 5 | . | 0 | 0 | 7 | 4 | 8 | 9 | 8 |

**5.** In .555 what figure stands for tenths? for hundredths? for thousandths? It is read 555 thousandths.

**6.** Is the decimal point named in reading a decimal? Observe that the decimal is read as an integer and that the last figure is given the required denomination.

Read:

| | | |
|---|---|---|
| **7.** .25 | **11.** .101 | **15.** .60745 |
| **8.** .05 | **12.** .0045 | **16.** .678705 |
| **9.** .005 | **13.** .4045 | **17.** .0065 |
| **10.** .375 | **14.** .0002 | **18.** .60005 |

**19.** 50.0745 is read 50 *and* 745 ten-thousandths. How are both the integer and the decimal read? How is the decimal point read? What name is given to the last decimal place?

**20.** When is the decimal point read? When is it not read?

**21.** What determines the value of any figure in a decimal?

**22.** In writing .5, .45, .075, .0075 as common fractions, what figure in each decimal tells us the size of the denominator?

A **mixed decimal** is a whole number and a decimal; as, 4.625.

**23.** What number in a mixed decimal is always read first?

**24.** How do the number of places in any decimal compare with the number of naughts in the denominator when the decimal is expressed as a common fraction?

Read:

| | | | | | |
|---|---|---|---|---|---|
| **25.** | 45.075 | **28.** | 72.003745 | **31.** | .00875 |
| **26.** | 50.3007 | **29.** | 1001.1001 | **32.** | .008090 |
| **27.** | 290.25387 | **30.** | 794.3085 | **33.** | 2.004890 |

**34.** 5 thousandths calls for how many decimal places? What part of the decimal (5 thousandths) stands for the numerator of the fraction? what part of this decimal stands for the denominator?

**35.** Name the numerator and the denominator in the following decimals: .05, .0006, .000025, .045.

*In* **writing a decimal** *write the numerator, and point off from the right as many decimal places as there are naughts in the denominator.*

### Written Work

Write:

1. 34 hundredths.
2. 675 ten-thousandths.
3. 16 and 75 millionths.
4. 400 and 45 thousandths.
5. 6006 and 66 ten-thousandths.
6. 89 and 5 thousandths.
7. Seven hundred forty-six ten-thousandths.
8. Nine hundred and 84 millionths.
9. 5 million 9 and 4 hundred 9 ten-millionths.
10. 5095 millionths.
11. 8 and 17 ten-thousandths.
12. 125 millionths.
13. 896 and 301 hundred-thousandths.
14. One thousand and one thousandth.
15. 18051 and 957 thousandths.
16. 97 and 3 ten-thousandths.
17. 9864 millionths.
18. 2135 and 32 millionths.
19. One and one millionth.
20. One million and one tenth.
21. 90 thousand and 71 thousandths.
22. 1830 and 11684 hundred-thousandths.
23. 429 thousand and 46 ten-thousandths.
24. 7035 and 97 hundredths.
25. 67375 and 35 hundred-thousandths.
26. 5815 hundred-thousandths.
27. 375 and 69 thousandths.

**28.** 419863 and 23456 millionths.

**29.** 81 and 921 hundred-thousandths.

**30.** 2986 and 298643 ten-millionths.

**31.** 3020 and 302 hundred-thousandths.

**32.** 70 and 7 hundredths.

**33.** 8 thousand and 8 thousandths.

**34.** 645 million and 9 millionths.

## COMPARISON OF COMMON FRACTIONS AND DECIMALS

**1.** $.5 = \frac{?}{10}$. $.5 = \frac{?}{100}$. $.5 = \frac{?}{1000}$.

**2.** Then observe that 5 tenths = 50 hundredths = 500 thousandths. $.5 = .50 = .500$.

**3.** 50 hundredths may be written .50, or .500. Does adding naughts to the right of a decimal change the value of the decimal?

**4.** What is the difference in value between \$.5 and \$.50? In writing decimal parts of a dollar, we always write two places for cents even if the last place is a naught.

**5.** Compare in value $\frac{1}{2}$ and $\frac{50}{100}$, $\frac{50}{100}$ and $\frac{500}{1000}$.

**6.** Does canceling the same number of naughts from both numerator and denominator change the value of a fraction?

**7.** Since $.5 = .50 = .500$, does canceling naughts from the right of a decimal change the value of the decimal?

**8.** Observe that canceling naughts from the right of a decimal really means canceling naughts from the numerator and the denominator.

Thus, $.50 = \frac{50}{100} = .5\cancel{0} = \frac{5\cancel{0}}{10\cancel{0}}$.

**9.** .400 is read 400 thousandths. How else may it be read?

**10.** Is the unit in .4 the largest decimal unit in which .400 can be expressed?

Read first as given, then as if the naughts at the right of the decimal were canceled:

| | | |
|---|---|---|
| **11.** .040 | **13.** .7500 | **15.** 10.0057 |
| **12.** 26.0050 | **14.** 8.0090 | **16.** 20.0900 |

**Changing decimals to common fractions.**

Write as common fractions and change to lowest terms:

| | | | |
|---|---|---|---|
| **1.** .25 | **3.** .045 | **5.** .50 | **7.** .0025 |
| **2.** .7 | **4.** .025 | **6.** .75 | **8.** .0775 |

**9.** Give the steps in changing a decimal to its fractional equivalent.

A **complex decimal** is a decimal and a fraction united; thus, $.16\frac{2}{3}$ is read $16\frac{2}{3}$ hundredths.

$$.16\tfrac{2}{3} = \frac{16\frac{2}{3} \times 3}{100 \times 3} = \frac{50}{300} = \frac{1}{6}.$$

**10.** Explain why multiplying both terms of the fraction $\frac{16\frac{2}{3}}{100}$ by 3 does not change the value of the fraction.

Change to common fractions:

| | | | |
|---|---|---|---|
| **11.** $.37\frac{1}{2}$ | **15.** $.83\frac{1}{3}$ | **19.** $.62\frac{1}{2}$ | **23.** $.12\frac{1}{2}$ |
| **12.** $.41\frac{2}{3}$ | **16.** $.06\frac{1}{4}$ | **20.** $.33\frac{1}{3}$ | **24.** $.14\frac{2}{7}$ |
| **13.** $.66\frac{2}{3}$ | **17.** $.04\frac{1}{6}$ | **21.** $.31\frac{1}{4}$ | **25.** $.58\frac{1}{3}$ |
| **14.** $.24\frac{1}{6}$ | **18.** $.08\frac{1}{3}$ | **22.** $.03\frac{1}{3}$ | **26.** $.88\frac{1}{3}$ |

### ADDITION AND SUBTRACTION OF DECIMALS

**1.** How must integers be written before they can be added? subtracted?

**2.** What *change* must be made in $\frac{2}{5}$ and $\frac{3}{4}$ before they can be added or subtracted?

In adding or subtracting decimals, tenths must be placed under tenths, hundredths under hundredths, thousandths under thousandths, etc.

### Written Work

**3.** Add .8085 and .005.

$$\begin{array}{r} .8085 \\ \underline{.005\phantom{0}} \\ .8135 \end{array}$$

**4.** Subtract .005 from .8085.

$$\begin{array}{r} .8085 \\ \underline{.005\phantom{0}} \\ .8035 \end{array}$$

Add as indicated and test:

| | **5.** | | **6.** | | **7.** | | **8.** | | |
|---|---|---|---|---|---|---|---|---|---|
| **9.** | .45 | + | 72.5 | + | 8.557 | + | 87. | = | —— |
| **10.** | 8.07 | + | 5.07 | + | 0.039 | + | 6.5 | = | —— |
| **11.** | 62.093 | + | 89.09 | + | 16.909 | + | 11.9 | = | —— |
| **12.** | 40.937 | + | 20.98 | + | 41.005 | + | 00.5 | = | —— |
| **13.** | | + | | + | | + | | = | —— |

Subtract examples 14 to 17 and add the remainders:

| | **14.** | | **15.** | | **16.** | | **17.** | | |
|---|---|---|---|---|---|---|---|---|---|
| | 40.275 | | 9.0098 | | 219.75 | | 28.7 | | |
| | 39.009 | | 6.7849 | | 8.95 | | 12.5 | | |
| **18.** | | + | | + | | + | | = | —— |

Add:

| **19.** | | **20.** | | **21.** | |
|---|---|---|---|---|---|
| | 3.7 | | .001 | | .65 |
| | 5.06 | | 12.3 | | .001 |
| | 8.023 | | 15.0248 | | 10.1 |
| | 9.04 | | 18.0149 | | 25.004 |

Add:

| **22.** | **23.** | **24.** |
|---|---|---|
| 1.1 | 120.2601 | 36.15 |
| 4.01 | 230.31002 | 9.00939 |
| 1.0101 | .05673 | 128.37 |
| 5.055 | 3.7 | 16.08753 |

| **25.** | **26.** | **27.** |
|---|---|---|
| 166.6 | .27 | 185.057 |
| 7.0425 | .0616 | 127.0348 |
| 28.318 | .010912 | 216.253 |
| 142.0101 | 1.940054 | 456.03456 |

Add:

**28.** 12.015, 26.01102, 126.0592, 134.00876.

**29.** 100.001, 9.99, 149.0492, 7.077.

**30.** 2.2, 28.18, 140.027, 284.0295.

**31.** 318.003, 33.33, 495.0485, 12.0012.

**32.** Find the weight of four silver bars weighing as follows: 15.75 pounds, .125 pounds, 14.3125 pounds, and 16.875 pounds.

**33.** Find the number of acres in four fields containing, respectively, 4.125 acres, .3125 acres, 8.8 acres, and 9.85 acres.

**34.** Find the sum of one hundred twenty-five and seven hundredths, eighty-nine and two hundred thirty-five thousandths, one hundred twenty-seven ten-thousandths, and sixteen and four tenths.

**35.** A farm cost \$4225.50; stock, \$745.25; buildings, \$1825.75; and implements, \$358.45. What was the total cost?

**36.** How many square feet are there in four floors measuring, respectively, $245\frac{1}{2}$ square feet, $278\frac{3}{4}$ square feet, 174.375 square feet, and 168.3125 square feet?

Find differences:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **37.** | .75 | **38.** | .3216 | **39.** | 4.205 | **40.** | 15. |
| | .1825 | | .275 | | 1.7856 | | 5.007 |
| **41.** | 38. | **42.** | \$45.67 | **43.** | $125\frac{1}{2}$ | **44.** | $249\frac{3}{4}$ |
| | 18.276 | | 12.09 | | 87.432 | | 178.625 |
| **45.** | 230.4897 | **46.** | 100.001 | **47.** | 1001.101 | **48.** | 105.55 |
| | 116.5988 | | 99.9 | | 900.909 | | 79.067 |
| **49.** | 1.1 | **50.** | 5.05 | **51.** | 8.25 | **52.** | 1000.00 |
| | .999 | | .6565 | | .0085 | | 999.99 |

**53.** $(9.5-2.25)+(15.28-12.056)+(22.089-19.063)$.

**54.** $(11.001-1.99)+(17.0107-14.014)+(29.3-23.2867)$.

**55.** The difference between two numbers is 1001.101, and the greater is 1101.011. What is the smaller number?

**56.** A is 35.875 years old, B is 48.25 years old, and C's age is 25.5 years less than the age of A and B combined. How old is C?

**57.** To the sum of .808 and 80.8 add their difference.

## MULTIPLICATION OF DECIMALS

**1.** Multiply .05 by .5.

**2.** What is the numerator in .05? in .5?

**3.** What is the denominator in .05? in .5? What shows the denominator in a decimal?

**4.** Multiply the numerators in $.5 \times .05$; thus, $5 \times 5 = 25$.

**5.** Multiply the denominators in $.5 \times .05$; thus, $10 \times 100 = 1000$.

**6.** Write the result as a common fraction; thus, $\frac{25}{1000}$.

**7.** What term of the fraction is expressed by the decimal point?

**8.** $\frac{5}{10} = .5$; $\frac{5}{100} = .05$; $\frac{5}{1000} = .005$; $\frac{5}{10000} = .0005$. Observe that there are as many decimal places in each decimal as there are *naughts* in the denominator of the equivalent common fraction.

Find the product of the following decimals by multiplying the numerators and the denominators, separately, and expressing the result as a decimal; thus,

$$.04 \times .7 = \frac{4}{100} \times \frac{7}{10} = \frac{28}{1000} = .028$$

| | | |
|---|---|---|
| **9.** $.2 \times .06$ | **12.** $.35 \times .05$ | **15.** $.02 \times .42$ |
| **10.** $.04 \times .5$ | **13.** $.07 \times .05$ | **16.** $.06 \times .004$ |
| **11.** $.25 \times .05$ | **14.** $.03 \times .02$ | **17.** $.05 \times .009$ |

*The product of two decimals contains as many decimal places as the sum of the decimal places in both factors.*

### Written Work

**1.** Multiply .25 by 5.

(*a*)

$$\begin{array}{r} .25 \\ 5 \\ \hline 1.25 \end{array}$$

$5 \times 5$ hundredths $= 25$ hundredths or 2 tenths and 5 *hundredths*. Write the 5 in hundredths' place, and carry the 2. $5 \times 2$ tenths $= 10$ tenths; 10 tenths $+ 2$ tenths $= 12$ tenths, or 1 and 2 tenths. Hence, $5 \times .25 = 1.25$.

**2.** Multiply .26 by .12.

(*b*)

$$\begin{array}{r} .26 \\ .12 \\ \hline 52 \\ 26\phantom{0} \\ \hline .0312 \end{array}$$

1. What is the sum of the decimal places in the two factors?

2. The product, then, must contain how many places?

*When the product has not enough decimal places, supply the deficiency by prefixing naughts.*

Find products:

| | | |
|---|---|---|
| **3.** $.25 \times .22$ | **20.** $.1232 \times .961$ | **37.** $.4986 \times .086$ |
| **4.** $.17 \times .28$ | **21.** $.2592 \times 8$ | **38.** $.006 \times 20$ |
| **5.** $.027 \times .03$ | **22.** $65.65 \times .65$ | **39.** $38.2 \times .75$ |
| **6.** $27 \times .12$ | **23.** $75.002 \times 16.04$ | **40.** $.0045 \times .05.$ |
| **7.** $.35 \times 42$ | **24.** $275 \times .007$ | **41.** $9.876 \times .786$ |
| **8.** $8.7 \times 9.22$ | **25.** $.0018 \times 720$ | **42.** $362.9 \times .0076$ |
| **9.** $.085 \times 50$ | **26.** $1500 \times .004$ | **43.** $119.8 \times 2.74$ |
| **10.** $.027 \times 18$ | **27.** $124 \times .064$ | **44.** $20.08 \times .006$ |
| **11.** $1.005 \times .011$ | **28.** $326 \times .096$ | **45.** $.375 \times 2.027$ |
| **12.** $26.8 \times 34$ | **29.** $627 \times .78$ | **46.** $98.64 \times 4.096$ |
| **13.** $28.25 \times 12$ | **30.** $246 \times .3$ | **47.** $.069 \times 8.92$ |
| **14.** $324.6 \times 8\frac{1}{2}$ | **31.** $29.4 \times .08$ | **48.** $6.34 \times 2.34$ |
| **15.** $39.10 \times 18.4$ | **32.** $9.86 \times 3.8$ | **49.** $12.34 \times .004$ |
| **16.** $.0214 \times .016$ | **33.** $39.75 \times .27$ | **50.** $6.08 \times .0001$ |
| **17.** $12.134 \times .0025$ | **34.** $8.708 \times 6.8$ | **51.** $1.002 \times 1.004$ |
| **18.** $15.684 \times 8$ | **35.** $368.9 \times 8.5$ | **52.** $.05 \times .005$ |
| **19.** $.1232 \times 345$ | **36.** $2009 \times .006$ | **53.** $6.876 \times 4.37$ |

**54.** How much must be paid for 85 acres of land at \$45.75 per acre?

**55.** Three brothers divided an estate worth \$9600. The first received .125 of it, the second .375 of it, and the third the remainder. How much did each receive?

**56.** A contractor furnished 2,626,000 bricks at \$7.75 a thousand, and a laborer for 65 days at \$2.75 a day. What was the amount of his bill?

**57.** If there are 39.37 inches in a meter, how many inches are there in 12 meters? how many yards?

**Multiplying by moving the decimal point.**

**1.** Multiply 6.385 by 10; by 100; by 1000.

$10 \times 6.385 = 63.85$

$100 \times 6.385 = 638.5$

$1000 \times 6.385 = 6385$

1. How do you multiply a number by 10?
2. How may you multiply a decimal by 10? by 100? by 1000?
3. How is the value of a number affected by moving the decimal point one place to the right? two places? three places?

*Moving the decimal point one place to the right multiplies the number by* 10; *two places by* 100; *three places by* 1000.

**2.** Multiply 6.1234 by 10; by 100; by 1000.

**3.** Multiply .0342 by 10; by 100; by 1000.

**4.** Multiply 1.3412 by 10; by 100; by 1000.

## DIVISION OF DECIMALS

**Dividing a decimal or a mixed decimal by an integer.**

**1.** Find $\frac{1}{8}$ of 48 hundredths; of 64 hundredths.

**2.** Find $\frac{1}{5}$ of .25; of .35; .45; .75.

**3.** Find $\frac{1}{6}$ of 6 and 36 hundredths; 12 and 24 hundredths.

**4.** Find $\frac{1}{6}$ of 12.36; 24.42; 48.06; 54.06.

Observe that in each problem a decimal or a mixed decimal when divided by an integer is simply separated or *partitioned*.

Give quotients at sight:

| | | | |
|---|---|---|---|
| **5.** $\frac{1}{4}$ of .16 | **9.** $\frac{1}{3}$ of 6.6 | **13.** $6.42 \div 6$ | **17.** $.006 \div 3$ |
| **6.** $\frac{1}{5}$ of .25 | **10.** $\frac{1}{4}$ of 8.08 | **14.** $12.04 \div 4$ | **18.** $.024 \div 6$ |
| **7.** $\frac{1}{2}$ of .08 | **11.** $\frac{1}{5}$ of 10.10 | **15.** $15.05 \div 5$ | **19.** $.008 \div 4$ |
| **8.** $\frac{1}{4}$ of .04 | **12.** $\frac{1}{6}$ of 12.06 | **16.** $24.18 \div 3$ | **20.** $.105 \div 5$ |

## Written Work

1. Divide 20.685 by 15.

```
     1.379
15)20.685
   15.
    5.6
    4.5
    1.18
    1.05
     .135
     .135
```

How many times is 15 contained in 20? in 5.6? in 1.18? in .135?

In practice we divide as in the second example, placing the decimal point directly above the point in the dividend, before beginning to divide, and dividing as in integers.

2. Divide 174.44 by 28.

```
     6.23
28)174.44
   168
     64
     56
      84
      84
```

When the divisor is an integer, division simply separates or partitions the dividend into equal parts. Thus, $\frac{1}{15}$ of 20 *and* 685 thousandths (20.685) = 1 *and* 379 thousandths (1.379).

*A* **decimal** *or a* **mixed decimal** *is divided by an integer by placing a decimal point above or below the decimal point in the dividend, before beginning to divide, and dividing as in the division of integers.*

3. $96.16 \div 8$
4. $849.6 \div 6$
5. $72.84 \div 12$
6. $22.5 \div 15$
7. $80.96 \div 16$
8. $2.5625 \div 25$
9. $.96064 \div 32$
10. $701.05 \div 35$
11. $2.268 \div 27$
12. $2.867 \div 47$
13. $36.54 \div 42$
14. $.666 \div 74$
15. $6.675 \div 89$
16. $12.312 \div 27$
17. $2.25 \div 15$
18. $809.6 \div 16$
19. $256.25 \div 25$
20. $96.064 \div 32$
21. $7010.5 \div 35$
22. $61.472 \div 68$
23. $27.8142 \div 307$
24. $425.92 \div 605$
25. $901.57 \div 97$
26. $.2322 \div 86$
27. $34.356 \div 409$
28. $45.76 \div 650$

## Making the divisor an integer.

1. $6.25 \div 1.25 = 5$
2. $62.5 \div 12.5 = 5$
3. $625. \div 125. = 5$

### Study of Problems

1. What is the first quotient? the second? the third?

2. What was done to the first problem to make the second? to the second to make the third?

3. How is a decimal affected by moving the decimal point one place to the right? two places?

4. How did moving the decimal point to the right the same number of places in both dividend and divisor of each problem affect the quotient?

*Multiplying both dividend and divisor by the same number does not change the quotient.*

### Written Work

Since multiplying both dividend and divisor by the same number does not change the value of the quotient, *make the divisor an integer before beginning to divide.*

**1.** $6.48 \div .4 = 64.8 \div 4.$

$$\begin{array}{r} 4\overline{)64.8} \\ \hline 16.2 \end{array}$$

1. Make the divisor an integer by moving the decimal point one place to the right in both dividend and divisor.

2. Show that this does not affect the quotient.

3. Solve, placing the decimal point directly below the point in the dividend, before beginning to divide.

**2.** $57.6 \div .024 = 57600 \div 24.$

$$\begin{array}{r} 2400. \\ 24\overline{)57600.} \\ \underline{48\phantom{00.}} \\ 96\phantom{0.} \\ \underline{96\phantom{0.}} \\ 000 \end{array}$$

1. Make the divisor an integer by moving the decimal point three places to the right in both dividend and divisor.

2. Solve, placing the decimal point directly above the point in the dividend, before beginning to divide.

*Divide as in integers, placing the decimal point directly above or below the decimal point in the dividend, before beginning to divide.*

**The use of the caret in division of decimals.**

Many teachers prefer to mark off by a *caret* as many decimal places from the right of the decimal point in the dividend as there are decimal places in the divisor, and divide as in integers, placing the decimal point directly below or above the caret in the dividend. Thus,

$$.8)5.68 = .8)\underline{5.6_{\wedge}8}$$
$$7.1$$

It is evident in the above problem that if both the dividend and divisor were changed so as to make the divisor a whole number, the decimal point in the dividend would be in the *place occupied by the caret*, and that the decimal point would be placed in the quotient immediately after the numbers to the left of the caret had been used in the process of division.

The use of the caret determines the *position* of the decimal point in the quotient, and at the same time retains the *identity* of the problem. Thus,

**1.** Divide 96.8 by .004.

$$.004)\underline{96.800_{\wedge}}$$
$$24\,200.$$

**2.** Divide 1.2864 by .032.

$$\begin{array}{r} .032)1.286_{\wedge}4(\underline{40.2} \\ \underline{1\,28\phantom{4}} \\ 64 \\ \underline{64} \end{array}$$

*Mark off by a* caret *the same number of decimal places from the right of the decimal point in the dividend as there are decimal places in the divisor. Divide as in integers, placing the decimal point in the quotient immediately after all the numbers to the left of the caret have been used in the process of division.*

Find quotients:

3. $4.05 \div .27$
4. $.252 \div .14$
5. $8.398 \div 3.8$
6. $2.173 \div 1.06$
7. $144 \div .12$
8. $.144 \div 12$
9. $31.36 \div .056$
10. $.41912 \div .338$
11. $3.125 \div .25$
12. $.3105 \div 15$
13. $.5 \div .625$
14. $6.705 \div .009$
15. $139.195 \div 14.35$
16. $46.5 \div .1875$
17. $.00522 \div .29$
18. $.001705 \div .31$
19. $1000 \div .001$
20. $.2375 \div .095$
21. $177.8028 \div 72.87$
22. $145.908 \div 1.26$
23. $.0656 \div .004$
24. $.1701 \div 63$
25. $85.75 \div .0049$
26. $.025641 \div .777$
27. $.0022 \div 200$
28. $222 \div .002$
29. $.025 \div .00025$
30. $.0003 \div 1.5$
31. $\$1.05 \div \$.005$
32. $.685 \div 500$
33. $.01058 \div 46$
34. $125.625 \div 1.005$

The division is frequently not exact. In such cases the sign + may be placed after the decimal to show that the division is not complete; thus, $1 \div 7 = .142 +$

Find the sum of the quotients:

| 35. | 36. | 37. |
|---|---|---|
| $1 \div .1 =$ | $3 \div .03 =$ | $.18 \div 72 =$ |
| $.1 \div 10 =$ | $30 \div .3 =$ | $.04 \div 50 =$ |
| $.25 \div 50 =$ | $6 \div .006 =$ | $2 \div .025 =$ |
| $2.5 \div .5 =$ | $16 \div .04 =$ | $20 \div .002 =$ |
| $25 \div 2.5 =$ | $60 \div 300 =$ | $200 \div 12.5 =$ |
| $.15 \div .15 =$ | $.6 \div 30 =$ | $64 \div .016 =$ |
| $1.5 \div 15 =$ | $.66 \div 1.1 =$ | $64 \div 160 =$ |
| $.15 \div 2.5 =$ | $.9 \div .009 =$ | $.4 \div 400 =$ |

**38.** Divide \$.10 by \$100.

**39.** If a stone cutter earns \$3.75 a day, how many days will it take him to earn \$311.25?

**40.** If 4275 acres of land cost \$1731.375, what is the price per acre?

**41.** At \$.22 a dozen, how many dozen eggs can be bought for \$19.47?

**42.** If 16 stoves are sold for \$292, what is the average price per stove?

**43.** Divide \$.18 by \$20.

**44.** If the wheel of a bicycle is 9.25 feet around, how many times does it turn in going a mile?

**45.** The product of two numbers is .9375. One of them is .75. What is the other?

**46.** There are $30\frac{1}{4}$ square yards in one square rod. How many square rods are there in a plot containing 559.625 square yards?

**47.** A merchant, in closing out his stock of goods, sold $.37\frac{1}{2}$ of the stock the first month, .35 the second month, and the remainder, \$5500 worth, the third month. What was the value of his stock of goods?

**Changing a common fraction to a decimal.**

## Written Work

Since $\frac{4}{5} = 4 \div 5$, to change a fraction to a decimal, consider it a problem in division of decimals. Thus, $\frac{4}{5} = 4 \div 5 = 5\overline{)4.0}$

$$\phantom{4 \div 5 = 5)}0.8$$

Change to decimals and test:

| | | | | |
|---|---|---|---|---|
| **1.** $\frac{5}{8}$ | **3.** $\frac{5}{12}$ | **5.** $\frac{7}{8}$ | **7.** $\frac{9}{16}$ | **9.** $\frac{7}{16}$ |
| **2.** $\frac{3}{5}$ | **4.** $\frac{5}{16}$ | **6.** $\frac{11}{25}$ | **8.** $\frac{19}{32}$ | **10.** $\frac{17}{25}$ |

When the division does not terminate, the quotient may be shown as a **complex decimal**. Thus, $\frac{3}{7} = 7\overline{)3.000}$ over $0.428\frac{4}{7}$ or as an **incomplete decimal**, $7\overline{)3.000}$ over $0.428+$.

Change to complex or incomplete decimals of not more than four places:

| | | | |
|---|---|---|---|
| **11.** $\frac{7}{11}$ | **15.** $\frac{17}{27}$ | **19.** $\frac{35}{27}$ | **23.** $\frac{1}{60}$ |
| **12.** $\frac{9}{31}$ | **16.** $\frac{9}{19}$ | **20.** $\frac{37}{49}$ | **24.** $\frac{6}{13}$ |
| **13.** $\frac{45}{73}$ | **17.** $\frac{11}{7}$ | **21.** $\frac{7}{13}$ | **25.** $\frac{27}{17}$ |
| **14.** $\frac{29}{53}$ | **18.** $\frac{25}{17}$ | **22.** $\frac{5}{11}$ | **26.** $\frac{18}{24}$ |

## REVIEW OF DECIMALS

1. Add $\frac{3}{8}$, .045, $.12\frac{1}{2}$, $18\frac{3}{4}$, .675.
2. Subtract $\frac{3}{7}$ from .5 of $3\frac{4}{5}$.
3. Multiply $(36.7 - 4\frac{3}{4})$ by 6.7.
4. Subtract $6\frac{1}{7}$ from 11.065.
5. Take $.003\frac{1}{3}$ from 6.
6. Divide $.047\frac{3}{8}$ by $2.3\frac{3}{5}$.

Add as indicated and test:

| | 7. | 8. | 9. | 10. | |
|---|---|---|---|---|---|
| **11.** | 8.375 | + .025 | + $6.24\frac{3}{4}$ | + $.87\frac{1}{2}$ | = —— |
| **12.** | $.05\frac{3}{5}$ | + $6.04\frac{1}{4}$ | + $98.005\frac{4}{5}$ | + $.05\frac{7}{8}$ | = —— |
| **13.** | $.45\frac{5}{16}$ | + $0.42\frac{4}{25}$ | + $8.7\frac{1}{2}$ | + $.95\frac{7}{12}$ | = —— |
| **14.** | 86.55 | + $9.05\frac{3}{10}$ | + $9.87\frac{3}{4}$ | + $.00\frac{3}{5}$ | = —— |
| **15.** | .875 | + 0.75 | + 7.7 | + $.41\frac{2}{3}$ | = —— |
| **16.** | | + | + | + | = —— |

### BUSINESS APPLICATIONS OF DECIMALS

In all business transactions *three* things must be considered :

(1) The **quantity of the commodity** bought or sold.

(2) The **price per unit** at which it is bought or sold.

(3) The **total amount** paid or received for the commodity.

Quantity is measured by *standard* units established by custom or law ; thus, the *pound* is a unit of weight ; the *foot* or the *yard*, a unit of length ; and the *gallon* or the *barrel*, a unit of liquid measurement.

The *price per unit* is the amount of money paid or received for a standard unit of the commodity ; thus, when butter is sold at 25 ¢ per pound, the standard *unit* is the pound and the *price* is 25 cents.

**1.** What standard is used in measuring grain? butter? eggs? milk? cloth? hay? oil?

**2.** What unit is used in measuring values in money? How many cents are there in \$1? in \$$\frac{1}{2}$? \$$\frac{1}{4}$? \$$\frac{3}{4}$? 50 ¢ is what part of \$1? 20 ¢ is what part of \$1? 25 ¢? 10 ¢? 5 ¢?

**Parts of \$1 that Should be Known**

| | |
|---|---|
| 1¢ = $\frac{1}{100}$ of \$1. | 25¢ = $\frac{1}{4}$ of \$1. |
| 2¢ = $\frac{1}{50}$ of \$1. | $33\frac{1}{3}$¢ = $\frac{1}{3}$ of \$1. |
| $2\frac{1}{2}$¢ = $\frac{1}{40}$ of \$1. | $37\frac{1}{2}$¢ = $\frac{3}{8}$ of \$1. |
| 4¢ = $\frac{1}{25}$ of \$1. | 40¢ = $\frac{2}{5}$ of \$1. |
| 5¢ = $\frac{1}{20}$ of \$1. | 50¢ = $\frac{1}{2}$ of \$1. |
| $6\frac{1}{4}$¢ = $\frac{1}{16}$ of \$1. | $62\frac{1}{2}$¢ = $\frac{5}{8}$ of \$1. |
| $8\frac{1}{3}$¢ = $\frac{1}{12}$ of \$1. | $66\frac{2}{3}$¢ = $\frac{2}{3}$ of \$1. |
| 10¢ = $\frac{1}{10}$ of \$1. | 75¢ = $\frac{3}{4}$ of \$1. |
| $12\frac{1}{2}$¢ = $\frac{1}{8}$ of \$1. | 80¢ = $\frac{4}{5}$ of \$1. |
| $16\frac{2}{3}$¢ = $\frac{1}{6}$ of \$1. | $83\frac{1}{3}$¢ = $\frac{5}{6}$ of \$1. |
| 20¢ = $\frac{1}{5}$ of \$1. | $87\frac{1}{2}$¢ = $\frac{7}{8}$ of \$1. |

**Finding the total cost when the quantity purchased is given and the price of a unit is an even part of $1.**

## Written Work

Business computations may be shortened by knowing the relation that the price of a unit bears to $1 or to $100.

**1.** How much will 44 bushels of potatoes cost at $.25 per bushel?

DECIMAL METHOD

$$\begin{array}{rl} \$.25 & = \text{price} \\ \underline{44,} & \text{no. of bushels} \\ 100 & \\ \underline{100} & \\ \$11.00 & = \text{total cost} \end{array}$$

SHORT METHOD

$$\begin{array}{r} 4\overline{)\$44} \\ \$11. \end{array}$$

At $1 each, 44 bu. would cost $44.
At $\$\frac{1}{4}$ each, they cost $\frac{1}{4}$ of $44, or $11.

*Find the cost of the quantity at $1. Divide this by the quantity that can be purchased for $1.*

Find cost of:

**2.** 60 bu. apples at $33\frac{1}{3}$¢ per bushel.

**3.** 25 lb. butter at 25¢ per pound.

**4.** 960 yd. calico at $6\frac{1}{4}$¢ per yard.

**5.** 50 lb. lard at $12\frac{1}{2}$¢ per pound.

**6.** 80 lb. rice at $12\frac{1}{2}$¢ per pound.

**7.** 120 yd. ribbon at $37\frac{1}{2}$¢ per yard.

**8.** 500 books at 40¢ each.

**9.** 1200 doz. eggs at 25¢ per dozen.

**10.** 600 bu. oats at $33\frac{1}{3}$¢ per bushel.

**11.** 1600 bu. coal at $6\frac{1}{4}$¢ per bushel.

**12.** 86 qt. cherries at $6\frac{1}{4}$¢ per quart.

**13.** 2500 bu. corn at 40¢ per bushel.

**14.** 160 lb. beef at 10¢ per pound.

**15.** A merchant buys

240 lb. coffee at $12\frac{1}{2}$¢ per pound,
300 lamp chimneys at $8\frac{1}{3}$¢ each,
6000 qt. milk at $4\frac{1}{6}$¢ per quart,
560 bu. potatoes at 50¢ per bushel.

Find total cost.

**16.** A farmer sells

1260 heads of cabbage at 5¢ per head,
250 bu. potatoes at 50¢ per bushel,
2240 lb. beans at $6\frac{1}{4}$¢ per pound,
600 qt. cherries at $8\frac{1}{3}$¢ per quart,
1200 qt. strawberries at $12\frac{1}{2}$¢ per quart.

Find total receipts.

**17.** A milk dealer bought

300 bu. corn at 50¢ per bushel,
6000 lb. bran at $\frac{1}{2}$¢ per pound,
6000 lb. hay at $1 per hundred pounds.

He sold 4000 gal. milk at $6\frac{1}{4}$¢ per quart. How much did he make?

**Finding the quantity purchased when the total cost is given and the price of a unit is an even part of $1.**

### Written Work

**1.** How many yards of calico, at $6\frac{1}{4}$¢ per yard, can be purchased for $100?

DECIMAL METHOD

$$\begin{array}{r} 1600. \\ \$.0625\overline{)\$100.0000_\wedge} \end{array}$$

SHORT METHOD

100 × 16 yd. = 1600 yd.

Since $6\frac{1}{4}$¢ = $\$\frac{1}{16}$, $1 pays for 16 yd.; and $100 pays for 100 × 16 yd., or 1600 yd.

*Multiply the quantity purchased for $1 by a number equal to the number of dollars invested.*

Find the quantity of each article if a grocer invested

**2.** \$160 in sugar at $6\frac{1}{4}$ ¢ per pound.

**3.** \$120 in sugar at 4 ¢ per pound.

**4.** \$6.00 in rice at 10 ¢ per pound.

**5.** \$100 in cloth at 50 ¢ per yard.

**6.** \$13.00 in gingham at 5 ¢ per yard.

**7.** \$3.00 in cheese cloth at 2 ¢ per yard.

**8.** \$800 in milk at $6\frac{1}{4}$ ¢ per quart.

**9.** \$100 in meat at $12\frac{1}{2}$ ¢ per pound.

**10.** \$4.00 in collars at $12\frac{1}{2}$ ¢ each.

**11.** \$1.00 in bananas at 20 ¢ per dozen (find number of bananas).

**12.** A hotel keeper purchased

\$100 worth of sugar at $6\frac{1}{4}$ ¢ per pound,
\$250 worth of potatoes at $33\frac{1}{3}$ ¢ per bushel,
\$60 worth of soap at $2\frac{1}{2}$ ¢ a cake.

Find number of pounds, bushels, and cakes purchased.

**13.** A farmer sold to a merchant

10 bu. apples at 40 ¢ per bushel,
20 qt. beans at 10 ¢ per quart,
16 bu. potatoes at 75 ¢ per bushel.

He invested $\frac{2}{3}$ of the proceeds in cloth at 25 ¢ per yard and the balance in coffee at $12\frac{1}{2}$ ¢ per pound. How much of each did he purchase?

**14.** A grocer bought coffee at $12\frac{1}{2}$ ¢ a pound, and sold it for \$39, thereby gaining \$6.50. How many pounds did he buy?

## SIMPLE ACCOUNTS FOR BOYS AND GIRLS

An **account** is a statement of the receipts and disbursements of any person.

There are two sides to an account: the *first*, or **debit side**, on which are entered all *receipts;* the *second*, or **credit side**, on which are entered all *disbursements*, or *amounts paid out.*

**Dr.** indicates the *debit* side of an account; **Cr.** indicates the *credit* side.

The **balance** is the difference between the *debit* and *credit* sides.

SEPTEMBER 1, 1906

| | | Dr. | | Cr. | |
|---|---|---|---|---|---|
| Sept. 1 | Cash on hand | $12 | 10 | | |
| Sept. 1 | Note-book, $.15; pencil, $.05 | | | $ | 20 |
| Sept. 4 | Arithmetic | | | | 50 |
| Sept. 5 | Geography | | | 1 | 00 |
| Sept. 7 | Copy-book, $.10; ink and pens, $.08 | | | | 18 |
| Sept. 15 | History | | | 1 | 00 |
| Sept. 17 | Worked one day | 1 | 00 | | |
| Sept. 25 | Car fare | | | | 50 |
| Sept. 29 | Tools | | | 1 | 60 |
| Sept. 30 | Balance, Cash on hand | | | 8 | 12 |
| | | $13 | 10 | $13 | 10 |

Continue the *balance* of each month through the following months to September, 1907.

NOTE TO PARENTS. — Children should be encouraged to keep their own personal accounts.

**1.** *October.* Oct. 3, Bought 1 pair of shoes, \$2.50. 1 hat, \$1.50. Oct. 8, Repairs to bicycle, \$.75. Oct. 15, Earned \$1.50. Oct. 17, Worked for Mr. Black and received \$.75. Oct. 25, Saturday outing, \$.60.

**2.** *November.* Nov. 5, Bought a sled, \$.95. Nov. 9, Bought a cap, \$.75. Nov. 15, Shoveled snow off Mrs. Graham's walk, \$.30. Nov. 17, Sawed kindling wood for Mr. Goff, \$.50. Nov. 26, Bought a knife, \$.25. Nov. 30, Ran errands, \$.35.

**3.** *December.* Dec. 3, Bought 1 pair of skates, \$.75. Dec. 10, Received from Mr. Black for work in store, \$1.00. Dec. 17, Expense for school supplies, \$.17. Dec. 21, Received from Mrs. Williams for carrying in load of coal, \$.30. Dec. 22, Bought Christmas presents, \$3.75. Dec. 25, Christmas gift from Uncle James, \$1.00. Dec. 29, Expense for having skates sharpened, \$.10.

**4.** *January,* 1907. Jan. 5, Received from Mrs. Jones for fixing doorbell, \$.15. Jan. 8, Bought 1 pair mittens, \$.50. Jan. 15, Delivered bills around town for Mr. Black, \$.50. Jan. 25, Bought necktie, \$.25. Jan. 30, Bought "History of French Revolution," \$.75.

**5.** *February.* Feb. 6, Worked on Saturday for Mr. Black, \$.75. Feb. 11, Shoveled snow from sidewalk for Mr. Hart, \$.25. Feb. 16, Ran errands, \$.40. Feb. 20, Helped unload car of feed, \$1.00. Feb. 26, Copied 2 leases for Mr. Irwin, \$.75. Feb. 28, Bought pair of gloves, \$1.25.

**6.** *March.* March 1, Cleaned yard for Mrs. Williams, \$.50. March 6, Bought 2 pairs of socks, \$.30. March 11, Bought new umbrella for mother, \$1.75. March 15, Repaired fence for Mr. Jones, \$.25. March 27, Car fare, \$.30. March 30, Sold my old bicycle for \$5.00.

**7.** *April.* Apr. 1, Burned paper and refuse for Mr. Hart, \$.25. Apr. 8, Made garden for Mrs. Black, \$.50. Apr. 10, Whitewashed cellar for Mrs. Goff, \$.35. Apr. 15, Wheeled load of coal for Mr. Brown, \$.35. Apr. 25, Bought 4 collars and 2 pairs of cuffs, \$.90. Apr. 30, Bought necktie, \$.25.

**8.** *May.* May 3, Bought straw hat, \$1.00. May 7, Mowed lawn for Mrs. Jones, \$.25. May 13, Repaired Mr. Brown's sidewalk, \$.40; May 30, Bought baseball, \$.50. May 31, Received a reward of \$5.00 for finding a pocketbook containing \$50, which I returned to owner.

**9.** *June.* June 1, Made \$.20 selling papers. June 6, Worked a day for Mr. Black, \$.75. June 10, Delivered package, \$.25. June 17, Bought ball bat, \$.50. June 20, Wheeled a trunk for Mr. Hart, \$.25. June 29, Bought 1 pair of baseball shoes, \$1.00.

**10.** *July.* July 4, Fireworks, \$.50. July 6, Received from Mr. Black salary for week, \$5.00. July 12, Bought 2 shirts, \$1.50. July 13, Received week's salary, \$5.00. July 15, Bought outing suit, \$6.50. July 20, Received my salary, \$5.00. July 25, Expense for small articles, \$.95. July 27, Received my week's salary, \$5.00. July 30, Received for overtime, for month, \$7.50.

**11.** *August.* Aug. 3, Salary, \$5.00. Aug. 8, Bought 1 pair of tan shoes, \$2.50. Aug. 10, Received salary, \$5.00. Aug. 15, Bought fishing tackle, etc., \$3.75. Aug. 17, Received week's salary, \$5.00. Aug. 31, Expenses for 2 weeks' vacation, \$15.75.

Sept. 1, Balance, Cash on hand, ——.

Make out a statement at close of year, showing total receipts and disbursements, and proving final balance.

# DENOMINATE NUMBERS

**1.** Write from memory the following tables:

Liquid Measures, Dry Measures, Avoirdupois Weight, Time Measures, and Measures of Length or Distance.

**2.** 1 yr. = —— mo. = —— da. = —— hr. = —— min. = —— sec.

**3.** 1 mi. = —— rd. = —— yd. = —— ft. = —— in.

**4.** 1 T. = —— cwt. = —— lb. = —— oz.

**5.** 1 bu. = —— pk. = —— qt.

**6.** 1 gal. = —— qt. = —— pt.

The standard or principal units of measure are as follows:

| | |
|---|---|
| Liquid — **gallon.** | Length or distance — **yard.** |
| Dry — **bushel.** | Avoirdupois — **pound** (16 oz.). |
| Time — **day.** | |

All other measures are determined from the above unit measures. Thus, the ton is 2000 times 1 pound (16 oz.). The hour is $\frac{1}{24}$ of the day, the period of one revolution of the earth on its axis.

A **denominate number** is a concrete number whose unit is a measure established by custom or law; as, 10 feet, in which 1 foot is the unit of measure, or 5 pounds, in which 1 pound is the unit of measure.

A **simple denominate** number is a number of one denomination; as, 12 rods, 2 ounces, 5 days, etc.

A **compound denominate** number is composed of two or more concrete numbers that express one quantity; as, 6 yards, 2 feet, 4 inches. Here yards, feet, and inches are used to express but one quantity.

## REDUCTION OF DENOMINATE NUMBERS

Change :

1. $5\frac{1}{3}$ yd. to feet.
2. 90 in. to feet.
3. 3 yd. 2 ft. to feet.
4. .5 rd. to inches.
5. 25 ft. to yards.
6. 5.5 hr. to minutes.
7. .5 mi. to rods.
8. 3.5 gal. to pints.
9. $\frac{1}{3}$ day to minutes.
10. .25 bu. to quarts.
11. $\frac{3}{4}$ pk. to quarts.
12. 2 lb. 8 oz. to ounces.
13. $\frac{3}{4}$ cwt. to pounds.
14. .5 yd. 1 ft. to inches.
15. .75 mi. to rods.
16. .25 bu. to pints.
17. 3.5 pk. to quarts.
18. 2 yd. 1.5 ft. to inches.
19. 3.5 min. to seconds.
20. 48 qt. to pecks.
21. 64 pt. to bushels.
22. 64 oz. to pounds.

### Written Work

1. Change 3 gal. 3 qt. 1 pt. to pints.

$$
\begin{array}{lll}
\text{gal.} & \text{qt.} & \text{pt.}\\
3 & 3 & 1\\
\underline{4} & & \\
12 & & \\
\underline{+3} & & \\
15, \text{ number of quarts.} & & \\
\underline{2} & & \\
30 & & \\
\underline{+1} & & \\
31, \text{ number of pints.} & &
\end{array}
$$

Observe that 4 qt. is really the multiplicand and 3 the multiplier in finding the first product; and that 2 pt. is really the multiplicand and 15 the multiplier in finding the second product. In considering the numbers abstractly, however, either factor may be regarded as the multiplicand and the arrangement as indicated saves time.

**2.** Change .875 gallon to pints, etc.

$$\begin{array}{l} .875 \\ \underline{\quad 4} \\ 3.500, \text{ number of qt.} \\ \underline{\quad 2} \\ 1.00, \text{ number of pt.} \end{array}$$

Since there are 4 qt. in a gallon, in .875 of a gallon there are .875 of 4 qt., or 3.5 qt. Since there are 2 pt. in 1 qt., in .5 of a quart there is 1 pt. The answer is 3 qt. 1 pt.

Change:

**3.** 15 lb. 8 oz. to ounces.

**4.** 96 ft. 5 in. to inches.

**5.** 5.5 bu. to quarts.

**6.** 3.5 pk. to pints.

**7.** 18 cwt. 25 lb. to pounds.

**8.** 23 hr. 16 min. to minutes.

**9.** 8.3 mi. to yards.

**10.** .75 yd. to inches.

**11.** $4\frac{4}{5}$ T. to pounds.

**12.** $7\frac{3}{4}$ min. to seconds.

**13.** 6.5 L. T. to pounds.

**14.** 63.5 gal. to pints.

**15.** $\frac{7}{8}$ bu. to quarts.

**16.** $10\frac{3}{4}$ bu. to pecks.

**17.** Change 266 quarts to bushels, etc.

$$\begin{array}{l} 8\underline{)266} \\ \;\,4\underline{)33}, \text{ no. of pk.} + 2 \text{ qt.} \\ \qquad 8, \text{ no. of bu.} + 1 \text{ pk.} \\ 8 \text{ bu. } 1 \text{ pk. } 2 \text{ qt.} \end{array}$$

There are $\frac{1}{8}$ as many pecks as quarts, that is, 33 pk. + 2 qt. There are $\frac{1}{4}$ as many bushels as pecks, that is, 8 bu. + 1 pk. Hence, 266 qt. = 8 bu. 1 pk. 2 qt.

Change to higher denominations:

**18.** 342 inches.

**19.** 6625 yards.

**20.** 5281 feet.

**21.** 2043 seconds.

**22.** 1033 ounces Av.

**23.** 347 cwt.

**24.** 6095 pounds.

**25.** 16857 rods.

**26.** 11097 qt. (Dry).

**27.** 952 pt. (Liquid).

**28.** 43920 in.

**29.** 6875 sec.

**30.** 56.5 pk.

**31.** 684.5 rd.

**32.** $964\frac{3}{5}$ min.

**33.** How many gallons of milk will a family consume in 75 days, if they use 2 qt. 1 pt. daily?

**34.** How much is received for $1\frac{1}{2}$ bushels of chestnuts at 8 cents a quart?

**35.** How much will 15 turkeys, averaging $14\frac{3}{4}$ lb. each, cost at 18 cents a pound?

**36.** If 100 tons of coal are bought by the long ton, at \$2.24 a ton, and sold by the short ton at the same price, how much is gained?

**37.** At 20 cents an hour, how much will a man earn in 26 days, working each day from 8 A.M. to 5 P.M., allowing 1 hour for lunch?

**38.** If a flour mill grinds wheat at the rate of 1 pint in 5 seconds, in how many hours and minutes will it grind 21,600 bushels?

**39.** A train goes 104 miles in 3 hours and 15 minutes. What is the rate per hour?

**40.** At 2 cents a foot find the length in miles and rods of a telephone wire that costs \$4672.80.

**41.** If a man's step averages 2 ft. 6 in., how far will he travel in taking 6600 steps?

**Relations of denominate measures.**

**1.** $\frac{3}{4}$ pk. is what decimal part of a bushel?

$$\tfrac{3}{4}\text{ pk.} = 6\text{ qt.}$$
$$6\text{ qt.} = \frac{6}{32}\text{ bu.} = .1875\text{ bu.}$$

**2.** 3 ft. 2 in. is what fractional part of a rod?

$$3\text{ ft. }2\text{ in.} = 38\text{ in.}$$
$$1\text{ rod} = 198\text{ in.}$$
$$3\text{ ft. }2\text{ in.} = \tfrac{38}{198},\text{ or }\tfrac{19}{99}\text{ rd.}$$

Find the fractional part:

**3.** $2\frac{2}{3}$ hr. is of 1 day.

**4.** $7\frac{1}{2}$ ft. is of 1 rod.

**5.** $3\frac{1}{2}$ qt. is of 1 gallon.

**6.** $1\frac{1}{2}$ pt. is of 3 qt.

**7.** $2\frac{1}{2}$ in. is of 10 ft.

**8.** $1\frac{1}{2}$ qt. is of 2 gal.

Find the fractional part:

9. 15 hundredweight is of 1 ton.

10. 3.5 quarts is of 1 bushel.

11. 280 rods is of 1 mile.

12. 37 pounds 8 ounces is of 1 hundredweight.

13. 440 yards is of 1 mile.

Find the decimal part:

14. 16 hours 48 minutes is of 1 day.

15. 1 foot 8 inches is of 1 rod 2 in.

16. 180 pounds is of 1 ton.

17. 16 minutes 48 seconds is of 1 hour.

18. A machinist works 10 hr. per day in summer and $8\frac{4}{5}$ hr. per day in winter. If his wages in summer are \$3.35 per day, at the same rate find his wages per day in winter.

## ADDITION AND SUBTRACTION

1. Find the sum of 2 gal. 3 qt. 1 pt., 4 gal. 1 qt. 1 pt., 7 gal. 1 pt., 5 gal. 3 qt.

| gal. | qt. | pt. |
|---|---|---|
| 2 | 3 | 1 |
| 4 | 1 | 1 |
| 7 | 0 | 1 |
| 5 | 3 | 0 |
| 20 | 0 | 1 |

The sum of the pints = 3 pt. = 1 qt. and 1 pt.

The sum of the quarts + 1 qt. carried = 8 qt. = 2 gal. 0 qt.

The sum of the gallons + 2 gal. carried = 20 gal.

Add:

2. 14 bu. 2 pk., 5 bu. 6 qt., 7 qt. 1 pt., 9 bu. 6 qt.

3. 5 T. 11 cwt., 4 T. 15 cwt. 60 lb., 11 T. 80 lb., 19 T. 3 cwt. 64 lb.

4. 9 yr. 120 da. 8 hr., 12 yr. 104 da. 17 hr., 14 da.

5. 3 wk. 6 da. 15 hr., 4 wk. 3 da. 9 hr., 7 wk. 5 da. 14 hr.

**6.** From 7 bu. 3 pk. 3 qt. take 3 bu. 1 pk. 5 qt.

| bu. | pk. | qt. |
|---|---|---|
| 7 | 3 | 3 |
| 3 | 1 | 5 |
| 4 | 1 | 6 |

1 pk., or 8 qt., + 3 qt. = 11 qt.; 11 qt. – 5 qt. = 6 qt.; 2 pk. – 1 pk. = 1 pk.; 7 bu. – 3 bu. = 4 bu.

Subtract:

| | mi. | rd. | yd. | ft. |
|---|---|---|---|---|
| **7.** | 80 | 120 | 0 | 12 |
| | 57 | 245 | 0 | 14 |

| | gal. | qt. | pt. |
|---|---|---|---|
| **8.** | 23 | 0 | 1 |
| | 9 | 3 | 0 |

**9.** From 18 hr. take 9 hr. 16 min. 45 sec.

Finding the **difference in time** between two dates is the most practical application of subtraction of denominate numbers.

**10.** Find the difference in time between November 15, 1903, and August 12, 1905.

| yr. | mo. | da. |
|---|---|---|
| 1905 | 8 | 12 |
| 1903 | 11 | 15 |
| 1 | 8 | 27 |

Aug. 12, 1905, is represented as the 12th day of the 8th month of 1905, and Nov. 15, 1903, as the 15th day of the 11th month of 1903.

1 mo., or 30 da., + 12 da. = 42 da.; 42 da. – 15 da. = 27 da.; 1 yr., or 12 mo., + 7 mo. = 19 mo.; 19 mo. – 11 mo. = 8 mo.; 1904 yr. – 1903 yr. = 1 yr.

Subtract:

| | yr. | mo. | da. |
|---|---|---|---|
| **11.** | 1908 | 7 | 12 |
| | 1901 | 9 | 15 |

| | yr. | mo. | da. |
|---|---|---|---|
| **12.** | 1905 | 9 | 1 |
| | 1890 | 8 | 15 |

**13.** How many years, months, and days old is each pupil in the room?

**14.** General Robert E. Lee was born January 19, 1807, and General Ulysses S. Grant April 27, 1822. How old was each at the close of the Civil War, April 9, 1865? How much older was General Lee than General Grant?

**15.** How old is a man to-day who was born July 3, 1882?

## MULTIPLICATION AND DIVISION

**1.** Multiply 3 wk. 5 da. 9 hr. by 7.

| wk. | da. | hr. |
|---|---|---|
| 3 | 5 | 9 |
| | | 7 |
| 26 | 2 | 15 |

$7 \times 9$ hr. $= 63$ hr. $= 2$ da. and 15 hr.; $7 \times 5$ da. $= 35$ da.; 35 da. $+ 2$ da. $= 37$ da. $= 5$ wk. and 2 da.; $7 \times 3$ wk. $= 21$ wk.; 21 wk. $+ 5$ wk. $= 26$ wk. Hence, the answer is 26 wk. 2 da. 15 hr.

Multiply :

**2.** 3 gal. 2 qt. 1 pt. by 3.

**3.** 12 bu. 3 pk. 3 qt. by 6.

**4.** 15 T. 5 cwt. 12 oz. by 10.

**5.** 27 wk. 3 da. 14 hr. by 9.

**6.** 23 mi. 124 rd. 11 ft. 4 in. by 12.

**7.** Divide 54 T. 15 cwt. 72 lb. by 12.

| | T. | cwt. | lb. |
|---|---|---|---|
| 12) | 54 | 15 | 72 |
| | 4 | 11 | 31 |

54 T. $\div 12 = 4$ T. and 6 T. remaining; 6 T. $= 120$ cwt.; 120 cwt. $+$ 15 cwt. $= 135$ cwt.; 135 cwt. $\div 12$ $= 11$ cwt. and 3 cwt. remaining; 3 cwt. $= 300$ lb.; 300 lb. $+ 72$ lb. $=$ 372 lb.; 372 lb. $\div 12 = 31$ lb.

Divide :

**8.** 18 wk. 5 da. 21 hr. by 5.

**9.** 188 gal. 1 pt. by 7.

**10.** 88 bu. 3 pk. 4 qt. by 9.

**11.** 61 yr. 11 mo. 18 da. by 11.

**12.** 86 T. 3 cwt. 44 lb. by 6.

**13.** Find the cost of 19 gross of pencils at 10¢ a dozen.

**14.** A man digs 4 rods, 2 yards of ditch in a day. How many rods, etc., can he dig in 6 days?

**15.** How many packages, weighing 5 ounces each, can be made from 5 pounds of candy ?

## REVIEW

**1.** If a watch gains 18 seconds in a day, how much too fast will it be in three weeks?

**2.** How many barrels, each holding 2 bushels and 3 pecks, will be required to pack 88 bushels of apples?

**3.** How many bushels of potatoes are necessary to plant $8\frac{7}{8}$ acres, allowing 6 bu. 1 pk. to the acre?

**4.** A merchant sells linseed oil at 12¢ a pint that cost him 56¢ a gallon. Find his profits on 45 gallons 3 quarts.

**5.** 5 car loads of coal weigh: 57,698 lb., 49,875 lb., 63,545 lb., 49,897 lb., and 54,273 lb. Find the number of tons, hundredweight, and pounds in all.

**6.** 4 men buy a plot of land that has 222 feet 8 inches street frontage. Allowing for an alley 20 feet in width in the center, what is the width of each man's lot if they divide the plot equally?

**7.** A force pump in a coal mine lifts $76\frac{5}{8}$ gallons of water to the surface per minute. Find the number of gallons pumped out in one day.

**8.** If 3 pounds 4 ounces of coal are consumed in generating power to lift 5 gallons of water in problem 7, find the number of tons of coal consumed each day.

**9.** A Kentucky farmer clipped $241\frac{1}{2}$ pounds of mohair from 70 Angora goats. Find the average clip from each goat and its value at \$.$37\frac{1}{2}$ per pound.

**10.** An automobile runs $2\frac{7}{8}$ miles in 5 minutes. At that rate, find the distance in miles, rods, and feet it runs in 1 hour 35 minutes.

**11.** A pencil factory makes $6\frac{3}{4}$ gross of pencils per hour. Find the number of dozen made in 26 days of $9\frac{1}{2}$ hours each.

# PRACTICAL MEASUREMENTS

## MEASURES OF LENGTH

**1.** Measure the length of your desk; the length of the room; the length of the blackboard; the height of the window from the floor.

**2.** In what are these short lengths measured?

TO THE TEACHER. — Secure a tape measure 50 feet long.

**3.** Measure the distance around the schoolroom in feet and fractions of a foot. How many yards is it around the room?

**4.** Measure the distance around the school grounds in rods, feet, and inches.

**5.** Take $16\frac{1}{2}$ ft. of the tape measure and measure 10 rods along the public road or street.

**6.** $320 \times 16\frac{1}{2}$ ft. = how many feet?
$1760 \times 3$ ft. = how many feet?

**7.** How many feet equal a mile? how many yards?

**8.** James walks $1\frac{3}{4}$ miles to school each day. How many rods does he walk in going to and from school?

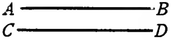
**9.** How many rods equal 5280 ft.? $\frac{3}{4}$ of a mile? 3560 ft.?

**10.** Mary walks $\frac{3}{4}$ of a mile to school each day. How many miles does she walk in going to and from school in 180 days?

**11.** Henry walks .8 of the number of miles Mary walks. Find the distance Henry walks in a term if he attends 160 days.

## MEASURES OF SURFACE

Observe that the straight lines $AB$ and $CD$ cannot meet, however far they may be extended. Such lines are called **parallel lines.**



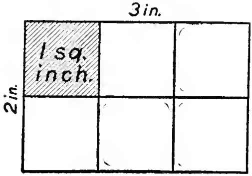

Lines that meet, making a square corner, form a **right angle.**

A figure that has four straight sides and four right angles is called a **rectangle.**

A rectangle having its four sides equal is called a **square.**

**1.** Name six different rectangles in the schoolroom. Are the opposite sides of a rectangle parallel lines?

**2.** How many dimensions has every rectangular surface? How does a surface differ from a line?




**3.** Draw, on a scale of $\frac{1}{2}$, a rectangle 3 inches long and 2 inches wide. Divide it by lines into square inches. How many square inches are there in the first row? in the second? in the rectangle? What is the *unit* of measure in this surface?

Observe that $2 \times 3 \times 1$ *sq. in.* $= 6$ sq. in.

*The* **area of a rectangle** *is found by multiplying its unit of measure by the product of its two dimensions, when expressed in like units.*

**4.** Draw a rectangle 2 ft. by 6 ft. Divide it into sq. ft.

**5.** Draw a square a foot on a side. Mark off the sides into 12 equal parts and connect them by straight lines. How many square inches equal a square foot?

**6.** Draw on the blackboard a line 4 feet long. From each end draw lines in the same direction 3 feet in length, making square corners with the 4-foot line. Connect by a straight line the ends of the 3-foot lines.

**7.** Are the sides of the figure straight? Are the corners equal in size? Find the area of the figure.

**8.** What is a right angle? a rectangle? (p. 124).

**9.** Show by a diagram the number of square feet in a square yard.

**10.** Draw a diagram on a scale of 1 inch to 3 feet to represent a rectangle 24 ft. long and 18 ft. wide. Find its area.

Draw diagrams on scales suitable to the size of your tablet or slate and find the surface of each of the following:

**11.** A rectangle 20 ft. by 24 ft.

**12.** A flower bed 16 ft. by 8 ft.

**13.** A floor 16 ft. long and 14 ft. wide.

**14.** A wall 15 yd. long and 5 yd. high.

**15.** By actual measurement find the number of square feet in the floor, the door, the blackboard, and the walls of the schoolroom.

**16.** In what denominations did we find the lengths and widths of the problems just given?

Land is measured in *acres*, *square rods*, *square feet*, etc.

**17.** Measure a square rod on your playground. How long is it? how wide?

**18.** Measure the length and width of your school grounds in rods and feet.

**19.** Since $16\frac{1}{2}$ feet equal 1 rod, how many yards equal 1 rod? How many square yards equal 1 square rod?

**20.** Since $16\frac{1}{2}$ feet equal 1 rod, how many square feet equal 1 square rod?

**21.** A field is 70 rods long and 40 rods wide. How many square rods are there in it? how many acres?

**22.** Memorize this table:

| | |
|---|---|
| **144 square inches (sq. in.)** | **= 1 square foot (sq. ft.)** |
| **9 square feet** | **= 1 square yard (sq. yd.)** |
| **$30\frac{1}{4}$ square yards** | **= 1 square rod (sq. rd.)** |
| **160 square rods** | **= 1 acre (A.)** |
| **640 acres** | **= 1 square mile (sq. mi.)** |

**1 A. = 160 sq. rd. = 4840 sq. yd. = 43,560 sq. ft.**

Change:

**23.** 2700 sq. yd. to sq. ft.

**24.** 50 sq. ft. to sq. in.

**25.** 1600 sq. rd. to A.

**26.** $1\frac{5}{8}$ A. to sq. rd.

**27.** 800 sq. yd. to sq. rd.

**28.** $5\frac{3}{4}$ A. to sq. ft.

**29.** A farm is 90 rods long and 60 rods wide. Find the number of acres in it. Find its cost at \$60 per acre.

**30.** A lot 100 ft. square has a house 36 ft. by 42 ft. located on it. The remaining space is lawn. Find the number of square feet of lawn. Draw diagram.

**31.** A concrete sidewalk in front of the lot is 4 ft. wide. Find its cost at 19¢ per square foot.

**32.** Find the cost of a flagstone walk, 135 ft. long and 6 ft. wide, at 21¢ per square foot.

**33.** City lots are sometimes sold by the square foot. Find the cost of a lot in Pittsburg 21 ft. by 70 ft. at \$27.50 per square foot.

**34.** A farm 160 rods long and 120 rods wide is sold in two pieces, $\frac{3}{8}$ of it at \$60 per acre, and the remainder at \$50 per acre. Find the amount of the entire sale.

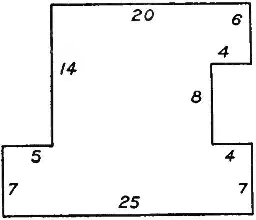
**35.** An Iowa farmer owns a farm a mile square. How many acres has he? Find its value at \$85 per acre.

**36.** A western wheat field 100 rods long and 80 rods wide yields 880 bushels of wheat. Find the average yield per acre.

**37.** City lots are usually sold by the front foot. Find the cost, at \$20 per foot front, of a lot 25 ft. front by 120 ft. deep. Find the cost per square foot.

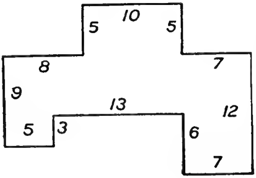
**38.** A four-room school building has a slate blackboard 24 ft. by 4 ft. in each room. Find the total cost of the blackboard at 23¢ per square foot.

**39.** The area of a field in the form of a rectangle is 8 acres. If one side is 32 rods, what is the other?




These diagrams represent pieces of land. The dimensions are given in rods, and the corners are all square.

**40.** Divide the first piece into 3 rectangles and find (1) how many square rods there are in each; (2) the perimeter of each; (3) the area of the entire piece in acres.




**41.** Divide the second piece into rectangular lots, and find (1) the perimeter of each; (2) the area of each; (3) the area of the entire piece.

## PAINTING AND PLASTERING

Painting, plastering, and kalsomining are generally measured by the **square yard.** In some localities an allowance is made for doors and windows, but there is no uniform rule in practice.

**1.** How much will it cost to paint a ceiling 18 ft. long and 15 ft. wide at 10¢ per square yard?

**2.** How much will it cost to kalsomine a hall 30 ft. long, 9 ft. wide, and 15 ft. high, at 5¢ per square yard? (Observe that the perimeter of the hall is 78 ft.)

**3.** How many square yards of plastering are there in a room 21 ft. long, 18 ft. wide, and 12 ft. high, making no allowance for openings?

**4.** How much will it cost, at 15¢ a square yard, to plaster a room 24 ft. $\times 19\frac{1}{2}$ ft. $\times$ 15 ft.?

**5.** A public hall is 120 ft. $\times$ 66 ft. $\times 22\frac{1}{2}$ ft. How much will it cost to paint the walls and ceiling at 10¢ per square yard?

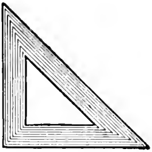
## THE RIGHT TRIANGLE

**1.** Draw on the blackboard a rectangle 12 inches long and 8 inches wide. Connect the opposite corners by a straight line.

This line is called the **diagonal** of the rectangle.

**2.** Into how many parts have we divided the rectangle? Shade one of the parts with chalk. How many angles are there in each part? how many right angles?

A **triangle** is a surface bounded by three straight lines.

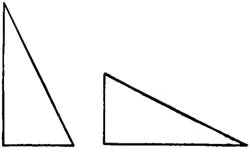


A **right triangle** is a triangle having one right angle.

The **base** of a triangle is the side on which it is assumed to stand.

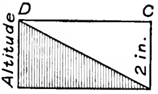
The **altitude** of a triangle is the line that meets the base line at a right angle.

To the Teacher. — As an aid in drawing have each pupil, if possible, get a right triangle as here shown.



**3.** Point out the base and altitude in the triangles at the right.

**4.** Fold a rectangular piece of paper, as $ABCD$, on its diagonal. Observe:




(1) That the rectangle $ABCD$ and the triangle $ABD$ have the same base and altitude.

(2) That the area of the triangle is just $\frac{1}{2}$ the area of the rectangle.

Hence, the area is $\frac{1}{2}$ of $4 \times 2 \times 1$ *sq. in.* = 4 sq. in.

*The* **area of a right triangle** *equals the unit of measure multiplied by $\frac{1}{2}$ the product of the base and altitude.*

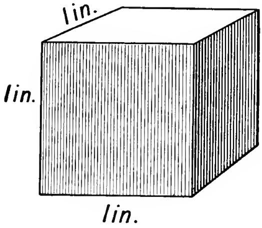
Draw on a scale suitable to your paper and find the area of the following right triangles in square inches:

**5.** Base 10 in., altitude 8 in. **7.** Base 25 in., altitude 18 in.

**6.** Base 12 in., altitude 6 in. **8.** Base 36 in., altitude 24 in.

**9.** Find the area of a field in the form of a right triangle whose base is 80 rods and altitude 40 rods.

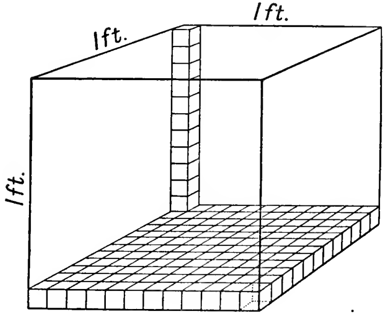
## MEASURES OF VOLUME




**1.** How many dimensions has the cube? Name them.

**2.** What dimensions has a line? What dimensions has a surface? a solid?

**3.** Name the different units of measure in which the length of a line may be expressed.




**4.** Name the different units of square measure in which surface may be expressed.

**5.** What cubic unit have we in the first cube?

**6.** If a cube is 1 foot on an edge, what is the cubic unit?

**7.** Draw a square 1 foot on a side. Show that it contains 144 square inches.

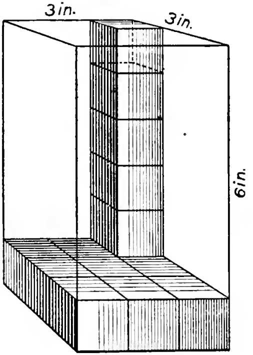
**8.** Observe that 144 cubes 1 inch on an edge can be placed on a surface 1 foot square. How many layers of such cubes will it take to make a cube 1 foot on an edge?

**9.** How many cubic inches equal 1 cubic foot?

**10.** How many surfaces has a cube?

**11.** Show that all the surfaces of an inch cube are the same in area; of a 2-inch cube; of a 9-inch cube.

**12.** Examine carefully the figure. Observe:




(1) That the surface of the face upon which it rests contains 9 square inches.

(2) That the first layer of units of volume contains 9 cubic inches.

(3) That the whole solid, if 6 inches high, contains $6 \times 9$ cubic inches, or 54 cubic inches.

**13.** How many 1-inch cubes are there in the first layer? how many in the solid?

**14.** What is the shape of the surfaces of the solid? Is each surface a rectangle?

**A rectangular solid** is a solid whose surfaces are all rectangles.

**15.** Observe that the number of inch cubes in the solid is equal to the product of its three dimensions.

**16.** What is the unit of measure in the solid?

Observe that $3 \times 3 \times 6 \times 1$ *cu. in.* = 54 cu. in.

*The* **contents, or volume, of a rectangular solid** *equals the unit of measure multiplied by the product of its three dimensions.*

TO THE TEACHER. — Secure 144 1-inch cubes.

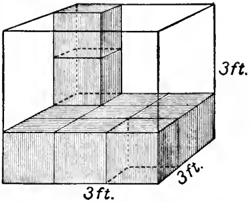
**17.** Build a cube 2 inches on an edge.

**18.** Build a cube 4 inches on an edge.

**19.** Compare the 4-inch cube with the 2-inch cube.

**20.** Give the different units of measure of surface; of length; of contents.

**21.** Find the contents of a box 3 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. high.




**22.** Observe the cube. What is its length? width? height?

**23.** How many 1-foot cubes does it contain?

A cube 3 ft. on an edge is called a **cubic yard.**

**24.** Memorize this table:

| | |
|---|---|
| **1728 cubic inches (cu. in.)** | **= 1 cubic foot (cu. ft.)** |
| **27 cubic feet** | **= 1 cubic yard (cu. yd.)** |

A **cart load** of earth is considered 1 **cubic yard.**

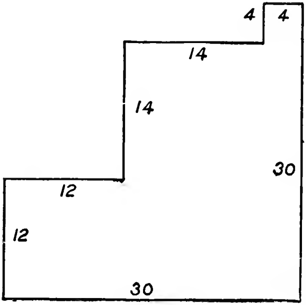
## PRACTICAL APPLICATIONS

Excavations are estimated by the **cubic yard.**

**1.** Find the cost, at 30¢ per cubic yard, of excavating a cellar 36 ft. in length, 24 ft. in width, and 4 ft. in depth.

$36 \times 24 \times 4 \times 1$ *cu. ft.* = 3456 cu. ft., the contents of the cellar.
3456 cu. ft. ÷ 27 = 128, number of cu. yd.
$128 \times \$.30 = \$38.40$, cost of excavation.

This diagram shows the outline of a cellar 5 ft. deep. Its dimensions are given in feet.




**2.** Find its area in square feet.

**3.** Find the length of its walls.

**4.** What is the cost of excavating it at 32¢ per cubic yard?

**5.** How much will it cost to cement the floor at \$.90 per square yard?

**6.** If a boy inhales 24 cubic inches of air at a breath, how many times must he breathe to inhale 1 cubic foot?

**7.** 29 pupils and their teacher occupy a schoolroom 30 ft. in length, 24 ft. in width, and 12 ft. in height. What is the average number of cubic feet of air for each person?

**8.** A city lot of $37\frac{1}{2}$ ft. by 120 ft. is to have a layer of earth 1 ft. thick over its surface. Find the number of loads needed and its cost at 25¢ per load.

**9.** A dining room is 13 ft. by 18 ft. and has a rug on it 9 ft. by 15 ft. Find the surface not covered by the rug.

**10.** If the rainfall on a certain day was $2\frac{1}{4}$ inches, find the number of cubic inches that fell on a lot 25 feet wide and 100 feet long. Find the number of gallons.

**11.** Find the cost of digging a ditch, 60 rods long, $3\frac{1}{2}$ feet wide, and 6 feet deep, at 60¢ per cubic yard.

**12.** Memorize:

**1 gallon = 231 cu. in.**
**1 bushel = 2150.42 cu. in.**
**1 bushel = $1\frac{1}{4}$ cu. ft. (nearly)**

**13.** Compare a 3-inch cube with a 4-inch cube. If a 2-inch cube weighs 6 ounces, how much will a 4-inch cube of the same material weigh?

**14.** A bin is 8 ft. long, 6 ft. wide, and 4 ft. deep. Estimate quickly about the number of bushels of wheat or oats it will hold.

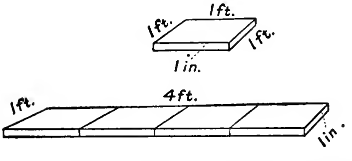
**15.** A farmer has a tank 12 ft. long, 8 ft. wide, and 6 ft. deep. How many gallons of water will it hold?

**16.** How much larger is a farm 80 rods square than a farm 60 rods square? Draw diagrams on a suitable scale to represent this.

**17.** The base of a rectangular tank is 48 sq. ft. and the volume is 192 cu. ft. Find the height.

**18.** What is the area of each surface of a cube 8 ft. on an edge? the entire surface?

## MEASUREMENT OF LUMBER




A board 1 foot square and 1 inch thick or less is a **board foot**.

A *board foot* is the *unit* in measuring lumber.

**1.** Draw on the blackboard a figure to represent a board 4 feet long, 1 foot wide, and 1 inch thick.

**2.** Show that this board contains 4 board feet.

**3.** How many board feet are there in a sill 4 ft. long, 1 ft. wide, and 4 in. thick?

Observe :

(1) That the sill is equal to 4 boards 4 ft. long, 1 ft. wide, and 1 in. thick.


(2) That each board contains 4 board feet.

Hence, the sill contains 4 × 4 × 1 *board foot* = 16 board feet.

*The* number of board feet in a piece of lumber *equals the number of board feet in one surface multiplied by the number of inches in thickness.*

Find the number of board feet in the following:

**4.** A plank 12 ft. long, 12 in. wide, and 2 in. thick.

**5.** A board 12 ft. long, 9 in. wide, and 1 in. thick.

**6.** A plank 15 ft. long, 12 in. wide, and 3 in. thick.

**7.** A plank 16 ft. long, 18 in. wide, and 2 in. thick.

**8.** A sill 20 ft. long, 10 in. wide, and 6 in. thick.

**9.** A sill 30 ft. long and 12 in. square.

**Buying and selling lumber.**

Lumber is usually sold at so much per 1000 (M.) *board feet.*

Find the cost of :

**1.** 5000 ft. poplar at $ 40 per M.

**2.** 500 ft. hemlock at $ 24 per M.

**3.** 10,850 ft. Georgia pine at $ 24 per M.

**4.** 8000 ft. white pine at $ 50 per M.

Small bills of lumber are usually estimated at so much per *board foot.*

**5.** Show that lumber at \$40 per M. = \$.04 per board foot; at \$27 per M. = \$.027 per board foot.

**6.** Make out a receipted bill to Henry James for the following: 365 ft. hemlock at \$25 per M., 780 ft. white pine at \$40 per M., 980 ft. yellow pine at \$29 per M.

The dimensions 10 ft. by 6 in. by 10 in. are commonly written $10' \times 6'' \times 10''$.

Estimate the cost of the following at \$28 per M.:

**7.** 4 sills $4'' \times 8'' \times 24'$

**8.** 6 girders $6'' \times 10'' \times 16'$

**9.** 2 posts $6'' \times 9'' \times 10'$

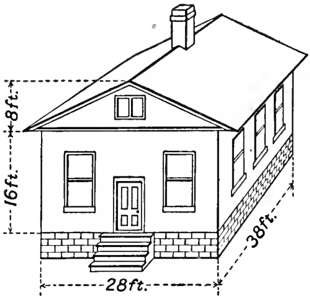
**10.** 8 beams $3'' \times 8'' \times 20'$

**11.** 60 joists $3'' \times 8'' \times 20'$

**12.** 90 studding $2'' \times 6'' \times 16'$

**13.** 90 planks $2'' \times 10'' \times 14'$

**14.** 60 rafters $2'' \times 4'' \times 24'$



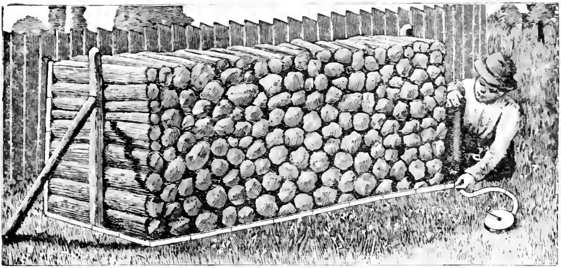

**15.** Observe the dimensions of the school building. What is the height of the sides of the building?

**16.** Find the number of board feet of siding needed for the sides and the two ends of the same height as the sides, making no allowance for openings.

**17.** The triangular parts at the top of the house, in front and in back, are called *gables*. Each gable can be divided by a line through the center of its base into two right triangles. How many board feet of siding are necessary for the two gables?

**18.** Find the cost of painting the siding at 10 cents per square yard.

## MEASURING WOOD



A pile of wood, of 4-foot sticks, 8 ft. in length and 4 ft. in height, is called a **cord of wood.**

$4 \times 4 \times 8 \times 1$ *cu. ft.* = 128 cu. ft. = 1 cord of wood.

**1.** How many cords are there in a pile of 4-foot wood, 160 feet long and 4 feet high?

**2.** Two men cut several piles of 4-foot wood that measure in all 640 feet in length and 4 feet in height. How many cords do they cut and how much do they receive for the work at $5.50 per cord?

Wood is frequently cut for house purposes into short lengths from 16 inches to 2 feet. The price of such a cord varies according to the length of the sticks.

*The* **number of cords of short wood** *in a pile is found by dividing the number of square feet in one side by* 32.

**3.** At a school building there is a pile of 16-inch wood 80 ft. long and 4 ft. high. Find its cost at $1.50 per cord.

**4.** Two men cut 4 cords of 2-foot wood each day for 16 days. Find the cost of the cutting at 70 cents per cord.

**5.** One side of a pile of 2-foot wood contains 400 square feet. Find the number of cords it contains.

## REVIEW OF PRACTICAL MEASUREMENTS

**1.** How many tiles 12 in. square will be required to lay a floor 36 ft. by 15 ft.?

**2.** What is the length of a board walk that is 4 ft. 8 in. wide and contains 1350 sq. ft.?

**3.** How many cubic yards of earth must be removed in digging a cellar 36 ft. long, 26 ft. wide, and 8 ft. deep?

**4.** Find the cost of covering the floor of a hall 45 ft. long and 30 ft. wide with matting, a yard wide, at 70 cents a yard.

**5.** How many times will the wheel of an engine 9 ft. in circumference turn in going 3000 miles?

**6.** Find the cost of 30 boards 16 ft. long, 12 in. wide, and 1 in. thick, at 5 ¢ a board foot.

**7.** At $.80 a bushel what is the value of a bin of wheat 16 ft. long, 8 ft. wide, and 4 ft. deep?

**8.** What is the number of gallons in a tank 12 ft. long, 10 ft. wide, and 8 ft. deep?

**9.** How much will it cost to cement the floor of a cellar 50 ft. long and 28 ft. wide at $1.08 a square yard?

**10.** At 7 ¢ a square yard, how much will it cost to paint the four sides of a building 50 ft. long, 20 ft. wide, and 15 ft. high?

**11.** My farm is in the form of a rectangle, and contains 40 acres. What is its width, if its length is 128 rods?

**12.** What will be the cost of plastering the ceiling of a room 22 ft. by 18 ft. at 11 ¢ a square yard?

**13.** A rectangular field contains 5 acres. If its length is 80 rods, what is its width?

**14.** How many cakes of soap 4 in. by 3 in. by 2 in. can be packed in a box whose inside dimensions are 2 ft., 3 ft., and 4 ft.?

**15.** Find the cost of digging a cellar 42 ft. long, 30 ft. wide, and 6 ft. 3 in. deep, at 40 cents a cubic yard.

**16.** How much flooring 1 inch thick will be required to lay the first floor of a house 22 ft. by 36 ft., no allowance being made for waste, and how much will it cost at $30 per M.?

**17.** The length of a field is 80 rods, and its width is 30 rods. How many acres are there in the field?

**18.** What is the number of bushels in a bin 20 ft. long, 16 ft. wide, and 8 ft. deep?

**19.** A tank 9 ft. square and 8 ft. deep contains how many gallons?

**20.** A building lot 100 foot front contains 15,000 sq. ft. What is its depth?

**21.** A baseball ground 160 yd. by 170 yd. has a tight board fence around it 8 ft. high. How much will the painting of the outside of the fence cost at $5\frac{1}{2}$ cents a square yard?

**22.** The area of a right triangle is 560 sq. ft., and its altitude is 28 ft. What is the base of the triangle?

**23.** How much will it cost to excavate a street 800 ft. long and 50 ft. wide, to a depth of 18 in., at 36 cents a load?

**24.** A plot of ground in the form of a square is 100 ft. on each side. A straight walk 8 ft. wide divides it into 2 equal parts — a lawn for flowers and a garden for vegetables. In the lawn there is a flower bed 5 ft. by 8 ft. Draw the plot.

**25.** Find the perimeter of the plot; of the lawn; of the garden; of the flower bed; of the walk.

Find the area in square yards:

**26.** Of the plot.

**27.** Of the lawn.

**28.** Of the flower bed.

**29.** Of the walk.

**30.** How much will it cost to fence the plot at $\$3\frac{3}{4}$ per rod?

**31.** How much will it cost to pave the walk at $1.55 per square yard?

**32.** How much will it cost to spade the flower bed at 5 cents per square yard?

**33.** How much will it cost to sod the lawn, excluding the flower bed, at $0.25 per square yard?

**34.** A board 16 ft. long contains 9 sq. ft. Find its width.

**35.** A room is 20 ft. long, 16 ft. wide, and 10 ft. high. How much will it cost to plaster the walls and ceiling at 20 cents a square yard?

**36.** How many gallons of water are there in a tank 12 ft. long, 8 ft. wide, and 6 ft. deep, if it is half full?

**37.** Find the cost of 40 boards, each 14 ft. long, 18 in. wide, and 1 in. thick, at $20 per M.

**38.** A city 5 miles long and 3 miles wide is equal in area to how many farms of 160 acres each?

**39.** How many sods 16 in. square will be required to turf a lawn 106 ft. 8 in. long and 50 ft. wide?

**40.** What will be the cost of painting the outside of a house 48 ft. long, 30 ft. wide, and 20 ft. high, at 18 cents a square yard?

# PERCENTAGE

**Per cent** means **by the hundred** or **hundredths.** The sign for it is %.

We may express the per cent of a number either as a *common fraction* or a *decimal.*

Thus, 6% = $\frac{6}{100}$ = .06; 6% of 500 means $\frac{6}{100}$ of 500, which equals 30; or, expressed decimally, .06 of 500 = 30.

2% of a number means $\frac{2}{100}$, or .02, of the number.

25% of a number means $\frac{25}{100}$, or .25, of the number.

**1.** What term in common fractions corresponds to the number before the sign %? to the sign %?

**2.** What expresses the *numerator* and what indicates the *denominator* of the fractions represented by the following:

| | | | |
|---|---|---|---|
| 1%? | 20%? | 40%? | 90%? |
| 6%? | 30%? | 75%? | 100%? |

**3.** Find 6% of 100.

$\frac{6}{100}$ of 100 = 6; or .06 × 100 = 6.

Find:

| | | |
|---|---|---|
| **4.** 5% of 100 | **10.** 10% of 100 | **16.** 8% of 75 |
| **5.** .05 of 100 | **11.** 25% of 100 | **17.** .08 of 75 |
| **6.** $\frac{5}{100}$ of 100 | **12.** .25 of 400 | **18.** $\frac{8}{100}$ of 75 |
| **7.** 6% of 150 | **13.** 3% of 60 | **19.** $33\frac{1}{3}$ of 300 |
| **8.** .06 of 150 | **14.** .03 of 60 | **20.** $\frac{33\frac{1}{3}}{100}$ of 300 |
| **9.** .10 of 100 | **15.** $\frac{3}{100}$ of 60 | **21.** $33\frac{1}{3}$% of 300 |

**Changing per cents to equivalents.**

Since $5\% = .05 = \frac{5}{100} = \frac{1}{20}$, these expressions may be called **equivalents.**

**1.** Give the fractional and decimal equivalents for $10\%$; $6\%$; $4\%$; $20\%$; $25\%$.

Read the following equivalents:

**2.** $\frac{20}{100}$, $20\%$, $.20$, $\frac{1}{5}$

**3.** $\frac{12\frac{1}{2}}{100}$, $12\frac{1}{2}\%$, $.12\frac{1}{2}$, $\frac{1}{8}$

**4.** $\frac{40}{100}$, $40\%$, $.40$, $\frac{2}{5}$

**5.** $\frac{37\frac{1}{2}}{100}$, $37\frac{1}{2}\%$, $.37\frac{1}{2}$, $\frac{3}{8}$

**6.** $\frac{80}{100}$, $80\%$, $.80$, $\frac{4}{5}$

**7.** $\frac{87\frac{1}{2}}{100}$, $87\frac{1}{2}\%$, $.87\frac{1}{2}$, $\frac{7}{8}$

**8.** Change the fractions $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ to their equivalent decimals and per cents; also $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

$\frac{1}{5} = 5\overline{)1.00}$
$\quad .20$, or $20\%$
$\frac{2}{5} = 2 \times .20 = .40$, or $40\%$
$\frac{3}{5} = 3 \times .20 = .60$, or $60\%$
$\frac{4}{5} = 4 \times .20 = .80$, or $80\%$

$\frac{1}{8} = 8\overline{)1.00}$
$\quad .12\frac{1}{2}$, or $12\frac{1}{2}\%$
$\frac{3}{8} = 3 \times .12\frac{1}{2} = .37\frac{1}{2}$, or $37\frac{1}{2}\%$
$\frac{5}{8} = 5 \times .12\frac{1}{2} = .62\frac{1}{2}$, or $62\frac{1}{2}\%$
$\frac{7}{8} = 7 \times .12\frac{1}{2} = .87\frac{1}{2}$, or $87\frac{1}{2}\%$

Change to their equivalent decimals and per cents:

| | | | | |
|---|---|---|---|---|
| **9.** $\frac{1}{2}$ | **13.** $\frac{3}{4}$ | **17.** $\frac{3}{10}$ | **21.** $\frac{2}{5}$ | **25.** $\frac{3}{8}$ |
| **10.** $\frac{1}{3}$ | **14.** $\frac{1}{6}$ | **18.** $\frac{7}{10}$ | **22.** $\frac{3}{5}$ | **26.** $\frac{5}{8}$ |
| **11.** $\frac{2}{3}$ | **15.** $\frac{5}{6}$ | **19.** $\frac{9}{10}$ | **23.** $\frac{4}{5}$ | **27.** $\frac{7}{8}$ |
| **12.** $\frac{1}{4}$ | **16.** $\frac{1}{10}$ | **20.** $\frac{1}{5}$ | **24.** $\frac{1}{8}$ | **28.** $\frac{1}{16}$ |

Give the products rapidly:

| | | | |
|---|---|---|---|
| **29.** $2 \times .33\frac{1}{3}$ | **32.** $5 \times .12\frac{1}{2}$ | **35.** $4 \times .12\frac{1}{2}$ | **38.** $4 \times .04\frac{1}{4}$ |
| **30.** $5 \times .16\frac{2}{3}$ | **33.** $7 \times .12\frac{1}{2}$ | **36.** $6 \times .12\frac{1}{2}$ | **39.** $3 \times .16\frac{2}{3}$ |
| **31.** $3 \times .12\frac{1}{2}$ | **34.** $3 \times .8\frac{1}{3}$ | **37.** $2 \times .15$ | **40.** $4 \times .16\frac{2}{3}$ |

Memorize the following table:

| | | | |
|---|---|---|---|
| $\frac{1}{2} = 50\%$ | $\frac{1}{5} = 20\%$ | $\frac{5}{6} = 83\frac{1}{3}\%$ | $\frac{1}{12} = 8\frac{1}{3}\%$ |
| $\frac{1}{3} = 33\frac{1}{3}\%$ | $\frac{2}{5} = 40\%$ | $\frac{1}{8} = 12\frac{1}{2}\%$ | $\frac{5}{12} = 41\frac{2}{3}\%$ |
| $\frac{2}{3} = 66\frac{2}{3}\%$ | $\frac{3}{5} = 60\%$ | $\frac{3}{8} = 37\frac{1}{2}\%$ | $\frac{1}{16} = 6\frac{1}{4}\%$ |
| $\frac{1}{4} = 25\%$ | $\frac{4}{5} = 80\%$ | $\frac{5}{8} = 62\frac{1}{2}\%$ | $\frac{1}{20} = 5\%$ |
| $\frac{3}{4} = 75\%$ | $\frac{1}{6} = 16\frac{2}{3}\%$ | $\frac{7}{8} = 87\frac{1}{2}\%$ | $\frac{1}{25} = 4\%$ |

Name rapidly the fractional equivalents of the following per cents:

| | | | |
|---|---|---|---|
| 41. 50% | 46. 20% | 51. $37\frac{1}{2}\%$ | 56. 90% |
| 42. $33\frac{1}{3}\%$ | 47. 40% | 52. $62\frac{1}{2}\%$ | 57. $12\frac{1}{2}\%$ |
| 43. $66\frac{2}{3}\%$ | 48. 60% | 53. $87\frac{1}{2}\%$ | 58. $16\frac{2}{3}\%$ |
| 44. 25% | 49. 16% | 54. 10% | 59. 80% |
| 45. 75% | 50. $83\frac{1}{3}\%$ | 55. 30% | 60. 70% |

Write the equivalents of the following in decimals, thus: $1\% = .01$; $32\% = .32$; $\frac{1}{2}\% = .00\frac{1}{2}$; etc.

| | | | |
|---|---|---|---|
| 61. 1% | 67. $\frac{5}{6}\%$ | 73. 50% | 79. 13% |
| 62. 32% | 68. 3% | 74. $\frac{2}{3}\%$ | 80. $13\frac{1}{3}\%$ |
| 63. $\frac{1}{2}\%$ | 69. 11% | 75. 6% | 81. $\frac{5}{8}\%$ |
| 64. 2% | 70. $\frac{1}{4}\%$ | 76. $\frac{3}{4}\%$ | 82. 100% |
| 65. $16\frac{1}{3}\%$ | 71. 4% | 77. 7% | 83. 125% |
| 66. $\frac{1}{3}\%$ | 72. $43\frac{1}{2}\%$ | 78. $\frac{7}{8}\%$ | 84. 127% |

**Finding a given per cent of a number.**

Recite the following thus: *Look at* "$66\frac{2}{3}\%$," *think* "$\frac{2}{3}$":

1. $66\frac{2}{3}\%$ of 18.
2. $33\frac{1}{3}\%$ of 90.
3. 50% of \$500.
4. 25% of \$2000.
5. 75% of 16 inches.
6. 20% of 100 yards.
7. 40% of 60 feet.
8. 60% of 40 miles.
9. 80% of 75 gallons.
10. $16\frac{2}{3}\%$ of \$6000.
11. $83\frac{1}{3}\%$ of \$1200.
12. $37\frac{1}{2}\%$ of \$7200.
13. $12\frac{1}{2}\%$ of \$6400.
14. 75% of \$4800.
15. $66\frac{2}{3}\%$ of \$999.
16. 80% of 60 sheep.
17. 60% of 75 horses.
18. 40% of 90 miles.
19. $87\frac{1}{2}\%$ of \$160.
20. $62\frac{1}{2}\%$ of \$240.
21. $37\frac{1}{2}\%$ of \$880.
22. $12\frac{1}{2}\%$ of 24.

### Written Work

1. A man had 100 cows and sold 25% of them. How many did he sell?

$$
\begin{array}{rl}
25\% = .25 & \\
100 & \text{cows} \\
\underline{.25} & \\
500 & \\
\underline{200\phantom{0}} & \\
25.00 & \text{number sold}
\end{array}
$$

As 25% = .25 we multiply 100 by .25. The result is 25, the number sold.

Find results decimally:

2. 50% of 750
3. 25% of 85.5
4. 40% of 8.75
5. 28% of 840
6. 32% of 1000
7. 75% of 980

8. John earns \$21.60 per month, and spends 75% for clothes. How much do his clothes cost him?

**9.** There are 780 pupils in school, and 40 % are males. How many are males?

**10.** If a man buys a horse for \$150 and sells it at a profit of 20 %, how much does he gain?

In each of the preceding problems we have two terms, a *per cent* and a *whole* or a *mixed* number. The per cent in each problem is the multiplier, and is called the *rate*. The whole or the mixed number is the multiplicand, and is called the *base*. The product is called the *percentage*.

The **base** is that number of which some per cent is to be taken; as, 5 % of \$200 (*base*).

The **rate** is the number of hundredths taken; as, 5% (*rate*) of 80 horses; that is, $\frac{5}{100}$ of 80 horses.

The **percentage** of a number is the result obtained by taking any per cent of it; as 10 % of 200 acres is $\frac{10}{100}$ of 200 acres, or 20 acres (*percentage*).

**11.** What is 75% of \$5.12?

| MULTIPLIER | | MULTIPLICAND | | PRODUCT |
|---|---|---|---|---|
| Rate | | Base | | Percentage |
| 75 % | of | \$5.12 | = | (  ) |

**Decimal Method**

$$75\,\% = .75.$$

$$\begin{array}{rl} \$5.12 & = \text{base} \\ .75 & = \text{rate} \\ \hline 2560 & \\ 3584\phantom{0} & \\ \hline \$3.8400 & = \text{percentage} \end{array}$$

**Fractional Method**

$$75\,\% = \tfrac{3}{4}$$

$$\tfrac{3}{4} \text{ of } \$5.12 = \$3.84$$

**Study of Problem**

What is the base? \$5.12. What is the rate? 75%.

To what do the *base* and *rate* correspond in simple multiplication? *Multiplicand* and *multiplier*.

To what does *percentage* correspond in simple multiplication? *Product*.

How is the *product* found in simple multiplication? *Multiplier × multiplicand*.

How is the *percentage* found? *Rate × base*.

*The* **percentage of a number** *equals the product of the base by the rate.*

Find:

**12.** 6 % of \$200.

**13.** $33\frac{1}{3}$ % of 6 months.

**14.** 60 % of 30 days.

**15.** 1 % of 100 acres.

**16.** 5 % of 100 acres.

**17.** 8 % of 400.

**18.** 7 % of 400 horses.

**19.** $3\frac{1}{3}$ % of 99.

**20.** 6 % of 150 lb.

**21.** $1\frac{1}{2}$ % of \$75.

**22.** 80 is the base, 25 % is the rate, find the percentage.

**23.** A house costs \$2500, and the damage by fire is 8 %. Find the amount of the damage.

**24.** John owes his tailor \$80, and pays $37\frac{1}{2}$ % of the debt. How much does he still owe?

**25.** Mary spells 90 % of 80 words correctly. How many does she miss?

**26.** A boy buys apples at \$1 per bushel, and sells them at a profit of 20 %. How much profit is that per bushel?

**27.** $6\frac{1}{4}$ % of 3680 equals what number?

**28.** A man buys a farm for \$2500, and sells it for 25 % more than it cost him. For how much does he sell the farm?

**29.** A man earns \$180 per month, and puts $33\frac{1}{3}$ % of it in the savings bank. What is his deposit each month?

**30.** If $37\frac{1}{2}$ % of a man's farm is in timber, and the total area is 240 acres, how much timber land has he?

**31.** A teacher who earned \$1200 a year spent $66\frac{2}{3}$ % of her salary. How much did she save?

**32.** Mr. Scott's horse is valued at \$250 and Mr. Hill's at 60 % of this. What is the value of Mr. Hill's horse?

**33.** The population of a town of 9672 inhabitants increased $12\frac{1}{2}$ % in a year. What was the increase in population?

## COMMISSION

An **agent** is a person who transacts business for another.

**Commission** is the sum charged by an agent or commission merchant for his services.

The **net proceeds** is the sum left after the commission and other expenses have been paid.

**1.** A real estate agent sold a house for \$5000, retaining 5% of this sum for his services. How much did he receive? How much did the owner receive?

$$\begin{array}{rl} \$5000 & = \text{selling price.} \\ .05 & = \text{rate charged by the agent.} \\ \hline \$250.00 & = \text{amount charged by the agent.} \end{array}$$

$$\$5000 - \$250 = \$4750, \text{ amount received by the owner.}$$

A commission merchant made the following sales. Find his commission for each day at 5%.

**2.** Monday, \$1800

**3.** Tuesday, \$1594

**4.** Wednesday, \$1954

**5.** Thursday, \$1400.80

**6.** Friday, \$1528

**7.** Saturday, \$2370.60

**8.** Find his total commission for the week.

**9.** A real estate agent sells a house and lot for \$6750, charging 2% commission. Find his commission and the net proceeds.

**10.** A traveling salesman sold \$50,000 worth of goods in a year at a commission of 8%. If his expenses for the year were \$2200, how much had he left?

**11.** An agent rents 12 houses at \$40 per month. If he receives 5% for collecting the rents, how much is remitted to the owners each month?

## COMMERCIAL DISCOUNT

Wholesale merchants and manufacturers usually publish printed price lists of their goods. The prices in these lists are higher than the wholesale prices and are subject to deductions called **trade discounts** or **commercial discounts.**

NOTE. — A **discount** is any deduction from a fixed price.

Sometimes several discounts are allowed. The first is a discount from the list price; the second, a discount from the remainder, etc.

The **net price** is the price less all trade discounts.

Find the selling price of goods marked:

1. \$15, less 20%.
2. \$20, less 40%.
3. \$6, less 50%.
4. \$25, less 20%.
5. \$7.50, less 20%.
6. \$12.50, less 40%.
7. \$40, less 60%.
8. \$48, less 25%.
9. \$6.80, less 25%.
10. \$4.50, less $33\frac{1}{3}$%.
11. \$9.60, less $16\frac{2}{3}$%.
12. \$4.80, less $37\frac{1}{2}$%.

### Written Work

Find the selling price of goods marked:

1. \$168.75, less 25%.
2. \$1374, less $16\frac{2}{3}$%.
3. \$1872, less $33\frac{1}{3}$%.
4. \$278.40, less $37\frac{1}{2}$%.
5. \$3030 less 40%.
6. \$225.65, less 20%.
7. \$875.50, less 30%.
8. \$278.90, less 10%.
9. \$2378.50, less 4%.
10. \$6775.20, less 5%.

Find the cost of:

Discount, 20%

11. 60 readers @ \$.40
150 geographies @ \$1
78 grammars @ \$.60

Discount, 4%

12. 160 lb. rice @ \$.06
300 lb. sugar @ \$.04$\frac{1}{2}$
200 lb. coffee @ \$.16

**13.** Find the net price of a bill of goods for \$ 75.40, trade discounts 20 %, 10 %.

| | |
|---|---|
| List price, | \$ 75.40 |
| Less 20 %, | 15.08 |
| First remainder, | 60.32 |
| Less 10 %, | 6.03 |
| Net price, | \$ 54.29 |

Observe that the *second* discount is reckoned on the *first* remainder. As there are only two discounts, the second remainder is the net price.

Find the net price of articles listed at:

**14.** \$ 400, less 20 %, 10 %.

**15.** 375.50, less 25 %, 5 %.

**16.** 290.80, less 40 %, 10 %.

**17.** \$ 10.75, less 40 %, 5 %.

**18.** \$ 6.80, less 25 %, 10 %.

**19.** \$ 12.75, less $33\frac{1}{3}$ %, 10 %.

Find the net price of the following bills of goods:

**20.** 36 dozen boys' caps @ \$ 6, discounts 25 %, 20 %.

**21.** 50 buggies @ \$ 120, discounts 20 %, 15 %.

**22.** 75 sets harness @ \$ 40, discounts 30 %, 10 %.

**23.** 25 grain drills @ \$ 95, discounts 40 %, 5 %.

**24.** 12 rubber hose, each 50 feet long at 15¢ per foot, discounts 30 %, 15 %.

**25.** Mr. Austin buys a wagon listed at \$ 95, less 20 %, 15 %. Find the amount paid for the wagon.

**26.** A merchant buys 12 stoves listed at \$ 45, less 40 %, 10 %. Find the net amount of the bill. Compare this with the net amount of the bill with only one discount of 50 %.

**27.** A hotel keeper buys 675 yards of carpet at \$ 1.25, less 20 %, 5 %. Find the cost of the carpet.

**28.** Compare the net price of an article listed at \$500, discounts of 20 %, 10 %, with the net price of a similar article listed at \$500, discounts of 10 %, 20 %.

Solve according to conditions:

**29.** The Packard Hardware Co. bought for cash from Jas. M. Armstrong Co., Chicago, Ill., 4 doz. Acme lawn mowers @ \$30 a dozen, 50 lb. lawn seed @ 15¢ a pound, $2\frac{1}{2}$ doz. brushes @ 40¢ a dozen. Trade discounts: 20%, 10%. Terms: 30 days net; 2% cash in 10 days.

Cost of bill of goods = \$128.50.

\$128.50 less trade discount of 20% = \$102.80; \$102.80 less trade discount of 10% = \$92.52, net price of bill if paid in 30 days. If the bill is paid within 10 days from date of purchase, the buyer gets a further discount of 2%. This is called a **cash discount.** \$92.52, less 2% for cash within 10 days, = \$90.68.

**30.** Jamison and Redmond, South Bend, Ind., bought for cash from the Acme Buggy Co., Cincinnati, O., 72 buggies @ \$105, 50 sets harness @ \$45, 15 sleighs @ \$60, 40 robes @ \$20. Trade discounts: 30%, 15%. Terms: 30 days net; 3% cash in 10 days.

**31.** James Cubbison, Greenville, O., buys for cash from Arbuthnot, Stevenson & Co., Pittsburg, Pa., 5 doz. handkerchiefs @ \$3.60; 5 bolts muslin, 40 yd. each, @ 8¢; 5 bolts prints, 42 yd., @ 7¢. Trade discount: $33\frac{1}{3}$%. Terms: 30 days net; 2% cash in 10 days.

**32.** S. H. Gardner Co., piano dealers, Detroit, Mich., order from the Harmonic Piano Co., Chicago, Ill., 2 Harmonic pianos #266 @ \$600, less 40%, 10% trade discount. Terms: 90 days net; 10% off 10 days. Find the cash price. Find the net price if paid in 30 days.

NOTE.—The sign #, when placed before a number, is read "number."

**33.** M. L. Smith, tailor, Brockton, Mass., orders from Bender & Co., New York, importers, 3 pieces suiting, 22 yd. each, @ \$3.15. Terms: 30 days net; 2% off 10 days. Find the net amount of bill if paid within 10 days.

# INTEREST

**1.** Mr. Johnston pays the liveryman $6 for the use of a horse and buggy for two days. What does he get in exchange for the $6?

**2.** Mr. Daniels pays $6 for the right to pasture his cow in a field for two months. What does he get in exchange for the $6?

**3.** Mr. Watson pays $6 for the use of $100 for one year. What does he get in exchange for the $6?

**4.** In the first two examples money is paid for the use of something that is not money. For what does Mr. Watson pay the money in the last example?

**Interest** is money paid for the use of money. Interest corresponds to the *percentage* in percentage.

**5.** How much does Mr. Watson pay for the use of the money? What is the $6 called?

**6.** On what is the interest reckoned? The $100 is called the *principal.*

The **principal** is the sum on which the interest is paid. The principal corresponds to the *base* in percentage.

The **rate of interest** is a certain number of hundredths of the principal paid for the use of the principal for *one year.*

**Time** is always a factor in *interest.* **Interest,** then, is the product of three factors: **principal, rate,** and **time.**

The **amount** is the sum of the principal and the interest.

## Interest for Years and Months

**1.** What part of a year are 6 months? 4 months? 3 months? 2 months? 1 month?

**2.** If the interest for a year is \$100, what should it be for 6 months? for 4 months? for 3 months? for 2 months? for 1 month?

## Written Work

**1.** What is the interest on \$200 for $2\frac{1}{2}$ years at 6%?

| | |
|---|---|
| \$200 | principal |
| .06 | rate |
| \$12.00 | interest for one year |
| $2\frac{1}{2}$ | |
| \$30.00 | interest for $2\frac{1}{2}$ years |

The interest for 1 year is .06 of the principal, or \$12. The interest for $2\frac{1}{2}$ years is $2\frac{1}{2} \times \$12$, or \$30.

*Multiply the principal by the rate and the product by the number of years.*

The year is usually considered as 360 days, that is, 12 months of 30 days each.

Find the interest on:

**2.** \$300 at 5% for 1 year.

**3.** \$800 at 8% for 2 years.

**4.** \$150 at $6\frac{1}{2}$% for 3 years.

**5.** \$700 at $4\frac{1}{2}$% for 4 years.

Find the interest of:

**6.** \$250 for $1\frac{1}{2}$ years at 4%.

**7.** \$75 for 2 years at 8%.

**8.** \$100 for $3\frac{2}{3}$ years at 7%.

**9.** \$80 for $4\frac{1}{2}$ years at 5%.

**10.** \$40 for $2\frac{1}{2}$ years at $6\frac{1}{2}$%.

**11.** \$500 for $2\frac{1}{3}$ years at $4\frac{1}{2}$%.

**12.** \$960 for 9 mo. at 6%.

**13.** \$900 for $2\frac{3}{4}$ years at 7%.

**14.** \$654 for $\frac{3}{4}$ year at 6%.

**15.** \$220 for $\frac{7}{8}$ year at 8%.

Find the interest at 6% on:

16. \$100 for 6 months.

17. \$500 for 4 months.

18. \$150 for 2 yr. 2 mo.

19. \$624 for 120 da.

20. \$170 for 8 mo.

21. \$355 for 130 da.

### Interest for Years, Months, and Days

**1.** What part of a month (30 days) are 15 days? 12 days? 20 days? 3 days? what part is 1 day?

**2.** If the interest for 1 year is \$360, what is the interest for 1 month? If the interest for 1 month is \$30, what is the interest for 1 day? for 15 days? for 12 days?

### Written Work

**1.** Find the amount of \$200 at 6% interest for 2 yr. 7 mo. 12 da.

| | | |
|---|---|---|
| Principal = | \$200 | |
| Rate = | .06 | |
| Int. for 1 yr. = | \$12.00 | |
| Int. for 2 yr. | = 2 × \$12.00, or | \$24.00 |
| Int. for 7 mo. | = $\frac{7}{12}$ of \$12.00, or | 7.00 |
| Int. for 12 da. | = $\frac{12}{30}$, or $\frac{2}{5}$, of \$1.00, or | .40 |
| Int. for 2 yr. 7 mo. 12 da. = | | \$31.40 |
| Principal = | | \$200.00 |
| Amount for 2 yr. 7 mo. 12 da. = | | \$231.40 |

### Study of Problem

*a.* What is the *first* step in the work? the *second* step?

*b.* How do we find the interest for 1 month? for 7 months? for 12 days?

*c.* What new term is introduced in interest? For what length of time is *rate* of interest always considered?

Find the interest and amount of :

2. \$300 for 3 yr. 6 mo. at 6 %.

3. \$250 for 2 yr. 4 mo. at 7 %.

4. \$160 for 4 yr. 3 mo. at 5 %.

5. \$50 for 1 yr. 8 mo. at 5 %.

6. \$800 for 3 yr. 2 mo. at 6 %.

7. \$50.80 for 9 mo. at 10 %.

8. \$16 for 8 mo. at 6 %.

9. \$75 for 8 mo. at 6 %.

10. \$420 for 10 mo. at 10 %.

11. \$40.50 for 1 yr. 1 mo. at 6 %.

12. \$300.40 for 5 mo. at 7 %.

13. \$100 for 7 mo. at 7 %.

14. \$500 for 11 mo. at 6 %.

15. \$1000 for 1 mo. at 6 %.

16. \$60.60 for 8 mo. at 8 %.

Find the interest and amount of :

17. \$250 at 8 % for 3 yr. 5 mo. 20 da.

18. \$75.80 at 5 % for 4 yr. 1 mo. 16 da.

19. \$1500 at 6 % for 2 yr. 9 mo. 15 da.

20. \$125.50 at 4 % for 4 yr. 11 mo. 12 da.

21. \$1140 at $5\frac{1}{2}$ % for 4 yr. 8 mo. 24 da.

22. \$912.60 at 5 % for 2 yr. 10 mo. 11 da.

23. \$3209 at 6 % for 3 yr. 7 mo. 21 da.

24. \$634.50 at 8 % for 11 mo. 12 da.

25. Henry Boydson borrows \$275 Sept. 1, 1906, at 6 % interest, and settles the note Jan. 1, 1908. Find the amount of the note at settlement.

## REVIEW OF PERCENTAGE AND INTEREST

**1.** A boy has \$30 in a savings bank and deposits a sum equal to 10% of it. What is the total amount he has in bank?

**2.** Mr. James's salary is \$1200 per year and he saves $33\frac{1}{3}$% of it. How much does he spend?

**3.** A boy spends \$8 for an overcoat and $37\frac{1}{2}$% of that sum for shoes. How much does he spend for shoes?

**4.** In a school of 45 pupils, $33\frac{1}{3}$% of the pupils are boys. What is the number of girls?

**5.** Find 25% of .05; of .5; of 5.5; of .25.

**6.** John earns \$50 during his vacation, and Margaret 25% as much as John. How much does Margaret earn?

**7.** A farmer sold a horse that cost him \$80 at a loss of 20%. Find the selling price.

**8.** What is the interest on \$150 for $2\frac{1}{2}$ years at 6%?

**9.** Find the interest on \$100 for 60 days at 5%.

**10.** My father borrows \$75 from his neighbor and promises to pay it in 4 months at 6%. Find the amount my father must pay at the end of four months.

**11.** A huckster buys eggs at \$.20 per dozen. For how much per dozen must he sell them to gain 20%?

**12.** If I borrow \$50 from Mr. James for 6 months at 6%, how much interest must I pay him?

**13.** A grocer sold flour last week at $1.20 per sack and this week at 10 % advance on last week's selling price. Find the price of flour per sack this week.

**14.** A huckster buys 150 dozen eggs at $.20 per dozen and sells them to a merchant at a gain of 25 %. The merchant sells them at a gain of 20 %. How much does the merchant receive for the eggs?

**15.** If I buy cloth at $1.50 a yard, for how much must I sell it to gain $33\frac{1}{3}$ %?

**16.** A grocer buys goods to the amount of $1200, 10 % off for cash. He sells them for $1500 cash. How much does he gain?

**17.** A mother took two boys and a girl to a store to buy clothes. The first boy's suit cost $10 less 10 % for cash. The second boy's suit cost $66\frac{2}{3}$ % of the cash price of the first boy's suit. The girl's coat cost $33\frac{1}{3}$ % of the money paid for both boys' suits. How much did the mother pay for the children's clothes?

**18.** Find 25 % of 200, and divide the result by $.00\frac{1}{4}$.

**19.** A man buys a house and lot for $5000. It costs every year $25 for repairs and $50 for taxes and insurance. He rents the house for 8 % of its cost. How much has he left after paying expenses?

**20.** A coal dealer bought 300 tons of coal for $600. The freight, storage, and delivery cost $33\frac{1}{3}$ % of the cost of the coal. What was the retail price per ton if he sold it at a gain of $12\frac{1}{2}$ %?

**21.** A real estate agent purchased a house for $1250. For how much per month must he rent the house to make 6 % after paying each year $18 for taxes and insurance and $15 for repairs?

## RECEIPTS AND CHECKS

John Watson pays James Adams $35.50 for work for one month, and asks Mr. Adams for a receipt. Write the receipt to show that the money was paid by Mr. Watson and received by Mr. Adams.

| | |
|---|---|
| $______________ | *Rochester, N. Y., June 1, 1907.* |

**Received from** __________________________________________

__________________________________________________ *Dollars*

*for* ______________________________________________________

_________________________________

1. What must every receipt show?

2. Write the receipt your grocer would give you in payment of $18.50 on account by your father or mother.

3. Your school district pays the National Book Company, New York, $ 25.75 for school books on Sept. 15, 1907. Make out the receipt of the National Book Company.

4. Henry Smith received $ 3.65 from James Brown for 3 months' water rent. Make out a receipt for the amount.

5. Ralph Taylor pays H. W. Henderson $5 for a month's tuition. Write the receipt Ralph Taylor should receive.

**6.** Write a receipt for \$75 which Nelson Page paid Edgar Poe for balance due on a buggy.

**7.** Make out and receipt the bill for the following articles bought by James Thomas from Jos. Horne & Co.:

| | | | | | |
|---|---|---|---|---|---|
| 3 shirts | @ | \$1.75 | 2 neckties | @ | \$.75 |
| 6 collars | @ | .20 | 4 pairs cuffs | @ | .20 |

**8.** Presuming that you are a collector for the *Gazette-Times*, Pittsburg, Pa., make out a receipt to a subscriber who has paid you \$2.60 in full of account.

A **check** is an order on a bank where a person keeps a deposit, ordering the bank to pay money.

| STUB | CHECK |
|---|---|
| *No. 875* | *No. 875* |
| | *Seattle, Wash., Jan. 10, 1907.* |
| *\$67$\frac{00}{}$* | The Yukon National Bank of Seattle. |
| | PAY TO THE |
| *Jan. 10, '07* | *Order of------James Ward-----\$67$\frac{00}{}$* |
| *To James Ward* | *Sixty-seven $\frac{no}{100}$~~~~~~Dollars.* |
| *For Labor* | *W. J. Moore.* |

**1.** Name the different things stated in this check.

**2.** Observe that this check is payable to the *order of* James Ward. He orders it paid by writing his name across the back of it. This is called **indorsing** the check.

**3.** Write the check your father would give your teacher in payment of \$3.50 for September tuition.

**4.** Emil Smith borrows from Joseph McLean \$240 to attend school and pays the same in 2 yr. 4 mo. 18 da. at 6%. Write the amount of the check that would pay the note.

## GENERAL REVIEW

**1.** The remainder is 92,568 and the minuend is 202,660. Find the subtrahend.

**2.** The dividend is 364,450 and the quotient is 9850. What is the divisor?

**3.** Add 3.5, .035, 45.006, and 2.06.

**4.** Write decimally twenty-five and sixty-one thousandths; one hundred twenty-five and five tenths; and three hundred and two ten-thousandths.

**5.** What number multiplied by one hundred seventy-nine is equal to 848,818?

**6.** From 2.0011 take 1.9892.

**7.** Explain the difference between $\frac{1}{5}\%$ and $\frac{1}{5}$.

**8.** Add $\frac{1}{2}$, $2\frac{3}{4}$, $\frac{1}{9}$, $\frac{23}{36}$, and $4\frac{5}{6}$.

**9.** Find 1%, $\frac{1}{5}$, $\frac{1}{5}\%$, $\frac{1}{20}$, $\frac{1}{20}\%$, 50%, and $\frac{1}{10}$ of 100.

**10.** The multiplicand is 1325 and the multiplier is .0416. What is the product?

**11.** If 38 dozen eggs cost $11.40, what is the cost per dozen?

**12.** A building is 46 ft. 3 in. wide, and twice as long as wide. Find the distance around the building.

**13.** From 86 miles and 3 inches, take 46 miles and 8 inches.

**14.** A man and his son together earn \$72 per month. If the man's earnings in 6 months amount to \$300, how much are the son's earnings in the same length of time?

**15.** A man bought 48 head of cattle, at \$36 per head, and sold them at a gain of 25%. What was the total amount received for the cattle?

**16.** Find the interest on \$370.50 for 4 yr. 8 mo. at 6%.

**17.** Divide $48\frac{4}{9}$ by $21\frac{4}{5}$.

**18.** Divide .65 by 6.5.

**19.** Reduce $187\frac{1}{2}$ rods to the fraction of a mile.

**20.** How much will it cost to ship a car load of wheat containing 42,000 lb. from Fargo, N.D., to Chicago, Ill., if the freight rate is \$.06 per bushel? (60 lb. = 1 bu.)

**21.** A train leaves Chicago at 8:15 A.M., and arrives at Pittsburg at 8:20 P.M. The distance is 468 miles. Find the number of miles per hour the train travels.

**22.** The steel rails on the Bessemer railroad weigh 100 pounds to the yard. Find the number of tons necessary to lay 5 rods of single track.

**23.** How much does an architect receive, at $4\frac{1}{2}$%, for the plans of a house that cost \$8350?

**24.** A man's salary is \$150 per month. He spends 40% of it for clothing and other expenses. How much does he save in a year?

**25.** A man purchases 80 acres of land for \$6400, and sells them at 25% gain. How much does he receive per acre?

**26.** Frank Stewart borrows \$250 Sept. 15, 1906, at 6% interest. Find the amount of the note if paid March 15, 1908.

**27.** Find the area in acres of a street 7 miles long and 66 feet wide.

**28.** A town lot is 43 ft. 3 in. wide and 120 ft. deep. How much is it worth at 75 ¢ per square foot?

**29.** A western farmer harvests 8960 bu. wheat from a field 320 rd. long and 160 rd. wide. If he sells the wheat at 60¢ per bushel, how much does he realize from each acre?

**30.** In 1 hour 20 minutes and 40 seconds, a train travels 60 miles. At that rate how long would the train be in traveling 1200 miles?

**31.** The average wages of a steel mill employing 3000 men are $2.50 per day. If a 10% reduction in wages is made, how much per day will the company's pay roll be reduced?

**32.** In a certain class the salary of the teacher for a year is $500. The books and supplies cost $90.65; fuel, $40, repairs and other expenses, $75.30. There are 35 pupils in the class. Find the average cost per pupil for the year.

**33.** Reduce to improper fractions: 6.25; 3.375; $4.66\frac{2}{3}$; and 2.05.

**34.** If it costs $72 to carpet a room 18 ft. long and 18 ft. wide, how much will it cost to carpet a room 36 ft. long and 36 ft. wide, with the same quality of carpet?

**35.** Mt. Rainier is 14,363 ft. high. Reduce the height to miles and the fractions of a mile.

**36.** How many cubic inches are there in a bin 9 ft. 7 in. long by 8 ft. 3 in. wide and 4 ft. 9 in. deep? how many cubic feet?

**37.** A grocer bought 225 bu. apples at $.50 per bushel. He sold 150 bu. at $.75 per bushel. The remainder, which were damaged, he sold at $.40 per bushel. Did he gain or lose and what per cent?

**38.** What is $33\frac{1}{3}\%$ of 24? of \$4.80? of \$62.50?

**39.** A piece of land 30 rods wide and 480 rods long was sold at \$62.50 per acre. Find the amount of the sale.

**40.** Time, 3 months; rate of interest, 5%; money borrowed, \$100. Find amount to be paid.

**41.** If $\frac{2}{3}$ of a bushel of potatoes cost \$.40, how much will $7\frac{1}{2}$ bu. cost?

**42.** A piece of land 40 rods long in the form of a rectangle contains 5 acres. Find its width in rods.

**43.** A farmer sold $12\frac{3}{4}$ acres of land at $\$55\frac{1}{2}$ per acre. How much did he receive for the land?

**44.** Houser Brothers sold the following bill of goods to William Pool:

| | |
|---|---|
| 12 lb. sugar | @ $\$.06\frac{1}{2}$ |
| 10 cans tomatoes | @ \$.15 |
| 6 lb. rice | @ $\$.07\frac{1}{2}$ |
| 11 lb. prunes | @ $\$.07\frac{1}{2}$ |
| 2 pair boots | @ \$3.50 |
| 1 overcoat | @ \$13.50 |
| 1 pair shoes | @ \$4.00 |

Mr. Pool at the same time sold Houser Brothers:

| | |
|---|---|
| 85 bu. potatoes | @ \$.65 |
| 50 bu. corn | @ $\$.42\frac{1}{2}$ |
| 16 lb. butter | @ \$.24 |
| 10 lb. butter | @ \$.28 |

Houser Brothers gave Mr. Pool the balance in cash. Make out the account.

**45.** A painter worked $17\frac{1}{2}$ days. After spending $\frac{4}{7}$ of his wages for board he had \$15 left. Find his daily wages.

**46.** I owe Frank Morrison, the grocer, \$32.50 and pay him \$23.75. Write the receipt that Mr. Morrison should give me.

NOTE.— When a debt is not paid in full, the receipt should read "On account."

**47.** A cellar 24 ft. by 32 ft. is to be excavated to an average depth of $5\frac{1}{2}$ ft. Find the number of cubic yards to be removed.

**48.** Express $22\frac{1}{2}$ yards as rods, feet, and inches.

**49.** The width of a rectangle is 20 rods and the area is 560 square rods. Find the length.

**50.** What is the difference between a square and a rectangle?

**51.** Give the rule for finding percentage. On what is gain or loss always reckoned?

**52.** A man's farm and personal property cost \$5600. The first year he cleared $12\frac{1}{2}\%$ of the money invested. The second year, on account of floods, he lost $5\%$ of the cost of the property. How much was his gain in the two years?

**53.** Of a bill of \$155 sent to a collector, $80\%$ was collected and the collector retained \$12.40. What per cent did he charge for collecting?

**54.** A boy receives \$1.20 per day and a man \$2.50 per day. How long will it take the boy to earn as much as the man can earn in 30 days?

**55.** The perimeter of a rectangle is 72 rods. The width is 12 rods. Find the length.

**56.** Estimating that 300 cu. ft. of air is required for each pupil, how many pupils, including the teacher, should occupy a room 40 ft. long, 30 ft. wide, and 12 ft. high?

**57.** Divide one thousand and one thousandth by one and one thousandth.

**58.** What is the interest on \$375 for 270 days at 6%?

**59.** Reduce $\frac{5}{9}$ of a mile to lower denominations.

**60.** A boy deposited half of his money in the savings bank; $\frac{1}{4}$ of the remainder he spent for clothes; and he had \$3 remaining. How much had he at first?

**61.** Reduce .025 cwt. to lower denominations.

**62.** A man bought two city lots costing him \$3500 and \$4100 respectively. He sold them at a gain of 25%. What was the gain in dollars?

**63.** How many gallons of water will a tank contain that is 11 ft. long, $3\frac{1}{2}$ ft. wide, and 4 ft. deep?

**64.** A barn floor is 20 ft. wide and 45 ft. long. How much will it cost to cover it with plank 2 inches thick at \$20 per thousand board feet?

**65.** Divide $\frac{3}{4}$ by $\frac{6}{9}$ of $\frac{2}{3}$.

**66.** John and James have together 165 acres of land, but James has twice as many acres as John. How many acres has each?

Suggestion. — 165 acres = twice John's + once John's or 3 times John's.

**67.** What fractional part of a day are 10 hours, 50 minutes, 40 seconds?

**68.** Divide nine ten-thousandths by one hundred twenty-five thousandths.

**69.** What is the interest on \$180 for 4 yr. 8 mo. at $5\frac{1}{2}$%?

**70.** I made \$1.95 by selling 15 dozen eggs at \$.31 per dozen. What was the cost of the eggs per dozen?

**71.** Find the net proceeds from the sale of 145 books, at \$2 each, on which a commission of $33\frac{1}{3}\%$ is paid.

**72.** A father divided his farm of 202 A. 16 sq. rd. equally among his four sons. How many acres did each receive?

**73.** .21 of a mile is equal to how many feet?

**74.** An automobile that cost \$2675 was sold at a loss of 28%. For how much was it sold?

**75.** What is the cost of 18 planks 20 ft. long, 12 in. wide, and 2 in. thick, at \$20 per M?

**76.** $\frac{1}{2}$ of 7 is what part of 9?

**77.** $\frac{5}{6}$ of a farm is worth \$7500. What is 20% of the farm worth?

**78.** What is the cost of a car load of bituminous coal weighing 84,000 pounds at \$2.65 per ton?

**79.** A farm in the form of a rectangle containing 120 acres is 60 rods wide. How long is it?

**80.** Express decimally the quotient of $\frac{7}{8} \div .35$.

**81.** If $\frac{4}{5}$ of a ton of hay is worth \$12, how much are 33,000 pounds of hay worth?

**82.** What is the value of a pile of 4-foot wood 48 ft. long and 6 ft. high, at \$4.50 per cord?

**83.** A dairyman owns a cow that averages 3 gal. 2 qt. 1 pt. of milk daily. If he sells the milk at \$.06 per quart, how much will he realize from the cow during the month of May?

**84.** I can buy an automobile at one store for \$3000, with discounts of 25%, 10%; or at another store for \$3000 with only one discount of 35%. Which is the cheaper?

# PART II — SEVENTH YEAR

## BILLS AND ACCOUNTS

### RECEIPTS

John Bentz rents a house in Boston, Mass., from James Smith for one year for $240, rent payable the first day of each month in advance.

1. Every receipt should state (1) the *place and date* of payment; (2) who *pays* the money; (3) who *receives* the money; (4) *for what* the money is paid; (5) the *amount* both in figures and in writing.
2. Every receipt in full should state *in full to date.*

Write the receipt given Mr. Bentz for September's rent.

$ ____________

____________________________

Received from ____________________________

______________________________________________

*For* ______________________________________________

____________________________

**1.** Providing John Bentz fails to pay the rent for August when due, but pays on September 1 the rent for both August and September, write the proper receipt.

**2.** Write the receipt for the tuition for the term of your school that any non-resident pupils would have to pay.

Write the receipt in full to date for each of the following bills which I owe:

3. John Thompson for milk, \$6.75.
4. Frank Jones for coal, \$16.85.
5. Smith & Co. for books, \$3.75.

## ORDERING GOODS

These forms of orders should be studied carefully, as they come into almost daily use in business life.

A

FURNEE AND KENNERDELL,
BOOKSELLERS.

KITTANNING, PA., *Oct. 10, 1907.*

*American Book Company,*
*100 Washington Square, New York.*
*Dear Sirs:*
*Please ship at once by Pennsylvania freight:*
*75 Steps in English Book 1.*
*300 Rose Primers.*
*Yours truly,*
*Furnee & Kennerdell.*

B

*Franklin, Pa., Jan. 1, 1907.*

*To Boggs & Buhl,*
*Allegheny, Pa.*
*Kindly send by Adams Express the following, as per sample, and charge the same to my account.*
*10 yd. of ribbon, at \$0.50*
*12 yd. of dress goods, at \$1.25*
*(Mrs.) Chas. A. Steele.*

**1.** Make out an order to each of the various schoolbook companies for the books you are studying.

**2.** Make out an order to McCreery & Co. of New York, for some goods to be sent to you C.O.D. (*cash on delivery*).

### RECEIPTED BILLS

Study the following:

NEW CASTLE, PA., *May 1, 1907.*

*Mr. James Grant,*
*24 County Line St.*

Bought of JOHN KNOX & SON,
196 E. WASHINGTON ST.,

FANCY GROCERS.

TERMS: *Cash*

| | | | | | |
|---|---|---|---|---|---|
| *1907.* | | | | | |
| *Apr.* | *1* | *1 bbl. Salt,* | *$1.50* | *$1* | *50* |
| *"* | *20* | *20 lb. Granulated Sugar,* | *6½¢* | *1* | *30* |
| *"* | *25* | *10 bu. Potatoes,* | *60¢* | *6* | *00* |
| *"* | *27* | *6 lb. Honey,* | *20¢* | *1* | *20* |
| | | | | *10* | *00* |
| | | *Received Payment, May 10, '07.* | | | |
| | | *John Knox & Son.* | | | |
| | | *Per Watson.* | | | |

Observe:

1. The *place* and *date* of sale. 2. The names of the *buyer* and the *seller*. 3. The *name*, *quantity*, and *price* of each article. 4. The *entire amount* of each separate item. 5. The *total* amount of the bill. 6. The *receipt* of the bill.

A **bill** is a written statement in detail of goods sold or of services rendered.

A bill is **receipted** when the words "Received payment" are written at the bottom of the bill, either by the *seller* or by some person authorized by him.

NOTE.—When the person authorized signs the name of the seller, he should always write on the next line below the word "by" or "per" and his own name or initials.

When a person purchases anything on time, the purchaser is called a **debtor**.

When the seller extends the time of payment to any one, the seller is said to give credit, and therefore is called a **creditor**.

Some abbreviations used in business:

| | | | |
|---|---|---|---|
| Acct. %, | account | mdse., | merchandise |
| Amt., | amount | No. (#), | number |
| bal., | balance | paymt., | payment |
| Co., | company | pd., | paid |
| Cr., | creditor | per, | by |
| Dr., | debtor | pc., | piece |
| do. ("), | the same | rec'd, | received |

The symbol # means *pounds* when placed *after* a number; but *number* if placed *before* a number.

Thus 6# means 6 pounds but #6 means Number 6.

Make receipted bills for the following transactions, performing all necessary operations:

**1.** Carter Bros., Elkins, W. Va., purchase from Bindley Hardware Co., Pittsburg, Pa., the following: 3 dozen locks @ \$4.80, 67 kegs of nails @ \$4.10, 6 dozen lanterns @ \$6.25, 1300 feet steel tracks @ 16¢, and 7 lawn mowers @ \$4.25.

**2.** John Dunn & Son, Akron, Ohio, bought from Thomas Townsend & Company, Cleveland, Ohio, 36 barrels of flour @ \$4.80, 4 boxes of prunes @ \$1.65, 500 pounds of coffee @ $11\frac{3}{8}$¢, 7 boxes of yeast @ 75¢, 50 pounds of Huyler's cocoa @ 32¢.

**3.** James Brown, Lincoln, Neb., purchases from May & Co., St. Louis, Mo., 22 bunches bananas @ \$1.75, 32 boxes oranges @ \$3.15, 17 boxes lemons @ \$2.80, 29 crates cranberries @ \$2.25, 6 boxes grape fruit @ \$2.90, and 35 bbl. apples @ \$2.75.

**4.** James Sweitzer, Peoria, Ill., bought from Swift & Co., Chicago, 1587 pounds of dressed beef @ $7\frac{1}{4}$¢, 267 pounds of mutton @ 9¢, 933 pounds of pork @ $5\frac{5}{8}$¢, and 180 pounds of lard @ 12¢.

**5.** Lyle Bros. & Co., Dubuque, Ia., bought of the Delaney-Brown Lumber Co., Grand Rapids, Mich., 28215 feet oak boards at \$32 per M., 147820 feet hemlock at \$27 per M., 92629 feet No. 1 white pine at \$60 per M., 63605 feet poplar boards at \$35 per M.

NOTE.— \$32 per M. equals \$.032 per board foot.

**6.** Mrs. James Thorpe bought of B. Altman & Co., New York, 1 pair gloves at \$2.75, 5 yd. ribbon at 39¢ a yard, $\frac{1}{2}$ dozen handkerchiefs at 25¢ each.

## ACCOUNTS

In the study of *bills* we simply found how much the debtor owed to the seller, or what one party owed to another for services rendered.

In an **account** we have a business transaction covering a period of time in which there is both a debtor's *bill* and a debtor's *payments*.

## Form of Account

PITTSBURG, PA., *May 1, 1907.*

*Mr. Samuel Bond,*
*Wheeling, W. Va.*

To JOSEPH HORNE CO., Dr.

| | | | | | | |
|---|---|---|---|---|---|---|
| | | *Dr.* | | | | |
| *Apr.* | *1* | *To Account rendered* | *$110* | *29* | | |
| *"* | *10* | *" 20 yd. silk @ $1.50* | *30* | *00* | | |
| *"* | *25* | *" 2 Ladies' suits @ 45.00* | *90* | *00* | | |
| *"* | *28* | *" Mdse.* | *7* | *80* | *238* | *09* |
| | | *Cr.* | | | | |
| *"* | *14* | *By Cash* | *100* | *00* | | |
| *"* | *24* | *By Cash* | *60* | *00* | *160* | *00* |
| | | *Balance due Joseph Horne Co.* | | | *78* | *09* |

NOTE.—If the above balance were paid in full May 1, the words "Received Payment" would be written.

JOSEPH HORNE CO.,
Per__________

It is customary for the creditor to send an itemized account to the debtor. If it is not paid, another form of bill called a *statement* is sent and contains only these words: "To account rendered" or "To Mdse." followed by the amount.

In the above account, what shows that there was a previous transaction?

Render the following statements:

**1**. Jan. 31, 1906, the debits and credits of George Weil in account with John Wanamaker, Philadelphia, Pa., were as follows:

DEBITS

Jan. 1, To account rendered, $295.63,
Jan. 7, To 3 overcoats @ $32,
Jan. 12, To 7 yd. dress goods @ $4.75,
Jan. 20, To 1 suite of furniture, $185.

CREDITS

Jan. 5, By cash, $250,
Jan. 20, By note for 30 days, $200.
Find balance due Wanamaker.

**2**. Nov. 30, 1905, the debits and credits of R. D. McClurg, with Stevenson & Co., Richmond, Va., were as follows:

DEBITS

Nov. 1, To account rendered, $86.25,
Nov. 10, To 12 cases corn @ $1.65,
Nov. 19, To 4 bbl. sugar, 1692 lb., @ $4\frac{1}{4}$¢,
Nov. 22, To 75 lb. dry beef at 19¢.

CREDITS

Nov. 5, By cash, $85,
Nov. 30, By note for balance due.

**3**. On Oct. 31, the account of Wm. B. Eager with H. A. Soltori, New York, was as follows:

| DEBITS | CREDITS |
|---|---|
| Oct. 2, To mdse., $93.37, | Oct. 10, By cash, $80, |
| Oct. 8, To mdse., $107.92, | Oct. 20, By 30-day note, $100. |
| Oct. 21, To mdse., $21.58. | Oct. 31, By cash, $25. |

## LEDGER ACCOUNTS

The orders or payments when received by a firm are first put in a **day book** in the order of their arrival. Each person's or firm's business is then placed in a book called the **ledger**, which is ordinarily *balanced* each month or when an account is paid.

A ledger account is headed by the name of the person and arranged so that the *purchases* appear on the *left* side as *debits*, and the *payments* or *services rendered* appear on the *right* side as *credits*.

The statement of account as given on page 171 is simply a copy of Mr. Bond's ledger account with Joseph Horne Co.

The following form shows a balanced ledger account at the close of the month with James Roberts, the balance being brought down to continue the account into the next month.

*Dr.* James Roberts *Cr.*

| 1907 | | | | | | 1907 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Feb. | 1 | Bal. bro't f'w'd | | $19 | 30 | Feb. | 6 | Cash | 6* | $30 | 00 |
| " | 2 | Lumber | 2* | 19 | 80 | " | 11 | Drayage | 45 | 15 | 75 |
| " | 4 | Cement | 8 | 40 | 50 | " | 18 | Drayage | 60 | 8 | 50 |
| " | 5 | Sand | 15 | 9 | 50 | " | 22 | Drayage | 70 | 26 | 50 |
| " | 9 | Tile | 30 | 15 | 70 | " | 25 | Cash | 74 | 82 | 50 |
| " | 13 | Plaster | 50 | 56 | 30 | " | 28 | Balance | | 38 | 75 |
| " | 18 | Sewer pipe | 60 | 20 | 90 | | | | | | |
| " | 26 | Lumber | 75 | 20 | 00 | | | | | | |
| | | | | 202 | 00 | | | | | 202 | 00 |
| Mch. | 1 | Bal. bro't f'w'd | | 38 | 75 | | | | | | |

**1.** What debts in this ledger account did Mr. Roberts incur during the month? What payments did he make?

* These numbers refer to the pages in the *day book* in which the accounts are found.

2. Find the sum of the debits and the sum of the credits. Does the difference equal the balance, $38.75?

3. Is the balance in favor of, or against Mr. Roberts?

4. Had the balance been in favor of Mr. Roberts, on which side would it have been entered?

5. How do you determine on which side to enter the balance? on which side to bring down the balance?

To **foot** a ledger is to add and set down (usually in pencil) the total debits and credits of the accounts.

TEST. — When the footing of one side equals the footing of the other side, the account is in balance.

### Written Work

The day book shows the following sales and receipts. Make a ledger for the year, enter each item, foot and close the accounts.

1. **William Stone.**

DEBITS. — Feb. 1, cook stove, $22; Feb. 4, 40 lb. nails, $2.40; Feb. 5, heater, $75.70; Feb. 12, tin roofing, $79.08; Feb. 19, hardware, $14; Feb. 23, lime and cement, $50.70; Feb. 28, tile, $22.

CREDITS. — Feb. 1, sand, $15; Feb. 4, drayage, $9.50; Feb. 12, cash, $50; Feb. 16, lumber, $74.25; Feb. 25, cash, $20; Feb. 28, cash, $105.

2. **Morris Brown & Co.**

DEBITS. — Apr. 4, mdse., $15.90; Apr. 5, lumber, $190.72; Apr. 11, mdse., $23.15; Apr. 12, wagon, $90; Apr. 27, mdse., $20.70; surrey, $129.70; May 7, mdse., $40.05; May 12, cash, $100; May 20, lumber, $189.

CREDITS. — Apr. 10, labor, $129.71; Apr. 14, cash, $75; Apr. 21, labor, $29.70; Apr. 28, cash, $147; May 7, labor, $270.10; May 19, stone work, $175.39; May 28, cash, $70.

# DENOMINATE NUMBERS

We measure the quantity of anything by finding how many times it contains some unit of the same kind, called the **unit of measure.**

Thus, the number of bushels in a load of apples is found by seeing how many times it contains the unit of measure, 1 *bushel.*

A **denominate number** is a concrete number whose *unit* is a *measure* established by custom or law; as, 5 yards or 8 bushels. In these numbers 1 yard and 1 bushel are the units of measure.

A **simple denominate number** is a number of one denomination; as, 3 feet, 5 pecks.

A **compound denominate number** is a number composed of two or more denominations that express one quantity; as, 8 yards, 2 feet, 3 inches (length).

## REDUCTION

**Reduction** of denominate numbers is changing their form without changing their value; thus,

2 bu. = 8 pk. = 64 qt.
or, 16 qt. = 2 pk. = .5 bu.

**1.** Review thoroughly these tables: Liquid Measures; Dry Measures; Measures of Length; Avoirdupois Weight; Troy Weight; Time Measures; Stationers Measures; Counting.

NOTE. — The other tables of denominate numbers are found on pages 433 to 436.

2. Change 64 qt. to bushels; to pints.

3. How many pecks in $\frac{1}{4}$ bu.? 3 bu.? $1\frac{3}{4}$ bu.?

4. How many days from June 28 to October 1?

5. Express 5 yards as feet; as the fraction of a rod.

6. How many rods equal $27\frac{1}{2}$ yards? $49\frac{1}{2}$ yards?

7. Change $2.50 to mills; to dimes.

8. Change 1.3 T. to pounds.

9. Express 27 pecks as bushels; as quarts.

10. How many ounces (avoir.) equal $\frac{3}{4}$ lb.? 2.5 lb.? $4\frac{3}{8}$ lb.? $7\frac{1}{4}$ lb.?

11. If a ring is 18 carats fine, what part of it is pure gold?

12. Express in minutes 1.5 hr.; 1.75 hr.; $3\frac{4}{5}$ hr.

13. Express in feet $1\frac{1}{2}$ rd.; $2\frac{1}{4}$ rd.; 8 yd.; and $5\frac{1}{2}$ yd.

14. How many minutes equal 720 seconds? 3600 seconds?

15. How many days is it from January 28, 1908 to March 5, 1908?

16. If a man works 8 hours a day, how many minutes does he work? How many minutes equal 9 hours?

17. How many sheets of paper are there in 2 reams of paper? in 5 reams?

18. I bought 2 gross of lead pencils. How many lead pencils did I buy?

19. If a horse eats 10 quarts of oats a day, how long will 5 bushels of oats last?

20. How many dozen equal 180 things? how many gross?

21. How many days is it from Memorial Day (May 30) to the fourth of July?

22. What is the perimeter of square that is 3.6 ft. on a side?

**23.** How many pecks does a 3-bushel bag hold? a 2-bushel bag? a 5-bushel box?

**24.** A bushel of wheat weighs 60 lb. How many bushels are there in a ton of wheat?

**25.** How many feet are there in 125 yards? in 120 inches?

**26.** Change 5 T. 4 cwt. to tons; to pounds; to hundredweight.

**27.** Find the weight in ounces of a dozen teaspoons, each weighing 12 pennyweights.

**Changing to smaller denominations.**

### Written Work

**1.** Change 7 gal. 3 qt. 1 pt. to pints.

7 gal. $= 7 \times 8$ pt. $= 56$ pt.
3 qt. $= 3 \times 2$ pt. $= 6$ pt.
1 pt. $= 1$ pt.
7 gal. 3 qt. 1 pt. $= 63$ pt.

Since there are 8 pt. in 1 gallon, in 7 gal. there are 7 times 8 pt., or 56 pt. Since there are 2 pt. in 1 qt., in 3 qt. there are 3 times 2 pt., or 6 pt. Hence, in 7 gal. 3 qt. 1 pt. there are 56 pt. + 6 pt. + 1 pt., or 63 pt.

**2.** A dairyman delivered to his customers in one morning 19 gal. 3 qt. 1 pt. of milk. Find the number of pints delivered.

19, number of gal.
4
76
+3
79, number of qt.
2
158
+1
159, number of pt.

The product is numerically the same whatever number is the multiplier. Thus, $4 \times 19 = 19 \times 4$; hence to shorten the work 4 and 2 may be *regarded* as multipliers, although in the explanation it must be remembered that 4 qt. and 2 pt. are really the multiplicands.

Reduce :

3. 28 bu. 3 pk. to quarts.

4. 27 gal. 1 qt. to pints.

5. 10 da. 7 hr. to hours.

6. 8 mi. 80 rd. to rods.

7. 15 da. 9 hr. 20 min. to minutes.

8. 16 T. 15 cwt. 85 lb. to pounds.

9. 1 solar year to hours.

10. 12 fathoms 5 ft. to inches.

11. 25 rd. 3 yd. 2 ft. to feet.

12. 5 knots 1050 ft. to feet.

13. 11 lb. 8 oz. to ounces (avoir.).

14. $\frac{5}{16}$ of a ton to a hundredweight and pounds.

$\frac{5}{16}$ T. = $\frac{5}{16}$ of 20 cwt. = $6\frac{1}{4}$ cwt.
$\frac{1}{4}$ cwt. = $\frac{1}{4}$ of 100 lb. = 25 lb.
$\frac{5}{16}$ of a ton = 6 cwt. 25 lb.

Change:

15. $1\frac{3}{4}$ lb. to ounces.

16. $1\frac{3}{4}$ bu. to pints.

17. 2 mi. 4020 ft. to inches.

18. $\frac{7}{8}$ mi. to rods.

19. $\frac{3}{4}$ long ton to hundredweight and pounds.

20. $\frac{5}{9}$ common years to days, hours, etc.

21. In an automobile race the fastest machine ran $\frac{7}{8}$ of a mile in 36 seconds. Find the number of feet it ran per second.

22. Change .75 week to days, etc.

23. School is in session 4.25 hours. Find the number of minutes it is in session.

24. James lives 1.85 miles from school. Find the number of feet he walks to school.

**Changing to larger denominations.**

### Written Work

**1.** An ice cream dealer retailed in one day 127 pints of ice cream. How many gallons, quarts, etc., did he sell?

2)127, no. of pt.
4)63, no. of qt. + 1 pt.
15, no. of gal. + 3 qt.
117 pt. = 15 gal., 3 qt., 1 pt.

Since 2 pt. = 1 qt., 127 pt. = 63 qt. and 1 pt. over. Since 4 qt. = 1 gal., 63 qt. = 15 gal. and 3 qt. over. Hence, 127 pt. = 15 gal. 3 qt. 1 pt.

NOTE. — The dividend and divisor are regarded as abstract numbers. Do not read 127 pt. ÷ 2 = 63 qt. Such a statement would be absurd.

Change:

**2.** 225 qt. to gallons, etc.
**3.** 2550 qt. to barrels, etc.
**4.** 1463 pk. to bushels, etc.
**5.** 15000 min. to days, etc.
**6.** 3184 in. to rods, etc.
**7.** 5675 sec. to minutes, etc.
**8.** 75000 in. to miles, etc.
**9.** 36481.62 ft. to leagues, etc.
**10.** 175680 oz. to long tons, etc.
**11.** 9049 in. to rods, etc.
**12.** 3 qt. 1 pt. to the decimal of a gallon.

2)1.0, no. pt.
.5
3.
4)3.5, no. qt.
.875, no. gal.

Since 2 pt. = 1 qt., 1 pt. = .5 qt.; 3 qt. + .5 qt. = 3.5 qt. Since 4 qt. = 1 gal., 3.5 qt. = .875 gal.

Change:

**13.** 45 yd. .6 ft. to the decimal of a mile.

**14.** 6 cwt. 8 oz. to the decimal of a ton.

**15.** $\frac{2}{3}$ ft. to the common fraction of a yard.

1 ft. = $\frac{1}{3}$ yd.
$\frac{2}{3}$ ft. = $\frac{2}{3}$ of $\frac{1}{3}$ yd., or $\frac{2}{9}$ yd.

**16.** $\frac{7}{8}$ in. to the fraction of a foot; of a yard.

**17.** 12 oz. to the fraction of an avoirdupois pound.

**18.** 240 rd. to the fraction of a mile.

**19.** The winner in an automobile race won in 21 hr. 3 min. 3.6 sec. What decimal of a day did it take?

**20.** How much did a merchant receive for 3 barrels (42 gallons each) 9 gallons 3 quarts of oil, retailed at 15 cents per gallon?

**21.** A grocer bought 150 bushels of potatoes at 60¢ a bushel. He lost $\frac{1}{5}$ of them by freezing, and retailed the remainder at 10¢ a half peck. How much did he gain?

**22.** How many boxes, each holding a quart, can be filled from 3 bu. 1 pk. 7 qt. of blackberries?

**23.** At $2.88 a bushel, how many quarts of chestnuts can be bought for $13.50?

**24.** What is the profit on 9 quires of paper, bought at $2.40 a ream and sold at a cent a sheet?

**25.** A rural mail carrier's route is 21 miles 176 rd. 4 yd. in length. Find the number of feet he travels in one delivery of mail.

**26.** A fruit grower sold in one season 23 bu. crates of cherries at 10 cents per basket; and 45 bu. crates and 17 baskets of strawberries at 13 cents per basket. Find the amount of the sales.

A crate contains 32 baskets.

**27.** A milk dealer put his milk in pint bottles. Find the number of bottles delivered in one evening if he sold 23 gal., 3 qt., and 1 pt.

**28.** How many 4-ounce packages of soda can be put up from 1 T., 3 cwt., and 75 lb. of soda?

**29.** A huckster bought 3 barrels of apples, each containing $2\frac{3}{4}$ bushels, for $8.25 and retailed them at 15¢ a half peck. Find his profits.

**30.** Find the length of a double-track railroad laid with 1640 rails, each 30 feet in length.

**31.** An ocean steamer in making a certain trip consumed 1920 tons of coal. If the time was 6 days, 5 hours, and 8 minutes, find the average number of pounds consumed per minute.

**32.** The report of a cannon was heard 1 minute 5 seconds after it was discharged. If sound travels 1120 feet per second, how many miles, rods, etc., was the hearer from the cannon?

**33.** A pupil pays $45 tuition in a term of 9 months of 20 days each, and is absent from school 16 days. Counting 6 school hours to a day, find the amount of tuition lost to him by his absence.

## FOREIGN MONEY

### English Money

The standard unit of English money is the **pound,** $4.8665.

| | | |
|---|---|---|
| **4 farthings (far.)** | = | **1 penny (*d.*)** |
| **12 pence** | = | **1 shilling (*s.*)** |
| **20 shillings** | = | **1 pound (£), or sovereign** |

In writing pounds, place the sign first; thus, £ 5.

A *farthing,* like a *mill,* is not coined, but is expressed as a fraction of a penny.

### French Money

The standard unit of French money is the **franc** = $.193.

100 centimes = 1 franc (fr.).

The **peseta** of Spain and the **lira** of Italy are of the same value as the franc. The franc is the standard unit of value also in Belgium and Switzerland.

### German Money

The standard unit of German money is the **mark** = $.238.

100 pfennigs (pf.) = 1 mark (m.).

All systems of money, except English money, are decimal systems. Canada has a decimal system of money like the United States. The denominations are the dollar, the dime, the cent, and the mill.

Memorize:

| | |
|---|---|
| **1 pound** | **= $4.8665** |
| **1 franc** | **= .193** |
| **1 peseta** | **= .193** |
| **1 lira** | **= .193** |
| **1 drachma** | **= .193** |
| **1 mark** | **= .238** |

APPROXIMATE VALUES

| | |
|---|---|
| **1 pound** | **= $5.00** |
| **1 mark** | **= .25** |
| **1 franc** | **= .20** |

Change:

**1.** £1 to *s.*; to *d.*

**2.** 200 centimes to francs.

**3.** £1 5*s.* to *s.*

**4.** 5 francs to centimes.

**5.** 675 centimes to francs.

**6.** 8.5 m. to pfennigs.

**7.** $6\frac{1}{2}$ francs to centimes.

**8.** 15 shillings to *d.*

**9.** A lady paid 6*d.* a yard for cloth. Find the cost, in shillings, of 20 yards.

**10.** An English merchant takes 1 pound and 5 half sovereigns to a bank to exchange for pennies. How many pennies does the merchant receive?

**11.** The admission to a German show costs 50 pfennigs. How many tickets can be bought for $5\frac{1}{2}$ marks?

**12.** In a French school a collection for charity contained 6 francs, 5 half francs, and 250 centimes. Find the value in francs.

## Written Work

Change to U.S. money, expressed in the nearest cent:

| | |
|---|---|
| **1.** £37.5. | **5.** 575 m. |
| **2.** 675 lira. | **6.** 380 drachmas. |
| **3.** 579 peseta. | **7.** 786.8 fr. |
| **4.** £35. | **8.** £6. |

**9.** Change £25 12*s.* 8*d.* to U.S. money. Change 12*s.* 8*d.* to the decimal of a pound; thus, $8d. = \frac{8}{12}$, or $\frac{2}{3}s.$; $12s. + \frac{2}{3}s. = 12\frac{2}{3}s.$ $12\frac{2}{3}s. = £\frac{12\frac{2}{3}}{20}$, or £.633⁺.

Hence, £25 12*s.* 8*d.* = £25.633.

1 pound = \$4.8665.
25.633 × \$4.8665 = \$124.74.

Change to U.S. money expressed to the nearest cent.

| | |
|---|---|
| **10.** £10 5*s.* 6*d.* | **14.** £87 10*s.* $3\frac{1}{2}d.$ |
| **11.** £27 11*d.* | **15.** 769 lira. |
| **12.** £98 19*s.* 5*d.* | **16.** 104 fr. 75 centimes. |
| **13.** 100 m. 60 pf. | **17.** 745 peseta. |

**18.** Change \$1685 to pounds, etc.

\$1685 ÷ \$4.8665 = 346.244+, the number of pounds.
.244 × 20*s.* = 4.88*s.*
.88 × 12*d.* = 10.56*d.* = to the nearest penny 11*d.*
Hence, \$1685 = £346 4*s.* 11*d.*

**19.** The cost of a bill of goods in England was \$780.50. Express the value to the nearest penny in English money.

**20.** A German clock cost in Strasburg \$119. Express the value to the nearest pfennig in German money.

**21.** An Italian workman saved on an average \$27.89 per month. Find the value to the nearest lira.

### ADDITION AND SUBTRACTION

In addition, subtraction, multiplication, and division of denominate numbers, the work is performed just as with other numbers. It is necessary, however, to bear in mind how many units of any denomination equal one unit of the next higher denomination.

#### Written Work

**1.** Add:

| yd. | ft. | in. |
|---|---|---|
| 6 | 2 | 4 |
| 3 | 2 | 6 |
| 4 | 2 | 8 |
| 15 | 1 | 6 |

The sum of the inches is 18 in. or 1 ft. 6 in. Write 6 in. and add 1 ft. to the feet. The sum of the feet is 7 ft., or 2 yd. 1 ft. Write 1 ft. and add 2 yd. to the sum of the yards. The sum of the yards is 15 yd. The sum is 15 yd. 1 ft. 6 in.

**2.** Subtract 54 lb. 9 oz. (av.) from 75 lb. 4 oz. (av.).

| lb. | oz. |
|---|---|
| 75 | 4 |
| 54 | 9 |
| 20 | 11 |

9 oz. cannot be subtracted from 4 oz. Change 75 lb. 4 oz. to 74 lb. 20 oz. 20 oz. – 9 oz. = 11 oz., which is written in the remainder. 74 lb. left in the minuend – 54 lb. = 20 lb. Hence, the remainder is 20 lb. 11 oz.

**3.** Add:

| bu. | pk. | qt. |
|---|---|---|
| 16 | 3 | 5 |
| 18 | 0 | 7 |
| 42 | 1 | 0 |
| 56 | 1 | 3 |

**4.** Add:

| hr. | min. | sec. |
|---|---|---|
| 18 | 44 | 48 |
| 21 | 39 | 29 |
| 27 | 51 | 58 |
| 29 | 36 | 24 |

**5.** Subtract:

| hr. | min. |
|---|---|
| 23 | 42 |
| 17 | 56 |

**6.** Subtract:

| yr. | mo. | wk. | da. |
|---|---|---|---|
| 21 | 5 | 3 | 4 |
| 9 | 7 | 3 | 6 |

The most important application of subtraction of denominate numbers is in finding the *difference* between two dates.

**7.** How long a time elapsed from April 19, 1775 to April 14, 1861?

| yr. | mo. | da. |
|---|---|---|
| 1861 | 4 | 14 |
| 1775 | 4 | 19 |
| 85 | 11 | 25 |

The later date is written in the minuend as the 14th day of the 4th month of 1861; and the earlier date in the subtrahend as the 19th day of the 4th month of 1775. In borrowing, consider 30 days to a month and 12 months to a year. The difference will be the time in years, months, and days.

**8.** Washington was born February 22, 1732; he was inaugurated president April 30, 1789. How old was he when he became president?

**9.** Find the time from the signing of the Declaration of Independence, July 4, 1776 to the beginning of the Civil War, April 14, 1861.

**10.** General Ulysses S. Grant was born April 27, 1822. How old was he when the Civil War closed April 9, 1865?

**11.** Find the sum of the collections at an English theater for four different evenings as follows: £ 479 10*s.* 8*d.*; £ 531 15*s.* 11*d.*; £ 594 9*d.*; £ 401 11*s.* 5*d.*

**12.** Find the difference between the largest collection in problem 11 and each of the others.

**13.** A United States mail agent leaves New York Monday on his trip to Buffalo at 9:35 A.M. and returns Tuesday at 6:15 P.M. How long is he gone?

**14.** A coal miner's 4 wagons for the day weighed as follows: 1 T. 6 cwt. 19 lb.; 1 T. 7 cwt. 13 lb.; 1 T. 5 cwt. 85 lb.; 1 T. 4 cwt. 98 lb. Find the total weight.

## MULTIPLICATION AND DIVISION

### Written Work

**1.** A square field is 40 rd. 5 yd. and 2 ft. on a side. Find the perimeter, or the distance around the field.

| rd. | yd. | ft. | |
|---|---|---|---|
| 40 | 5 | 2 | $4 \times 2$ ft. = 8 ft. or 2 yd. 2 ft. |
| | | 4 | $4 \times 5$ yd. = 20 yd.; 20 yd. + 2 yd. = 22 yd. or 4 rd. |
| | | | $4 \times 40$ rd. = 160 rd.; 160 rd. + 4 rd. = 164 rd. |
| 164 | 0 | 2 | The perimeter of the field is 164 rd. 2 ft. |

**2.** Six English workmen divide their profits equally. Find each one's share, if the total profits are £ 44 17*s.* 6*d.*

| | £ | *s.* | *d.* |
|---|---|---|---|
| 6) | 44 | 17 | 6 |
| | 7 | 9 | 7 |

£ 44 ÷ 6 = £ 7 and £ 2 remaining; £ 2 = 40*s.*; 40*s.* + 17*s.* = 57*s.*; 57*s.* ÷ 6 = 9*s.* and 3*s.* remaining; 3*s.* = 36*d.*; 36*d.* + 6*d.* = 42*d.* 42*d.* ÷ 6 = 7*d.* The share of each workman is £ 7 9*s.* 7*d.*

Division may be performed by changing all the denominations to a *common denomination;* thus, 164 rd. 2 ft. ÷ 40 rd. 5 yd. 2 ft. = 2708 ft. ÷ 677 ft. = 4, the quotient; or by expressing the *common denomination* as a mixed decimal; thus, 172 ft. 3 in. ÷ 6 ft. 6 in. = 172.25 ft. ÷ 6.5 ft. = 26.5, the quotient.

**3.** A gardener sold on an average 6 bushel crates and 17 baskets of blackberries each day for 8 days. Find the total number of bushels and baskets sold.

**4.** 57 students at a ball game each wore a badge $5\frac{1}{2}$ inches long. How many yards and inches of ribbon were needed to make the badges?

**5.** A wire fence inclosing a square field 40 rd. 5 yd. on a side has 4 wires. Find the total length of wire in feet.

**6.** A 45 horse-power automobile used on an average 3 gal. 3 qt. 1 pt. of gasoline daily on a certain trip of 11 days. Find the number of gallons, etc., used.

**7.** An English estate of £8000 10*s.* 8*d.* was divided as follows: the widow received $\frac{1}{4}$ of the estate and the remainder was equally divided among 5 children. Find the amount to the nearest cent in United States money that each received.

**8.** A Greek confectioner bought 12 bushels of chestnuts at \$2.50 per bushel and retailed them at 5¢ per pint. Find his gain.

**9.** How many times will a wheel 10 feet 8 inches in circumference turn in going 12 miles?

**10.** A ball room is $46\frac{1}{2}$ feet long and $30\frac{2}{3}$ feet wide. Find the cost of a picture molding around it at 9¢ per foot.

**11.** A bicyclist traveled 63 miles 170 rods 2 yards in a forenoon, and 30 miles 56 rods less in the afternoon. Find the distance he traveled that day.

**12.** A silver dollar weighs $412\frac{1}{2}$ grains. Find in tons, etc., the weight of \$5,600,000 silver dollars.

**13.** A horse is fed $1\frac{1}{2}$ pecks of oats per day. How much will the oats for the horse cost at 40¢ per bushel for December and January?

**14.** Find $\frac{7}{8}$ of 275 ft. 4 in.; $\frac{3}{4}$ of 11 lb. 8 oz.; $\frac{3}{4}$ of 36 bu. 3 pk. 4 qt.

**15.** A dairyman's sales for each of 4 weeks were as follows: 1st week, 115 gal. 3 qt. 1 pt.; 2d week, 105 gal. 3 qt. 1 pt.; 3d week, 113 gal. 1 qt.; 4th week, 103 gal. 1 qt. 1 pt. If $\frac{1}{8}$ of the total sales were uncollectable, find the cash amount of his sales for the four weeks at $7\frac{1}{2}$¢ per quart.

**16.** A city weighman one morning recorded the weight of 4 loads of hay as follows: 1 T. 5 cwt. 19 lb.; 1 T. 7 cwt. 29 lb.; 1 T. 98 lb.; 1 T. 9 cwt. 3 lb. Find the total amount of the sales at $\$14\frac{1}{2}$ per ton.

## REVIEW PROBLEMS

**1.** What is the sum of \$8.45, \$.55, \$.37$\frac{1}{2}$, \$15.055, and \$49.45?

**2.** If .9 of a ton of structural iron is worth \$23.40, how much are 11.75 tons worth?

**3.** Express as a common fraction in its lowest terms .15625.

**4.** A huckster sold 29$\frac{1}{2}$ bushels of potatoes at \$.80 a bushel and bought apples with the proceeds at \$2.36 a barrel. How many barrels of apples did he receive?

**5.** A carpenter worked 235$\frac{1}{2}$ days for \$2.75 a day. He received \$480.75. How much was still due him?

**6.** Divide .0001 by .00001, and multiply the quotient by 1000.

**7.** If shovels are worth \$15 a dozen, how many dozen can be bought for \$37.50?

**8.** Find the sum of 85 ones, 85 tenths, 85 hundredths, and 85 thousandths.

**9.** The fastest long-distance train in the world (1906) runs from New York to Chicago, a distance of 979.52 miles, in 18 hours. Find the average rate per hour that it runs.

**10.** The fastest English train runs for 8 hours at a speed of 50.18 miles per hour. What distance does it run?

**11.** The regular fare from New York to Chicago is \$21.50, but on this fast train a total excess fare of \$9 is charged. What is the actual cost per mile to the passenger?

**12.** The train in problem 9 makes 6 stops, averaging 5 minutes each. At what rate must the train actually run so as to reach Chicago from New York in 18 hours?

**13.** A gallon of water weighs $8\frac{1}{3}$ pounds. Cast iron is 7.08 times as heavy as water. How much would a piece of iron equal in volume to 3.25 gallons of water weigh?

**14.** Gold is 19.3 times as heavy as water. How much would the same volume of gold weigh?

**15.** A bushel of shelled corn weighs 56 pounds. How many bushels are required to fill an ordinary freight car whose capacity is 60000 pounds?

**16.** The freight on such a car shipped from Kansas City to Albany, N.Y., was \$75. How much was that per bushel?

**17.** This corn cost 56¢ per bushel in Kansas City. How much did this car load of corn cost delivered at Albany?

**18.** The dealer's other expenses in handling this car of corn were \$27.50. He sold it for 70 cents per bushel. How much did he gain?

**19.** In an automobile race 297 miles were covered in 290.173 minutes. What was the time per mile?

**20.** The speed of one machine in this race was 67.63 miles per hour. At this rate how far could the machine travel in 1.25 hours?

**21.** How many rods of hedge surround a school ground 8.625 rods long and 5.75 rods wide?

**22.** If .76 of a pound of gunpowder is niter, 1425.76 pounds of niter is necessary in making how many pounds of gunpowder?

**23.** $\frac{7}{8} \div (.125 \times 7) + (16 \times .375) = ?$

**24.** If the price of gas is $.27 per thousand cubic feet, how much is the average gas bill per month, when 85620 cubic feet are consumed in 6 months?

**25.** Allowing 2.75 bushels to the barrel, how many barrels of apples, at $.65 a bushel, can be bought for $178.75?

**26.** A teacher paid .35 of his salary for board, .18 for clothing, .07 for travel, .1 for incidentals, and had $300 left. How much was his salary?

**27.** From $3\frac{3}{4}$ take the sum of $3\frac{3}{4}$ thousandths and $3\frac{3}{4}$ millionths.

**28.** If seventy-five hundredths of a number is 372, what is eighty-three and one third hundredths of it?

**29.** What is the value of $\left(\frac{2.6}{.4} - \frac{10.8}{4.5} + \frac{1}{2}\right) \times .03$?

**30.** A real estate agent bought 4 lots at $650.50 each. He sold two of them at a loss of $67.25. If he sells the other two at a profit of $75.75, how much does he gain on his investment?

**31.** A school board paid $144 for books: $\frac{2}{5}$ of the amount was paid for General Histories at $1.20 each; $\frac{1}{3}$ of it for Algebras at $1 each; and the remainder for Rhetorics at $.96 each. How many books were purchased?

**32.** A man invested .32 of his money in mining stock, .48 in railroad stock, and had $12000 remaining. How much had he at first?

**33.** An engineer took a contract to build a bridge for $18500. The material cost $10575; he employed 40 men $7\frac{1}{2}$ weeks of six days each at $2.25 per day, and 30 men for $4\frac{1}{2}$ weeks of five days each at $2.75 a day. How much did he gain?

**34.** A merchant bought 140 boys' suits at \$6.75 a suit. He sold $\frac{3}{4}$ of them at \$9.50 a suit and the remainder at \$8.50 a suit. How much did he gain?

**35.** Four men purchased an oil property, the first paying for .3 of it, the second for .375 of it, the third for $\frac{1}{5}$ of it, and the fourth the remainder, which was \$3000. How much did the property cost them?

**36.** A merchant bought 975 pounds of sugar for \$48.75. He sold $\frac{2}{5}$ of it at \$.055 a pound, $\frac{1}{3}$ of it at \$.06 a pound, and the remainder at \$.065 a pound. Find his gain.

**37.** I invested .4 of my money in a farm and deposited .75 of the remainder in a bank. If the amount paid for the farm was \$300 less than the amount deposited in the bank, how much money had I?

Find the sum of the quotients:

**38.**

$5 \div 5$
$5 \div .5$
$5 \div .05$
$5 \div .005$
$.005 \div 5$
$.5 \div 5$
$500 \div .5$

**39.**

$.6 \div .2$
$66 \div .22$
$11 \div .022$
$.08 \div .2$
$.088 \div 22$
$.16 \div .004$
$.078 \div .013$

**40.**

$.001 \div .01$
$.72 \div .004$
$.096 \div .32$
$.198 \div 6.6$
$.05 \div .125$
$8.1 \div .09$
$216 \div .036$
$.8 \div 160$

**41.**

$3.24 \div 18$
$.5 \div .0125$
$675 \div .75$
$9 \div .225$
$.288 \div 32$
$100 \div .001$
$39.2 \div .14$
$4.4 \div 55$

**42.**

$.002 \div 20$
$31.11 \div 6.1$
$.003 \div 37.5$
$16 \div .016$
$.4 \div 250$
$40.04 \div 1.43$
$.576 \div .8$
$43.3 \div 100$

# PRACTICAL MEASUREMENTS

## LENGTH AND SURFACE

To the Teacher. — Secure a 50-foot tape measure and have pupils make as many actual measurements as possible.

**1.** Find the dimensions of the school ground.

**2.** If your school is in the city, measure the length and width of some square near your school, or if in the country, some field.

**3.** Measure $\frac{1}{4}$ of a mile along a street or a public road. Compare a mile with a rod; with a yard; with a foot.

After some practice in actual measurements, the pupils should be able to give quite accurate estimates of short distances.

**4.** Have the pupils estimate the length and the height of the schoolroom; the height of the school building; the length and the width of the playground, etc.

**5.** Have each pupil draw on the blackboard, without the aid of a rule, an inch, a foot, a yard.

**6.** Show by diagram the number of square inches in a square foot.

**7.** Show by diagram the number of square feet in a square yard.

**8.** Draw a square rod on a scale of 3 inches to 1 yard.

**9.** Show by a diagram, on a scale of 1 inch to the yard, the number of square yards or square feet in a square rod.

160 square rods of land is called an **acre.**

A square mile of land is called a **section of land.**

**10.** How many yards are there in the perimeter of a field a mile square?

NOTE. — The perimeter of a figure is the line that bounds it, or the sum of its sides.

**11.** Name the different units of long measure and the different units of surface measure.

**12.** What is the shape of the figure that represents a square inch? a square foot? a square yard? a square rod?

**13.** Draw two figures of different dimensions to represent an acre. How do you show that 160 square rods equals each surface?

**14.** What unit of surface measure is *not* a square unit?

**15.** Have each pupil draw a unit surface, without the aid of a rule, to show a *square inch*, a *square foot*, a *square yard*.

Learn this table of surface or square measure:

| | |
|---|---|
| 144 square inches (sq. in.) | = 1 square foot |
| 9 square feet (sq. ft.) | = 1 square yard (sq. yd.) |
| $30\frac{1}{4}$ square yards (sq. yd.)<br>$272\frac{1}{4}$ square feet (sq. ft.) | = 1 square rod (sq. rd.) |
| 160 square rods (sq. rd.)<br>43560 square feet (sq. ft.) | = 1 acre (A.) |
| 1 mile square | = 1 section |
| 640 acres | = 1 square mile |
| 36 square miles | = 1 western township |
| 100 square feet | = 1 square in roofing and flooring |

## Written Work

Change:

**1.** 3 sq. ft. 48 sq. in. to square inches.

**2.** 4 A. 35 sq. rd. to square rods.

**3.** 125 sq. rd. to a decimal of an acre.

**4.** .375 of an acre to square rods.

**5.** 4 A. to square feet.

**6.** $\frac{3}{4}$ sq. ft. to square inches.

**7.** 4.5 sq. rd. to square inches.

**8.** $2\frac{1}{2}$ sq. mi. to square rods.

**9.** 1800 sq. rd. to acres, etc.

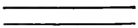
**10.** 1584 sq. in. to square feet, etc.

**11.** 6.75 A. to square feet.

**12.** Mr. Jamison's farm contains 125 A. 120.8 sq. rd. Three fourths of it is purchased at $312.50 per acre, to be laid out in town lots. Find the number of acres sold and the amount received from the sale.

## LINES AND ANGLES

**1.** Observe the two lines. How do they compare in direction?

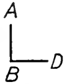


**Parallel lines** are lines that cannot meet however far they may be extended.

An **angle** is the difference in direction of two lines that meet.

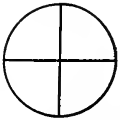
When two lines meet each other forming a square corner, they form a **right angle**; thus, ∟

Lines drawn at right angles to each other are **perpendicular**; thus, *AB* and *BD* in the cut are perpendicular to each other.




**2.** Draw a circle and divide it into 4 equal parts by diameters perpendicular to each other.

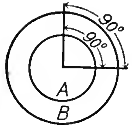
**3.** How many angles are there at the center of the circle? What is each angle called?



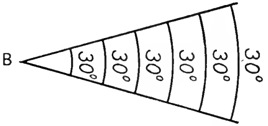
The **circumference** of a circle is the perimeter or distance around it. The circumference of a circle is measured in **degrees**; every circum-

ference, whether large or small, is divided into 360 degrees (written 360°).

$\frac{1}{4}$ of a circle is 90°. Observe that the angle between two lines that meet at a point is measured by the part of the circumference cut by the lines extended.




**4.** How do you explain that $\frac{1}{4}$ of circumference *A* contains as many degrees as $\frac{1}{4}$ of circumference *B*?

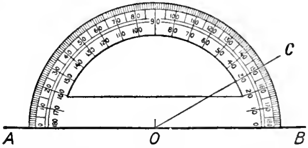



**5.** Observe the figure. Show that the curved lines are simply parts of circumferences of circles that could be formed about the point *B*. How do you show that each curved line measures an angle of 30°? Show the angles on the figure.

**6.** Show that an angle is the difference in direction of two lines that meet at a point and that the angle remains the same, however far the lines may be extended.

## Angular Measure

Angles are measured by an instrument called a **protractor**.




When the center *O* of the protractor is placed at the vertex of the angle to be measured, the size of the angle may be seen on the scale between the lines that form the angle. Thus, *BOC* is an angle of 30°, and *AOC* is an angle of 150°.

Every circumference contains 360 degrees (360°), each degree, 60 minutes (60′), and each minute, 60 seconds (60″).

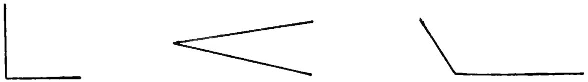
### Table of Angular Measure

| | | |
|---|---|---|
| 60 seconds ($''$) | = | 1 minute ($'$) |
| 60 minutes | = | 1 degree ($^\circ$) |
| 360 degrees | = | 1 circumference (C) |

The length of a degree at the **equator** is $69\frac{1}{5}$ miles.

Draw an angle of 90°; 45°; 60°; 120°; 30°.

### Kinds of Angles



Which one of these angles is a right angle? Why? Which is less than a right angle? Which is greater than a right angle?

A **right angle** is an angle of 90°.

An **acute angle** is an angle less than 90°.

An **obtuse angle** is an angle greater than 90°.

## TRIANGLES

A **triangle** is a surface bounded by three straight lines. (*Tri* means *three.*)

A **vertex** of a triangle is a point where two sides meet.

The **base** of a triangle is the side on which it seems to rest.

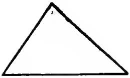
The **altitude** of a triangle is the perpendicular distance from the vertex opposite the base to the base, or the base extended.

Triangles are named in *two* ways:

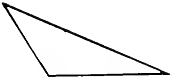
I. From their angles:

(1) **Right-angled** triangles. (One right angle.)
(2) **Acute-angled** triangles. (All angles less than a right angle.)
(3) **Obtuse-angled** triangles. (One angle greater than a right angle.)

RIGHT-ANGLED



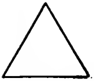
ACUTE-ANGLED



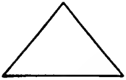
OBTUSE-ANGLED

II. From their sides:

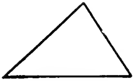
(1) **Equilateral.** (Having three sides equal.)
(2) **Isosceles.** (Having two sides equal.)
(3) **Scalene.** (Having no two sides equal.)



EQUILATERAL



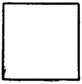
ISOSCELES



SCALENE

**Measuring degrees and angles.**

**1.** How many right angles are there in the square?



SQUARE

**2.** How many right angles are there in the rectangle?

RECTANGLE

**3.** Cut from paper a square. Fold it on the line connecting the opposite corners, and cut it into two triangles.

**4.** How many degrees are there in each angle of each triangle thus formed?

**5.** Every right triangle contains how many degrees?

By Geometry it is shown that the sum of the angles in any triangle is equal to 180°. This can also be shown by measuring the angles with a protractor.

*The sum of all the angles of any triangle is equal to two right angles, or* 180°.

The following numbers in each case represent the size of two angles of a triangle. Find the size of the third angle:

**6.** 90° and 45°

**7.** 90° and 60°

**8.** 120° and 30°

**9.** $120\frac{1}{2}$° and $60\frac{1}{2}$°

**10.** 60° and 40°

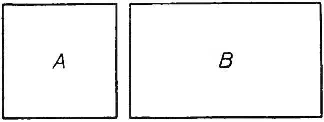
**11.** 100° 45′ and 37°

**12.** 75° 10′ and 95° 30′

**13.** 100° and 45° 40′

## QUADRILATERALS

A **quadrilateral** is a surface having four straight sides. (Quadrilateral means having *four sides.*)




SQUARE RECTANGLE

**1.** Examine the quadrilaterals. What are the essential features of *A* ?

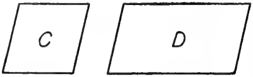
A **square** is a quadrilateral having four equal sides and four right angles.

**2.** What are the essential features of *B*? In what way does figure *B* differ from figure *A* ?

A **rectangle** is a quadrilateral having four straight sides and four right angles.

**3.** Show that the opposite sides of a rectangle must be equal and parallel. Is a square a rectangle?

A **parallelogram** is a quadrilateral whose opposite sides are parallel.



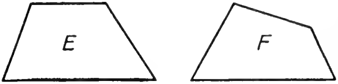

RHOMBUS RHOMBOID

**4.** Examine these quadrilaterals. Why are they parallelograms? How do the sides of surface *C* compare in length? Show that its angles are *not* right angles.

A **rhombus** is a quadrilateral whose sides are *equal*, and whose angles are *not* right angles.

**5.** Why is surface *D* a parallelogram? Show that its angles are *not* right angles. Show that its sides are *not* equal.

A **rhomboid** is a quadrilateral whose *opposite* sides are equal and whose angles are *not* right angles.




TRAPEZOID TRAPEZIUM

**6.** Why is surface *E* a quadrilateral? Why is it not a parallelogram? How many of its sides are parallel?

A **trapezoid** is a quadrilateral having but *two* sides parallel.

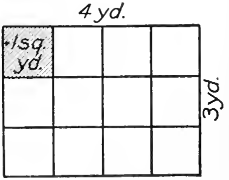
**7.** Why is surface *F* not a trapezoid? What is its name?

A **trapezium** is a quadrilateral having *no* two sides parallel.

**8.** Describe each of the six quadrilaterals named above with reference to its sides and angles. How many of these quadrilaterals are parallelograms? Give reasons.

## AREAS OF RECTANGLES

**Finding the area of a rectangle.**




Find the area of a rectangle 4 yd. long and 3 yd. wide. How long is this rectangle? how wide? What is the *unit* of measure? How many such units are in the first row? in the second? in the entire surface?

If the length and width of a rectangle are expressed in inches, the unit of measure is 1 *sq. in.;* if expressed in feet, the unit of measure is 1 *sq. ft.;* if expressed in rods, the unit of measure is 1 *sq. rd.* If the length and width are expressed in related units, as feet and inches, or yards, feet, etc., the dimensions must be changed to like units before finding the **area**, that is the number of square units it contains.

### Written Work

**1.** Find the area of a flower bed 20 feet 8 inches in length by 10 feet 6 inches in width.

Length $= 20\frac{2}{3}$ ft.; width $= 10\frac{1}{2}$ ft.
Area $= 20\frac{2}{3} \times 10\frac{1}{2} \times 1$ *sq. ft.*, or
$\frac{62}{3} \times \frac{21}{2} \times 1$ *sq. ft.* $= 217$ sq. ft.

*The* **area of a rectangle** *is found by multiplying its unit of measure by the product of its two dimensions when expressed in like units.*

Find the areas of rectangles having the following dimensions:

**2.** 20.5 ft. by 12 ft.

**3.** 21 ft. by 6.9 ft.

**4.** 72 yd. by $40\frac{1}{2}$ yd.

**5.** 6 yd. 1 ft. by 3 yd. $2\frac{1}{2}$ ft.

**6.** 115 ft. by 54 in.

**7.** 45 yd. by 7 ft.

**8.** 108 in. by 3 ft.

**9.** 54 ft. by 108 in.

**10.** How many square yards are there in a lawn 45 feet long and 36 feet wide?

**11.** A square ball-park 600 feet on a side is inclosed with a tight board fence 9 feet in height. Find the outside surface of the fence in square yards.

**12.** Compare in area a surface 8 inches square and a surface 2 inches square; a surface 20 rods square and a surface 40 rods square.

**13.** Bricks are generally 8 in. $\times$ 4 in. $\times$ 2 in. in size. Estimate the number necessary to lay a sidewalk 100 ft. long and 5 ft. wide, if the bricks are laid on the flat side. Find the cost of the bricks needed at \$13.75 per thousand.

**14.** A surveyor finds a field in the form of a rectangle to be 680 ft. long and 330 ft. wide. Find its area without changing feet to rods.

**15.** A field in the form of a rectangle contains 1200 sq. rd. and the length is 40 rods. Find the width.

**16.** How many lots, each 30 ft. by 120 ft., can be made from a plot of ground 120 ft. in depth and containing 10800 sq. ft.? (Make a diagram.)

## PLASTERING AND PAINTING

In plastering, painting, and kalsomining, the unit of measure is the **square yard.**

In some localities an allowance is made for openings and baseboards, but there is no uniform rule in practice. Any allowance should always be specified in the contract.

There are either 50 or 100 laths in a **bundle.** A bundle of 100 is generally estimated to cover 5 square yards of surface.

### Written Work

**1.** How many square yards of plaster are necessary to cover the ceiling of your classroom?

**2.** Find the cost of painting both sides of a tight board fence, 150 ft. long and 8 ft. high, at 15¢ per square yard.

**3.** Allowing nothing for openings, how much will it cost to kalsomine the walls and ceiling of a room 20 ft. long, 16 ft. wide, and 12 ft. high, at 6¢ per square yard?

**4.** A store room is 75 ft. long, 20 ft. wide, and 15 ft. from floor to ceiling. It has a door in the rear 7 ft. by $3\frac{1}{2}$ ft., and a window 8 ft. by 3 ft. How many bundles of laths, each containing 100, are required for the sides, rear, and ceiling, making full allowance for openings?

**5.** How much will it cost to plaster the walls and ceiling of a store room, 40 ft. by 18 ft. and 12 ft. high, at 6¢ per square yard for lathing, and 18¢ per square yard for plastering, deducting $\frac{1}{2}$ the area of 2 doors, each 9 ft. by 4 ft., and of 4 windows, each $6\frac{1}{2}$ ft. by $3\frac{1}{2}$ ft.?

**6.** A building 90 ft. by 24 ft. contains 3 stories, each 13 ft. high. The first story is plastered on the sides and rear. The second and third stories each have 3 windows in the front, each 8 ft. by $3\frac{1}{3}$ ft., and each 2 windows in the rear, each 8 ft. by 3 ft. If the ceilings are sheet iron, find the cost of the plastering, at 33¢ per square yard, deducting for all openings.

**7.** In modern business buildings metal laths are used. Estimate the cost of metal laths, for the building in example 6, at 25¢ per square yard.

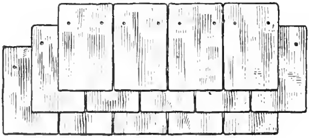
## ROOFING AND FLOORING

In roofing, tiling, and flooring, the unit of measure is the **square** of **100 square feet.**

### Written Work

**1.** Each of the two slopes of a roof is 60 ft. long and 20 ft. wide. Find the cost of covering them with tar paper at $5.60 per square.

**2.** The floor of a hallway 30 ft. by 12 ft. is inlaid with 2-inch square tile. Find the number necessary.



In **roofing** with slate, each course of slate is partly overlapped. Each slate as here shown is 10 in. by 16 in. and has 4 in. exposed to the weather.

**3.** How many square inches of each slate are exposed?

**4.** If a 10-inch by 16-inch slate is exposed 4 inches to the weather, find the number of slates necessary to lay a square (10 ft. by 10 ft.).

**5.** If slate 10 in. by 16 in. is laid 6 in. to the weather, find the number necessary to lay a square. Find the weight of a square of slate at $4\frac{1}{2}$ lb. per square foot.

**6.** Each slope of a roof is 40 ft. by 20 ft. Find the number of slates, 10 in. by 16 in., exposed 4 in. to the weather, required for this roof, allowing nothing for breakage. Find the cost of the slates at $5.50 per square.

There are 250 shingles in a **bunch.**

Shingles average 16 inches in length and 4 inches in width. The exposed surface of a shingle laid $4\frac{1}{2}$ inches to the weather is, therefore, 18 square inches. Without waste 8 shingles will lay one square foot, and 800 shingles will lay 100 square feet, or 1 *square.* Allowing for waste, 4 bunches, or 1000 shingles are estimated to lay a square.

**7.** Allowing nothing for waste, how many bunches of shingles are required to cover a barn roof 35 ft. in width on each side and 70 ft. in length. Find the cost at $4.00 per thousand shingles.

**8.** Adding $\frac{1}{4}$ for waste, estimate the cost at $3.50 per thousand of 157 bunches of shingles required to cover the roof in example 7.

**Flooring** is frequently estimated by the *square.*

**9.** How much will it cost, at $5.00 per square, to lay the floor of a hall 30 ft. by 60 ft., adding $\frac{1}{6}$ for waste?

**10.** Estimate the number of squares of flooring required for two floors of a store room 25 ft. by 60 ft.

## PAPERING AND CARPETING

The unit of measure in wall paper is the **single roll**, which is 8 yards in length and usually 18 inches in width. A **double roll** is 16 yards in length.

In approximating the number of rolls, paper hangers generally deduct from the perimeter of the room the width of the doors and windows. The remaining number of feet divided by $1\frac{1}{2}$ ft. (18 in. = $1\frac{1}{2}$ ft.) gives the number of strips required for the surface of the wall. Dividing the total number of strips by the number that can be cut from a double roll gives the number of double rolls required. Fractional parts of a roll are not sold. The ends of the rolls are generally sufficient to paper the surfaces above and below the doors and windows. Border is sold by the *linear yard.*

Carpet, matting, and border are sold by the **linear yard.** Oil cloth and linoleum are sold by the linear yard or by the **square yard**. Ingrain carpets are usually 1 yard wide, other carpets are generally 27 inches wide.

Liberal allowance must be made for loss in matching.

### Written Work

**1.** Estimate the number of double rolls of paper required for a ceiling 18 ft. by 22 ft., strips running lengthwise.

22 ft. = length of one strip.
16 yd. = 48 ft.; 48 ft. ÷ 22 ft. = 2, the number of *whole* strips in a double roll.
18 ft. ÷ $1\frac{1}{2}$ ft. = 12, the number of strips required.
12 ÷ 2 = 6, the number of double rolls required.

**2.** A dining room 15 ft. by 22 ft. is 11 ft. from baseboard to ceiling. It has four openings $3\frac{1}{2}$ ft. by 7 ft. Estimate the paper required for it, strips on ceiling running lengthwise.

**3.** The dining room in problem 2 has a plate rail extending around it between the openings. Find the cost of this rail at 30¢ per foot.

**4.** How much carpet 27 in. wide, laid the long way of the room, is required for a room 18 ft. long and 15 ft. wide, allowing 12 in. on each strip except the first for matching?

$$\begin{aligned}
6 \text{ yd.} &= \text{the length of one strip.}\\
27 \text{ in.} &= 2\tfrac{1}{4} \text{ ft.; and } 15 \text{ ft.} \div 2\tfrac{1}{4} \text{ ft.} = 6\tfrac{2}{3}, \text{ therefore}\\
7 \quad &= \text{the number of strips.}\\
7 \times 6 \text{ yd.} &= 42 \text{ yd.}\\
6 \times 12 \text{ in.} &= 72 \text{ in., or } 2 \text{ yd., the waste on 6 strips.}\\
42 \text{ yd.} + 2 \text{ yd.} &= 44 \text{ yd. of carpet required.}
\end{aligned}$$

**5.** Explain why it takes fewer yards of carpet to cover a room 18 ft. by 27 ft. with ingrain carpet (1 yard wide) than with Brussels carpet (27 inches wide). Laying the carpet the long way of the room, how many yards of each would it take, if 10 in. were allowed on each strip, except the first, for matching?

**6.** The widths of certain floors are 15 ft., $13\frac{1}{2}$ ft., $15\frac{3}{4}$ ft., 18 ft., 16 ft. Estimate the number of strips of ingrain carpet necessary to cover each room.

**7.** Estimate the number of strips of Brussels carpet necessary to cover each room described in example 6.

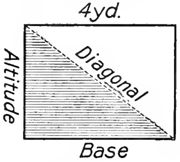
**8.** Find the cost of covering a kitchen $13\frac{1}{2}$ ft. by 12 ft. with linoleum at \$1.60 per yard double width, if $\frac{1}{6}$ of a yard is allowed for matching and the linoleum is laid the long way of the room.

**9.** Estimate the difference in cost between covering a room 18 ft. by $20\frac{1}{4}$ ft. with Axminster carpet 27 inches wide, at \$1.45 per yard, laid lengthwise, allowing 12 inches on each strip except the first for matching, and covering the room with ingrain carpet at 85¢ per yard, laid in the same way, allowing 12 inches on each strip, except the first, for matching.

## AREAS

### Finding the area of a right triangle.

**1.** Find the area of a right triangle whose base is 4 yards and whose altitude is 3 yards.




Observe: 1. That the diagonal divides the rectangle into two equal *right* triangles.

2. That the *unit* of measure is 1 *sq. yd.*

3. That the area of one of the right triangles is $\frac{1}{2}$ of the area of the rectangle; that is, $\frac{1}{2}$ of $4 \times 3 \times 1$ *sq. yd.*, or 6 *sq. yd.*

*The* **area of a right triangle** *is found by multiplying the unit of measure by half the product of the base and the altitude.*

Name the unit of measure, and find the area of each of the following right triangles:

**2.** Base 10 in., altitude 6 in.

**3.** Base 12 yd., altitude 8 yd.

**4.** Base 9 ft., altitude 6 ft.

**5.** Base 10 ft., altitude 7 ft.

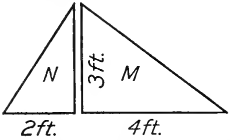
**6.** Base 14 ft., altitude 10 ft.

**7.** Base 6 ft., altitude 20 ft.

### Finding the area of any triangle.

#### Written Work

**1.** Find the area of two right triangles, the base of one being 2 ft. and of the other 4 ft. and the alt. of each 3 ft.



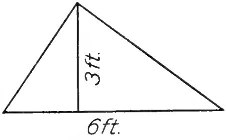

Draw the triangles as shown in the figure.

Observe: 1. That the *unit* of measure is 1 *sq. ft.*

2. That the area of the right triangle $N$ is equal to $\frac{1}{2}$ of $2 \times 3 \times 1$ *sq. ft.*, or 3 sq. ft.

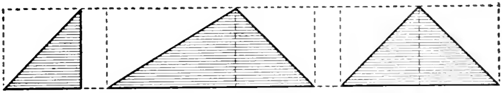
3. That the area of the right triangle $M$ is equal to $\frac{1}{2}$ of $4 \times 3 \times 1$ *sq. ft.*, or 6 sq. ft. Therefore, the area of $N$ plus the area of $M$ is equal to $\frac{1}{2}$ of $6 \times 3 \times 1$ *sq. ft.*, or 9 sq. ft.

**2.** Find the area of a triangle whose base is 6 ft. and whose altitude is 3 ft.




Observe that the area of the triangle in example 2 is equal to the area of the two right triangles in example 1, and is, therefore, equal to $\frac{1}{2}$ of $6 \times 3 \times 1$ *sq. ft.*, or 9 sq. ft.

Show by cutting and folding paper, as indicated in the following figures, that the area of each triangle is equal to one half the area of a rectangle, having the same base and altitude.



*The* **area of any triangle** *is found by multiplying the unit of measure by one half the product of the base and altitude.*

Find the area of the following triangles:

**3.** Base 20 ft., altitude 14 ft.

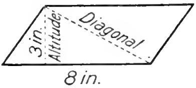
**4.** Altitude 8 ft., base 15 ft.

**5.** Base 10 ft., altitude 30 ft.

**6.** Altitude 50 ft., base 18 ft.

**Finding the area of a parallelogram.**

Find the area of a parallelogram whose base is 8 in. and altitude 3 in.




Observe: 1. That the diagonal of the parallelogram divides it into two equal triangles.

2. That the area of each triangle is equal to $\frac{1}{2}$ of $8 \times 3 \times 1$ *sq. in.*, and the area of the parallelogram is equal to $\frac{2}{2}$, or once the product of the base and altitude; that is, $8 \times 3 \times 1$ *sq. in.*, or 24 sq. in.

*The* **area of a parallelogram** *is found by multiplying the unit of measure by the product of the base and altitude.*

### Written Work

Find the area in acres of:

**1.** A parallelogram whose base is 140 rd. and altitude 60 rd.

**2.** A rhomboid whose base is 90 rods and altitude $50\frac{1}{2}$ rods.

**3.** A rhombus whose base is 120 rods and altitude 100 rods.

Find the altitude of:

**4.** A rhomboid whose area is 7.5 A., base 48 rd.

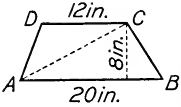
**5.** A rhomboid whose area is 6.125 A., base 140 rd.

**6.** Find the base of a parallelogram whose altitude is $60\frac{1}{2}$ rods and whose area is 30.25 acres.

**Finding the area of a trapezoid.**

### Written Work

**1.** Find the area of a trapezoid whose parallel sides are 20 inches and 12 inches, and whose altitude is 8 inches.




Examine the trapezoid *ABCD*. Draw the diagonal *AC*, dividing it into two triangles.

Observe: 1. That the area of the trapezoid is equal to the area of its two triangles *ABC* and *ACD*.

2. That the area of triangle *ABC* equals $\frac{1}{2}$ of $20 \times 8 \times 1$ *sq. in.*

3. That the area of the triangle *ACD* equals $\frac{1}{2}$ of $12 \times 8 \times 1$ *sq. in.*

4. That the area of the trapezoid equals $\frac{1}{2}$ of $(20 + 12) \times 8 \times 1$ *sq. in.*, or 128 square inches.

*The* **area of a trapezoid** *is found by multiplying the unit of measure by the product of the altitude and $\frac{1}{2}$ the sum of the parallel sides.*

**2.** The parallel sides of a trapezoid are 38 inches and 62 inches respectively, and its altitude is 21 inches Find its area.

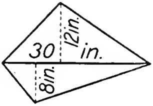
**3.** The area of a trapezoid is 2.5 acres. The sum of its parallel sides is 80 rods. Find its altitude.

**4.** The area of a trapezoid is $4\frac{1}{4}$ A. If its altitude is 20 rd., and one of its parallel sides 38 rd., what is the other?

**Finding the area of a trapezium.**

### Written Work

**1.** Find the area of a trapezium whose diagonal is 30 inches, and whose altitudes are 12 inches and 8 inches respectively.




Observe: 1. That the area of the trapezium equals the area of its two triangles.

2. That the area of one triangle equals $\frac{1}{2}$ of $30 \times 8 \times 1$ *sq. in.*

3. That the area of the other triangle equals $\frac{1}{2}$ of $30 \times 12 \times 1$ *sq. in.*

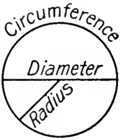
4. That the area of the trapezium equals $\frac{1}{2}$ of $30 \times 20 \times 1$ *sq. in.*, or 300 sq. in.

*The* **area of a trapezium** *is found by dividing it into triangles and finding the sum of their areas.*

**2.** The base line dividing a trapezium into two triangles is 40 ft. The altitude of one triangle is 10 ft., of the other is 12 ft. Find the area of the trapezium.

**3.** A trapezium is divided into two triangles by a line 28 ft. long. Find the area of the trapezium, if the altitude of one triangle is 8 ft. and of the other triangle 14 ft.

## THE CIRCLE




Observe the figure. What is its shape? Observe that its boundary line changes its direction regularly at every point.

A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the *center*.

The **circumference** of a circle is its bounding line.

A **diameter** is a straight line passing through the center with both ends terminating in the circumference.

A **radius** is a straight line extending from the center to the circumference.

Measure carefully with a cord the distance around a circle 1 foot in diameter, and you will find it is about 3.1416 ft. in circumference. This relation of diameter to circumference is true of all circles.

*The* **circumference of a circle** *is found by multiplying the diameter by* 3.1416. *This ratio is represented by the symbol* $\pi$ (*pī*).

*The* **diameter of a circle** *is found by dividing the circumference by* 3.1416.

### Written Work

Find the circumference if the diameter is:

**1.** 15 ft. **4.** $10\frac{2}{3}$ ft. **7.** 40 ft. 4 in.

**2.** 25 ft. **5.** 12 ft. 6 in. **8.** 30 in.

**3.** 60 ft. **6.** 25 yd. **9.** 8 yd. 2 ft. 4 in.

**10.** Circumference equals 25.1328 ft. Radius = ?

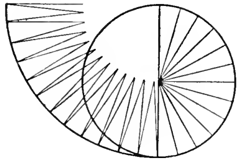
**11.** Circumference equals 125.664 yd. Diameter = ?

**Finding the area of a circle.**

**1st Method.**

Examine the figure:

Observe: 1. That the circle may be considered as made up of triangles whose bases form the circumference.



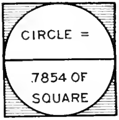
2. That the radius of the circle is equal to the altitudes of the triangles.

3. That the area of the circle is equal to the areas of all the triangles, or $\frac{1}{2}$ of the sum of their bases (circumference) by their altitude (radius).

*The* **area of a circle** *is found by multiplying the circumference by one half the radius.*

**2d Method.**

Examine the circle inscribed in the square.




Observe: 1. That the diameter of the circle is just equal to the side of the square.

2. That *much* of the surface of the square, but *not all of it* lies within the circumference. Careful measurement shows that about .7854 of the surface of any square lies within the circumference of the inscribed circle.

*The* **area of a circle** *equals* .7854 *of the surface of the circumscribed square.*

### Written Work

Circumference $= C$. Diameter $= D$. Radius $= R$. Area $= A$.

Find the area if:

| | | |
|---|---|---|
| **1.** $D = 10$ rd. | **4.** $R = 18$ ft. | **7.** $R = 40$ rd. |
| **2.** $R = 10$ rd. | **5.** $D = 20$ in. | **8.** $D = 25$ yd. |
| **3.** $D = 18$ ft. | **6.** $R = 20$ in. | **9.** $R = 40$ ft. |

Find the area to two decimal places if:

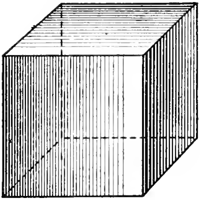
| | | |
|---|---|---|
| **10.** $C = 3.1416$ ft. | **13.** $D = 35$ yd. | **16.** $R = 125$ ft. |
| **11.** $C = 6.2832$ rd. | **14.** $R = 10$ ft. | **17.** $D = 120$ yd. |
| **12.** $C = 94.248$ in. | **15.** $D = 10$ yd. | **18.** $R = 19$ yd. |

**19.** A circle 20 ft. in diameter is inscribed in a square. What is the area of one of the corners within the square, but outside the limits of the circle?

**20.** A circular fountain 20 ft. in diameter is surrounded by a cement walk 4 ft. wide. How much will the walk cost at \$1.50 per square yard?

Note. — Find the difference between the areas of the two circles, the first bounded by the circumference of the fountain, and the second, by the circumference of the walk.

## SOLIDS



1. How many faces has this solid? What is their shape? How do they compare in size?

A **cube** is a solid bounded by six equal square faces.

2. Every 1-inch cube rests on how many square inches of surface?

3. Show that 144 one-inch cubes may be placed on 1 square foot of surface.

4. How many cubes would make 12 such layers?

5. Show that 1728 cubic inches equal 1 cubic foot.

6. How many 1-inch cubes can be put into a cubical box whose edge is 3 inches?

7. How many 1-foot cubes can be placed on 9 square feet of surface? (Make diagram.)

8. How many cubes are there in three such layers?

9. Show that 27 cubic feet equal 1 cubic yard.

Learn this table of solid or cubic measure:

| | |
|---|---|
| **1728 cubic inches (cu. in.)** | **= 1 cubic foot (cu. ft.)** |
| **27 cubic feet (cu. ft.)** | **= 1 cubic yard (cu. yd.)** |
| **128 cubic feet (cu. ft.)** | **= 1 cord of wood or tanbark** |
| **100 cubic feet (cu. ft.)** | **= 1 cord of stone** |

1 cubic yard of earth equals 1 **load.**

The unit of cubic measure is a cube whose edge is one of the linear units; thus, a cube each edge of which is one inch in length is a **cubic inch.**

A **cubic foot** is a cube whose edge is one foot.

A **cubic yard** is a cube whose edge is one yard.

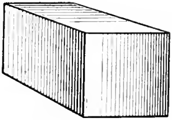
A **cord of wood or tanbark** is a pile of 4-foot wood or tanbark 8 feet long and 4 feet high.

A **cord of short wood** is a pile of short lengths 8 feet long and 4 feet high.

*The* **number of cords of short wood** *in a pile is found by dividing the number of square feet in one side by* 32.

## Surface of Rectangular Solids

How is the surface of each face found? How many faces has this solid? Show that the sum of the faces in this solid is the surface of the solid.



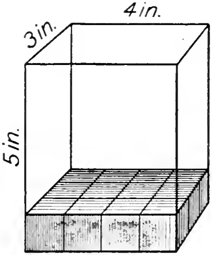
A **rectangular solid** is a solid bounded by six rectangular surfaces.

### Written Work

Find the entire surface of:

| Rectangular Solids | Cubes |
|---|---|
| **1.** 12 ft. by 8 ft. by 6 ft. | **8.** 4 inches on an edge. |
| **2.** 20 ft. by 10 in. by 10 in. | **9.** 12 inches on an edge. |
| **3.** 16 ft. by 2 ft. by $1\frac{1}{2}$ ft. | **10.** 2 feet on an edge. |
| **4.** 10 ft. by 8 ft. by 7 ft. | **11.** $12\frac{1}{2}$ inches on an edge. |
| **5.** 6 ft. by 5 ft. by 5 ft. | **12.** 14 inches on an edge. |
| **6.** 13 ft. by 8 ft. by 3 ft. | **13.** $11\frac{1}{2}$ feet on an edge. |
| **7.** 20 ft. by 9 ft. by 7 ft. | **14.** $14\frac{1}{2}$ inches on an edge. |

## Volume of Rectangular Solids




Scale: $\frac{1}{4}$ inch = 1 in.

Each cube in the solid represents **one cubic inch.** How many cubic inches are there in the first layer? How many such layers does this solid contain? How many cubic inches does the solid contain? Observe that the product of the three dimensions expresses the number of *cubic units*.

The **volume** of a solid is the number of cubic units it contains.

If the dimensions are expressed in inches, the unit of measure is 1 *cubic inch;* if expressed in feet, the unit of measure is 1 *cubic foot;* if expressed in yards, the unit of measure is 1 *cubic yard*. If the dimensions are expressed in related units, as feet and inches, or yards and feet, they must first be changed to like units.

### Written Work

1. Find the volume of a rectangular solid 8 ft. 6 in. square and 12 ft. 4 in. in length.

Thickness = 8.5 ft.; width = 8.5 ft.; length = $12\frac{1}{3}$ ft.
Contents or volume = $8.5 \times 8.5 \times 12\frac{1}{3} \times 1$ *cu. ft.*, or $891.08\frac{1}{3}$ cu. ft.

*The* **volume of a rectangular solid** *is found by multiplying the unit of measure by the product of its three dimensions when expressed in like units.*

Find the contents or volume of the following solids:

2. 10 ft. by 6 ft. 3 in. by 4 ft.
3. 12 ft. by 9 ft. 6 in. by 6 ft.
4. 10 ft. square and 8 ft. high.
5. 1 yd. by 2 ft. by 18 in.
6. 68 in. by 1 ft. by 10 in.
7. 5 yd. by $1\frac{1}{2}$ yd. by 2 ft.

**8.** How many loads of earth must be removed in excavating for a cellar 30 ft. by 24 ft. and 8 ft. in depth?

**9.** Estimate the number of cakes of soap 3 inches by 2 inches by 2 inches that can be packed in a box 3 feet by 2 feet by 2 feet.

**10.** A schoolroom is 40 ft. by 28 ft. by 16 ft. How many cubic feet of air space are there for each of 39 pupils and their teacher?

**11.** How many cords of 4-foot wood are there in a pile 40 ft. long and 4 ft. high?

**12.** Estimate the number of cords of 18-inch wood in 3 piles each 60 ft. long and 4 ft. high.

**13.** How many cubical boxes 3 ft. 6 in. on an edge can be placed in a storage room 14 ft. in length, width, and height?

In a certain township, the piles of 20-inch wood in the yards of four schools were as follows:

| | No. of Piles | Length of Piles | Height of Piles |
|---|---|---|---|
| **14.** Sykes, | 1 | 60 ft. | 4 ft. |
| | 2 | 40 ft. | 6 ft. |
| **15.** Graham, | 2 | 50 ft. | 5 ft. |
| | 1 | 60 ft. | 4 ft. |
| **16.** Wilson, | 1 | 72 ft. | 5 ft. |
| | 2 | 45 ft. | 6 ft. |
| **17.** Clark, | 2 | 36 ft. | 5 ft. |
| | 1 | 60 ft. | 4 ft. |

Estimate the number of cords at each school and the value of the wood at $1.85 per cord.

**18.** Find the number of loads of earth removed in excavating for a cellar 16 feet wide, 30 feet long, and 6 feet in depth.

**19.** A pile of tanbark is 8 ft. wide, 9 ft. high, and 100 ft. long. Find the number of cords.

**20.** A cubical block of granite 2 ft. on an edge is what part of a cubical block of granite 6 ft. on an edge?

When possible, use cancellation in the following problems:

**21.** Cape Cod cranberries are shipped in a crate whose inside dimensions are 20 in. $\times$ $10\frac{1}{2}$ in. $\times$ $6\frac{3}{4}$ in. How many cubic inches are there in a crate?

**22.** Sweet potatoes are sometimes sold in a box $19\frac{1}{4}$ in. $\times$ $11\frac{3}{4}$ in. $\times$ 10 in. How much does this differ from a bushel (2150.4 cu. in.)?

**23.** Colorado apples are sometimes shipped in a box 18 in. $\times$ $11\frac{1}{2}$ in. $\times$ 11 in. How many cubic inches more or less than a bushel does such a box contain?

**24.** Colorado apples are sometimes shipped in a box 16 in. $\times$ $11\frac{3}{4}$ in. $\times$ $8\frac{5}{8}$ in. How many cubic inches does such a box contain?

**25.** California celery is shipped in crates $24\frac{1}{4}$ in. $\times$ 22 in. $\times$ $20\frac{1}{2}$ in. How many cubic inches are there in such a crate?

**26.** California dates are sold in boxes $17\frac{1}{4}$ in. $\times$ 10 in. $\times$ $9\frac{1}{4}$ in. How many cubic inches are there in such a box?

**27.** Figs are packed solid in a box 12 in. $\times$ 9 in. $\times$ $1\frac{3}{4}$ in. How many cubic inches are there in such a box?

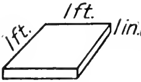
**28.** The standard California orange box is now 24 in. $\times$ $11\frac{1}{2}$ in. $\times$ 12 in. Tangerines are shipped in boxes 24 in. $\times$ 12 in. $\times$ $6\frac{1}{2}$ in. Which is the larger, and how many cubic inches larger is it?

**29.** How many cubic inches are there in a box $5\frac{1}{2}$ in. $\times$ $8\frac{1}{4}$ in. $\times$ $2\frac{3}{4}$ in.? Find the number of cubic inches in a box $8\frac{1}{2}$ in. $\times$ $12\frac{1}{2}$ in. $\times$ $4\frac{1}{4}$ in.

## LUMBER

### Measurement of lumber.

**Lumber** is any kind of sawed timber as boards, planks, sills, etc. The unit of lumber measure is the **board foot**; it is a board 1 foot long, 1 foot wide, and 1 inch thick. Draw it.



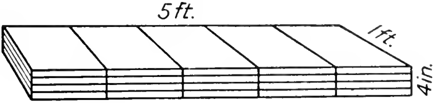
ONE BOARD FOOT

NOTE. — Boards less than 1 inch in thickness are measured as if they were 1 inch thick; boards over 1 inch in thickness are measured by their actual thickness in inches and fractions of an inch.

**1.** How many board feet are there in a board 1 foot wide, 1 inch thick, and 3 feet long? 5 feet long? 9 feet long?

**2.** How many board feet are there in a board 6 inches wide, 1 inch thick, and 3 feet long? 10 feet long? In a board 6 inches wide, $\frac{1}{2}$ inch thick, and 12 feet long?

**3.** How many board feet are there in a sill 5 feet long, 1 foot wide, and 4 inches thick?




### Written Work

**1.** Find the number of board feet in a sill 18 ft. long, 10 in. wide, and 8 in. thick.

10 in. = $\frac{5}{6}$ ft. One surface = $18 \times \frac{5}{6} \times 1$ *board foot*, or 15 board feet. The sill contains $8 \times 15$ board feet, or 120 board feet.

*The* **number of board feet in a piece of lumber** *is found by multiplying the number of board feet in one surface by the number of inches in thickness.*

Find the number of board feet in the following:

2. 1 board, 10 ft. long, $1\frac{1}{2}$ ft. wide, and 1 in. thick.

3. 1 board, 16 ft. long, $1\frac{1}{2}$ ft. wide, and $\frac{3}{4}$ in. thick.

4. 2 boards, each 16 ft. long, 1 ft. wide, and $\frac{1}{2}$ in. thick.

5. 6 boards, 15 ft. $\times$ 2 ft. $\times$ 1 in.

6. 4 boards, 16 ft. $\times$ $1\frac{1}{2}$ ft. $\times$ $\frac{1}{2}$ in.

How many feet of lumber are there in:

7. 1 plank, 12 ft. long, 1 ft. wide, and 3 in. thick?

8. 1 sill, 15 ft. long, $1\frac{1}{4}$ ft. wide, and 8 in. thick?

9. 4 planks, 12 ft. long, $1\frac{1}{2}$ ft. wide, and 2 in. thick?

10. 2 pieces, 18 ft. by 1 ft. by 1 ft.?

Find the number of feet of lumber in:

11. 10 planks, each 8 ft. long, $1\frac{1}{2}$ ft. wide, and 3 in. thick.

12. 12 sills, each 20 ft. long and 10 in. square.

13. 20 joists, each 12 ft. long, 12 in. wide, and 3 in. thick.

14. 3 beams, each 40 ft. long and 10 in. by 12 in.

15. 30 scantlings, each 16 ft. long and 2 in. by 3 in.

16. How much will the flooring for two rooms, each 18 ft. $\times$ 20 ft., cost at \$30 per M.?

**Buying and selling lumber.**

Lumber is bought and sold by the *thousand board feet.* In practice the cost is computed at so much per board foot; thus, \$20 per thousand feet (M.) is \$.02 *per board foot.*

Show that \$35 per M. = \$.035 per board foot.
\$60 per M. = \$.06 per board foot.

### Written Work

**1.** Estimate the cost of 378 feet of oak at \$26 per M.; of 6389 ft. white pine at \$48 per M.; of 972 ft. cherry at \$72 per M.; of 693 ft. white ash at \$47 per M.

Find the cost at \$35 per M. of:

**2.** 50 boards, 16 ft. long, 12 in. wide, and 1 in. thick.

**3.** 60 boards, 12 ft. long, 15 in. wide, and $1\frac{1}{2}$ in. thick.

**4.** 100 boards, 15 ft. long, 6 in. wide, and $\frac{3}{4}$ in. thick.

**5.** 75 boards, 18 ft. long, 10 in. wide, and 1 in. thick.

**6.** 45 boards, 16 ft. long, 5 in. wide, and 1 in. thick.

Short forms are used by carpenters, architects, and mechanics; thus, one mark (′) represents feet, and two marks (″) represent inches.

Find the number of board feet:

**7.** 120 studding, $2'' \times 4'' \times 12'$.

**8.** 400 planks, $2'' \times 1' \times 16'$.

**9.** 300 boards, $1'' \times 10'' \times 14'$.

**10.** 600 boards, $1'' \times 6'' \times 16'$.

**11.** 100 boards, $\frac{3}{4}'' \times 12'' \times 16'$.

**12.** 15 sills, $6'' \times 10'' \times 20'$.

**13.** 250 joists, $2'' \times 8'' \times 24'$.

**14.** 70 sills, $10'' \times 12'' \times 30'$.

**15.** 125 sleepers, $3'' \times 10'' \times 28'$.

**16.** 200 boards, $\frac{1}{2}'' \times 4\frac{1}{2}'' \times 16'$.

**17.** 500 joists, $2\frac{1}{2}'' \times 8'' \times 20'$.

**18.** 325 planks, $3'' \times 14'' \times 16'$.

**19.** 300 sills, $5'' \times 8'' \times 24'$.

**20.** 50 posts, $10'' \times 12'' \times 14'$.

**21.** 400 studding, $2'' \times 3'' \times 18'$.

**22.** 500 boards, $1\frac{1}{2}'' \times 10'' \times 16'$.

**23.** Estimate the cost of the planks in examples 8 and 18, at $.027 per board foot.

**24.** Estimate the cost of the sills in examples 12, 14, and 19, at $.032 per board foot.

**25.** Estimate the cost of the studding in examples 7 and 21, at $.026 per board foot.

### CONCRETE, STONE, AND BRICKWORK

**Concrete work** is estimated by the *cubic yard.*

**Stone work** is estimated by the *perch,* of 24.75 cu. ft., or by the *cord* of 100 cu. ft. Stones are often sold by the pound. 3200 pounds are estimated to lay 1 perch.

In estimating either *contract work* or *cost of labor,* in concrete and stone work, the distance around the wall is considered the *length.* In cases where there are inside corners, however, as at *a* and *b* in the figure on p. 226, add, for each inside corner, *twice* the thickness of the wall. This measures all corners *twice.* In estimating *material,* deduct for openings and measure the corners but *once.*

Range work and lintels are measured by the *linear foot.*

**Brickwork** is estimated by the *thousand.* Bricks vary in size, but they are usually 8″ by 4″ by 2″.

In estimating the number of bricks in a wall, measure the corners once, deduct all openings, and multiply the number of square feet remaining in the surface by 7 when the wall is 1 brick thick; by 14 when the wall is 2 bricks thick; and by 21 when the wall is 3 bricks thick.

### Written Work

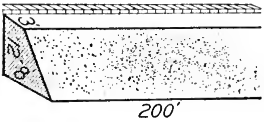
**1.** Find the number of cords of stone in a breakwater 200 ft. long, 14 ft. wide, and 16 ft. high.

**2.** A building 150 ft. by 130 ft. has a concrete foundation 4 ft. in width and 10 ft. in depth below the structural iron. Estimate the number of cubic yards of material used.

3. If the cement, the gravel, and the sand are in the ratio of 1, 5, and 2, find the number of loads of gravel and of sand used in the construction of the foundation in example 2.

4. Estimate the contract cost of the concrete work at \$7.75 per cubic yard.

5. Estimate the number of cubic yards of concrete in this retaining wall.

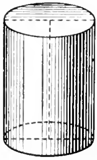



6. The walls of a brick house 36 ft. long, 24 ft. wide, and 18 ft. high are 13 in. or 3 bricks thick. Estimate the number of bricks required for the walls, allowing for 11 windows averaging $3\frac{1}{2}$ ft. by 6 ft., and 2 doors averaging $3\frac{1}{2}$ ft. by 7 ft.

7. A house whose walls are 9 in. or 2 bricks thick is 40 ft. long, 30 ft. wide, and 24 ft. high. Estimate the number of bricks required for the walls, allowing for 12 windows 3 ft. by 7 ft., and 3 doors $3\frac{1}{2}$ ft. by 8 ft.

8. The stone work for the foundation of a house 28 ft. by 38 ft. is $1\frac{1}{2}$ ft. in thickness and 6 ft. in height to the range work. Estimate the cost of the stone work at \$6.30 a perch, and the range work along the two sides and the rear at 60 cents per linear foot.

## THE CYLINDER

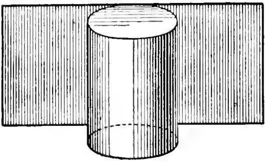


Examine this solid.

How many ends or bases has it? What is the shape of each? Are the bases equal and parallel? Describe the shape of the body.

A **cylinder** is a solid whose two bases are equal and parallel circles and whose diameter is uniform.

The **convex surface** of a cylinder is the lateral or curved surface. The **altitude** is the perpendicular distance between its two bases.



Examine this cylinder.

Observe: 1. That if a piece of paper is fitted to cover its convex surface and then unrolled, its form will be that of a rectangle.

2. That the circumference of the base is the *length* of the rectangle, and the altitude of the cylinder is the *width* of the rectangle.

*The* **convex surface of a cylinder** *is found by multiplying the unit of measure by the product of the circumference and the altitude.*

*The* **entire surface of a cylinder** *is found by adding the area of the bases to the convex surface.*

### Written Work

Find the convex surface of a cylinder:

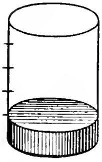
**1.** *D.* 10 in., height 24 in.
**2.** *D.* 15 in., height 30 in.
**3.** *D.* 2 ft., height 10 ft.
**4.** *D.* 20 in., height 4 ft.
**5.** *D.* 8 in., height 4 ft.
**6.** *D.* 6 ft., height 15 ft.

Find the entire surface of:

**7.** A water tank 12 ft. in diameter and 12 ft. in height.

**8.** A steam boiler 15 ft. long and 3 ft. in diameter.

**9.** Find the volume of a cylinder 3 ft. in diameter and 5 ft. high.



Observe: 1. That the area of the base is $3^2 \times .7854 \times 1$ *sq. ft.*, or 7.0686 sq. ft.

2. That the first row of cubic units contains 7.0686 cu. ft.

3. That the cylinder contains 5 times 7.0686 cu. ft., or 35.343 cu. ft.

*The* **volume of a cylinder** *is found by multiplying the unit of measure by the area of the base and this product by the height of the cylinder.*

Find the volume of a gas tank, silo, cistern, etc.:

**10.** *D.* 15 ft., height 18 ft.

**11.** *D.* 25 ft., height 30 ft.

**12.** *D.* 16 ft., height 20 ft.

**13.** *D.* 20 ft., depth 15 ft.

**14.** *R.* 2 ft., depth 8 ft.

**15.** *R.* 8 ft., height 30 ft.

**16.** *D.* 1 ft., length 16 ft.

**17.** *D.* 5 ft., length 12 ft.

## BINS, TANKS, AND CISTERNS

Wheat and other grains are generally sold by weight, but the capacity of bins is often estimated in bushels. The capacity of tanks and cisterns is estimated in gallons or barrels.

NOTE. — The *standard bushel* in the United States contains **2150.42 cubic inches**, stricken measure, and **2747.71 cubic inches** heaped measure.

**231 cu. in. = 1 gal.** **$31\frac{1}{2}$ gal. = 1 bbl.** when estimating contents.

### Written Work

Find contents in bushels of:

**1.** A bin 20 ft. by 10 ft. by 5 ft.

**2.** A box 12 ft. × 9 ft. × 6 ft.

**3.** A metal trough for watering cattle is 12 ft. long, 3 ft. wide, and 20 in. deep. Estimate the number of gallons it holds.

**4.** A cistern tank for a windmill pump is 8 ft. in diameter and 10 ft. in depth. Estimate the number of barrels of water it holds.

**5.** The rainfall on a certain day was $1\frac{1}{4}$ inches. Find the number of barrels of water that fell on Mr. Anderson's flower plot which is 20 ft. long and 10 ft. wide.

## APPROXIMATE MEASUREMENTS

Approximate equivalents of the following measures are:

| | | |
|---|---|---|
| 1 bu. shelled grains | = | $1\frac{1}{4}$ cu. ft. |
| 1 bu. apples, coal, roots, corn in ear, etc. | = | $1\frac{5}{9}$ cu. ft. |
| 1 bbl. in estimating contents | = | $4\frac{1}{2}$ cu. ft. |
| 1 cu. ft. of water | = | $62\frac{1}{2}$ lb. |
| 1 gal. of water | = | $8\frac{1}{3}$ lb. |
| 1 cu. ft. of any liquid | = | $7\frac{1}{2}$ gal. |
| 1 ton of hay well packed | = | 450 cu. ft. |
| 1 ton of clover hay | = | 550 cu. ft. |
| 1 ton of bituminous coal | = | 42 cu. ft. |
| 1 ton of hard coal | = | 35 cu. ft. |

### Written Work

**1.** A water meter registered 900 gallons of water consumed in a month. Estimate the weight of the water used, and its volume in cubic feet.

**2.** The inside measurement of a wagon box is 12 ft. 4 in. by 3 ft. 6 in. by 16 in. Estimate the number of tons, etc., of anthracite coal it would contain; the number of tons, etc., of soft coal.

**3.** The rainfall on a roof 20 ft. by 30 ft. during April and May was 9.5 in. Find the weight of the water that fell on the roof during that time.

**4.** A swimming pool is 80 ft. long, 60 ft. wide, and 5 ft. deep. Estimate the number of barrels of water in the pool.

**5.** Estimate the number of bushels in an oat bin 14 ft. long and 10 ft. wide if the bin is filled with oats to a depth of 6 feet.

**6.** How many tons of hard coal are there in a bin 16 ft. × 12 ft., when the pile is 1 ft. high?

**7.** There are 5 ft. of water in a cistern 4 ft. in diameter. How many gallons of water are there in the cistern?

**8.** Find the weight of the water in a railroad tank 12 ft. in diameter and 16 ft. in depth, if the tank has 12 ft. of water in it.

**9.** How many bushels of wheat can be shipped in a car whose inside measurements are 36 ft. by 8 ft. 6 in. by 8 ft.?

**10.** Estimate the number of tons of clover hay in a mow 60 ft. by 18 ft. by 16 ft. Estimate the number of tons of timothy hay in the same mow.

## REVIEW PROBLEMS

**1.** A field containing 20 acres is 64 rods long. How wide is it?

**2.** The area of the floor of a schoolroom contains 1120 sq. ft. The air in the room occupies 16800 cu. ft. What is the height of the ceiling?

**3.** How many tiles 6 in. square are required for a hall 40 ft. by 20 ft. 6 in.?

**4.** The side of a square is 20 inches. Find its area; its perimeter.

**5.** The edge of a cube is 18 inches. Find its surface; its contents.

**6.** How many cubical boxes whose edges are 6 in. can be put into a box 8 ft. 6 in. by 4 ft. 6 in. by 3 ft.?

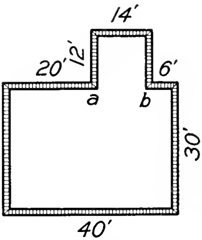
**7.** How many cakes of soap 2 in. × 2 in. × 4 in. may be packed in a box 2 ft. long, 1 ft. wide, and 1 ft. high?

**8.** The edges of two cubes are respectively 10 inches and 12 inches. How much more surface has one than the other?

**9.** Find the cost of a farm, 480 rods long and 320 rods wide, at \$60 per acre.

**10.** How much will it cost to put a wire fence around this farm at 50¢ per rod?

**11.** A ranchman bought one square mile of land at \$10 per acre. He put a fence around it and then divided it by fences into four equal square farms for his sons. Find the entire cost if the fence cost \$.40 per rod.




**12.** Estimate the contract price of building this cellar wall 18 in. thick and $6\frac{1}{2}$ ft. in height at \$5.95 a perch.

Distance around the wall = 164 ft.

Add to this distance *twice* the thickness of the wall for each of the inside corners, *a* and *b*; that is, *twice* 2 × 18 in., or 6 ft. Then, the length of the wall with 8 corners counted *twice* = 164 ft. + 6 ft. = 170 ft. The volume of the wall = $170 \times 6\frac{1}{2} \times 1\frac{1}{2} \times 1$ *cu. ft.* = 1657.5 cu. ft. The number of perch of stone = $1657\frac{1}{2} \div 24\frac{3}{4} = 66\frac{32}{33}$ perch. The cost of the wall = $66\frac{32}{33} \times$ \$5.95 = \$398.47.

QUERY. — Why are the 8 corners measured *twice*?

**13.** Estimate the number of bricks necessary for a dwelling erected on the foundation as given in example 12, if the walls are 3 bricks (13 in.) thick and 20 feet in height, making an allowance of 150 square feet for openings.

QUERY. — Why are the 8 corners measured *once*?

**14.** Estimate the cost of the face brick for the above dwelling at \$16.50 per thousand and \$9.00 per thousand for laying.

NOTE. — Consider the face brick as a wall 4 in. thick and measure the 8 corners *once*.

**15.** A level lot 60 ft. by 120 ft. has erected on it a dwelling 36 ft. by 42 ft. If the excavating averages 5 ft. and the removed earth is placed on the lot, to what height will it raise the grade of the lot?

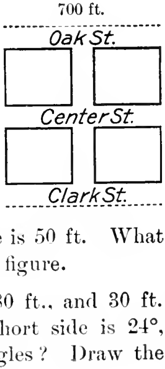
**16.** A certain town has a cylindrical water tank 20 ft. in diameter and 45 ft. in height. The gauge shows 30 ft. of water in the tank. Estimate the weight of the water.

**17.** How many yards of carpet, 27 in. wide, are required for a room 24 ft. by 20 ft. 3 in.? Strips are to run lengthwise.

**18.** How much will it cost to carpet a room 24 feet square with carpet 27 inches wide, at \$1.25 per yard, allowing 10 inches on each strip except the first for matching?

**19.** How much will it cost to plaster a room 20 ft. by 16 ft., and 12 ft. to the ceiling, at 20 ¢ per square yard, allowing for one door $3\frac{1}{2}$ ft. by 7 ft., and 2 windows, each 4 ft. by 6 ft.?

**20.** This plot of ground is 700 ft. long and 600 ft. wide. Find the cost of grading streets 40 ft. in width run through the center each way, as here shown, at \$1.90 per linear yard. Find the amount from the sale of lots 30′ × 140′ facing on Oak, Center, and Clark Streets, at \$20 per front foot.



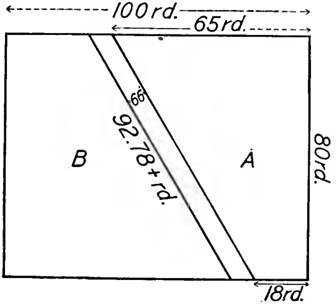

**21.** Each of the three sides of a triangle is 50 ft. What is the size of each of its angles? Draw the figure.

**22.** The sides of a triangle are 80 ft., 80 ft., and 30 ft. respectively. If the angle opposite the short side is 24°, what is the size of each of the other angles? Draw the figure.

**23.** The angle opposite one of the equal sides of an isosceles triangle is 75°. Find the size of the other two angles.

**24.** One angle in a right triangle is 13.75°. Find the other two angles.

**25.** The angle opposite the base in an isosceles triangle is $18\frac{1}{2}$°. What is the size of the other two angles?




**26.** A railroad company owns a strip of land in the form of a parallelogram, 66 ft. wide and 92.78 + rd. long through this farm. Find the area of A and the area of the part owned by the railroad. How can the area of B be found?

**27.** A wheel is 3 ft. in diameter. How many revolutions will it make in moving forward 942.48 ft.?

**28.** The speed of a vessel for 5 hours was 23.17 knots per hour. Find her average speed per hour in statute miles.

**29.** Lead is 11.35 times as heavy as water. Find the value of a cubic foot of lead at 5 ¢ per pound.

**30.** How many rolls of paper are required for a room 20 ft. × 18 ft., and 13 ft. 3 in. from the top of the baseboard to the ceiling, allowing for 2 windows $3\frac{1}{2}$ ft. in width and one door $3\frac{1}{2}$ ft. wide (papering the ceiling lengthwise)?

**31.** The base of a triangle is 30 ft. and its altitude 23 ft. Find its area.

**32.** A barn is 80 ft. by 50 ft. It is 40 ft. to the base of the gable and 58 ft. to the top of the gable. How much will it cost to paint it at 8 ¢ per square yard?

**33.** If the slope of the roof of the barn in problem 32 is 32 ft. long, and it projects $1\frac{1}{2}$ ft. at each end, how much will it cost to roof it at \$8 per square?

**34.** A schoolroom is 40 ft. long and 30 ft. wide. Estimating 450 cubic feet of air to each person, what should be the height of the room to accommodate 39 pupils and their teacher?

**35.** Find the cost of a stone wall 30 ft. long, $2\frac{1}{2}$ ft. thick, and 6 ft. high, at \$5.30 a perch.

**36.** How much will it cost to cement the floor of a cellar, 40 ft. by 20 ft., at 90¢ a square yard?

**37.** A street 50 ft. from curb to curb is opened for a distance of 300 yards. How much will it cost to excavate it to a depth of 1 foot at 40¢ per cubic yard?

**38.** How much will the curb of this street cost at 26¢ per linear foot?

**39.** The sidewalk on this street is 12 ft. wide, including a curb of 8 inches. How much will the brick for the walk cost, at \$9 per thousand, if the exposed surface of a brick is 4 in. $\times$ 8 in.?

**40.** A farmer built a circular silo 12 ft. in diameter and 24 feet high. Find its contents in cubic feet.

**41.** How many blocks of ice, 2 ft. $\times$ 1 ft. $\times$ 1 ft., can be packed in a car 6 ft. $\times$ 8 ft. $\times$ 40 ft.? Ice is .92 as heavy as water. Find the weight of the ice if a cubic foot of water weighs $62\frac{1}{2}$ lb.

**42.** A cistern is 4 ft. in diameter and 6 ft. deep. How many barrels of water will it contain?

**43.** Estimate the weight of the water in a tank 8 ft. long, 6 ft. wide, and 2 ft. deep.

**44.** A vault is 5 ft. square and 6 ft. deep. How much will it cost to cement the sides and bottom at \$.50 per sq. ft.?

**45.** A circular amusement park is 80 rods in diameter. Find the cost of the boards for a tight board fence 8 ft. high, inclosing the park, at $ 20 per M.

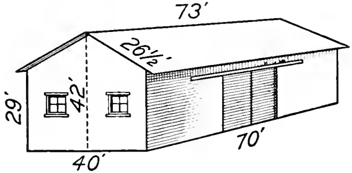
**46.** A corner lot in Seattle is 25 ft. by 100 ft. At $ 25 per M., what will be the cost for 2-inch plank for a 10-foot sidewalk in front and on the side, including the corner?

NOTE. — Illustrate by diagram.

**47.** Mr. Ames owns a 50-ft. lot fronting on a street 60 ft. wide from curb to curb. The law compels him to pay $\frac{1}{3}$ of the cost of paving the street in front of his lot. How much will it cost at $ 2.90 per square yard?

**48.** A tank open at the top is 50 ft. long, 4 ft. wide, and 3 ft. deep. How much will the lead for lining it cost, at 8 ¢ per pound, estimating 4 pounds to a square foot?

**49.** Clay weighs 1.2 as much as the same volume of water. Estimate the weight of a load of clay.




**50.** Find the cost of painting the sides and ends of this hay barn at 15 ¢ per square yard, and the cost of staining the roof at 12 ¢ per square yard.

**51.** In excavating for a cellar 60 ft. long, 30 ft. wide, and 8 ft. deep, the material was evenly distributed over a lot 90 ft. by 40 ft. To what depth was the lot covered?

**52.** A two-story school building has 8 rooms 30 ft. × 32 ft. and a hallway 28 ft. × 15 ft., on each floor. How much will the flooring for the building cost at $ 44 per M.?

**53.** In digging a sewer $3\frac{1}{2}$ ft. in width and 8 ft. deep, $1244\frac{4}{9}$ cubic yards of earth were excavated. Find the length of the sewer in feet.

# ANALYSIS

## THE EQUATION

**1.** $8 = 8$ **2.** $8 + 4 = 12$ **3.** $8 = 12 - 4$

In example (1) we have an equal number on each side of the equality sign. In example (2) we have $8 + 4 = 12$; but in example (3), in order to preserve the equality, when we take 4 from the left of the equality sign in example (2), we must subtract 4 from the number on the right of the equality sign. Thus, $8 = 12 - 4$.

**Test:**

$$\begin{array}{rcr} 8 + 4 & = & 12 \\ -4 & & -4 \\ \hline 8 & = & 12 - 4 \end{array}$$

**4.** $8 = 6 + 2$ **5.** $8 - 2 = 6$

*Observe that a number may be moved from one side of an equation to another by changing its sign.*

### Written Work

Change the following so that the first number in each problem will stand alone at the left of the equality sign:

**6.** $20 - 10 = 10$

**7.** $40 - 15 = 25$

**8.** $80 + 15 = 95$

**9.** $100 + 75 = 175$

**10.** $75 - 20 = 55$

**11.** $85 - 5 - 10 = 70$

**12.** $90 - 10 + 5 = 85$

**13.** $100 + 10 - 20 = 90$

**14.** First add, then subtract, 5 from each member of the equation $10 = 10$.

(*a.*)
$$\begin{array}{rcr} 10 & = & 10 \\ +5 & = & +5 \\ \hline 15 & = & 15 \end{array}$$

(*b.*)
$$\begin{array}{rcr} 10 & = & 10 \\ -5 & = & -5 \\ \hline 5 & = & 5 \end{array}$$

*The same number may be added to or subtracted from both sides of an equation without destroying the equality.*

### Factors and their Product

**1.** 5 times a certain number is 35. What is the number?

$$\begin{array}{c} \text{FACTORS} \qquad \text{PRODUCT} \\ 5 \times \text{the number} = 35 \\ \text{The number} = 35 \div 5 = 7 \end{array}$$

*When the product of two factors is divided by one of the factors, the quotient is the other factor. When one of the factors is unknown, it may be found by dividing the product by the known factor.*

State the factors and solve:

**2.** 5 times John's money = \$40. How many dollars has he?

**3.** 2 times A's sheep are 60. How many sheep has he?

**4.** 6 times B's age is 360 years. How old is he?

**5.** $\frac{1}{2}$ of a number is 150. What is the number?

**6.** 1.25 times a number is 30. What is the number?

**7.** .75 of a number is 75. Find the number.

**8.** $\frac{1}{3}$ of a number is 75. Find the number.

$$\begin{aligned} \tfrac{1}{3} \text{ of the number} &= 75 \\ \text{The number} &= 3 \times 75 = 225 \end{aligned}$$

The work may be shortened by calling the unknown factor $x$. For example,

**9.** Mr. Brown's profits equal 4 times Mr. Long's profits, and together their profits are \$125. Find the profits of each.

$$\begin{aligned} \text{Let } x &= \text{Mr. Long's profits.} \\ 4x &= \text{Mr. Brown's profits} \\ \hline 5x &= \$125, \text{ or the profits of both.} \\ x &= \$25, \text{ Mr. Long's profits.} \\ 4x &= \$100, \text{ Mr. Brown's profits.} \end{aligned}$$

**10.** Mr. Byers and Mr. Boydson together have 240 acres of land, and Mr. Byers has 40 acres more than Mr. Boydson. How many acres has each?

Let $x$ = the number of acres in Mr. Boydson's farm.
$x + 40$ = the number of acres in Mr. Byers's farm.

$2x + 40$ = the number of acres in both farms, or 240 acres.

$$2x = 240 - 40$$
$$2x = 200$$

$x = 100$, the number of acres in Mr. Boydson's farm.
$x + 40 = 140$, the number of acres in Mr. Byers's farm.

Solve first by written analysis, then orally :

**11.** Four times my money and \$6 more is \$50. How much money have I?

**12.** \$80 is \$5 more than twice the cost of a bicycle. Find the cost.

**13.** Harry's age plus $\frac{1}{2}$ his age plus 6 years equals 30 years. How old is he?

**14.** $2\frac{1}{2}$ times the number of books in Henry's library, less 5, equals 70. How many books has he?

**15.** James spent $\frac{1}{4}$ of his money for a top, $\frac{5}{8}$ of it for a ball, and had 10 cents remaining. How much money had he at first?

**16.** After paying $\frac{1}{2}$ and $\frac{1}{4}$ of my debts, I still owed \$25. How much did I owe at first?

**17.** A merchant lost $\frac{1}{5}$ of his capital, then gained $\frac{1}{2}$ as much as he had left, and then had \$10800. How much was his capital at first?

**18.** Robert's money, diminished by $\frac{1}{2}$ and $\frac{1}{3}$ of itself, equals \$1.25. How much money has he?

**19.** After a fruit dealer had sold $\frac{3}{5}$ of his apples, and $\frac{1}{2}$ of the remainder, he had 12 bushels left. How many bushels had he at first?

**20.** If $\frac{3}{8}$ of Wilbur's money is increased by $\frac{1}{4}$ of $\frac{3}{4}$ of his money, the sum will be \$54. How much money has he?

**21.** A banker gave a $\frac{3}{8}$ interest in a bank to one son, a $\frac{1}{2}$ interest to another son, and the remaining interest, valued at $10000, to his wife. What was the value of the bank?

**22.** A lot was sold for $360, which was $\frac{3}{4}$ of what it cost. Find the cost.

**23.** Mr. Amos sold his farm for $3300, which was $\frac{3}{8}$ more than it cost him. Find the cost.

**24.** A typewriter spends $\frac{3}{5}$ of his income and saves $400. How much is his income?

**25.** A suit of clothes was sold for $18, which was $\frac{1}{4}$ less than it cost. Find the cost.

**26.** A merchant sold apples at $1.80 a barrel, which was $\frac{1}{5}$ more than they cost him. How much did they cost per barrel?

**27.** There are 1200 pupils in a certain school. The number of boys is $\frac{3}{5}$ of the number of girls. How many girls are there in the school?

**28.** The united ages of Alice and Mary are 28 years; Alice is $\frac{3}{4}$ as old as Mary. How old is Alice?

**29.** A lady paid $30 for a watch, which was $\frac{1}{4}$ more than it cost. Find the cost.

**30.** A house that cost $1200 was sold for $\frac{1}{4}$ more than the cost. How much was gained?

**31.** A has 45 cents, which is $\frac{1}{2}$ more than B has. How much has B?

**32.** How much will a two-thirds interest in a store cost, when a four-fifths interest sells for $6000?

**33.** There are 40 pupils in a school, and $\frac{1}{4}$ of them are boys. How many girls are there in the school?

**34.** If a man owns $\frac{5}{8}$ of a mill, and sells $\frac{3}{5}$ of his interest for $3000, what is the value of the mill?

**35.** A lady paid \$35 for a cloak. $\frac{6}{7}$ of the cost of the cloak was $\frac{3}{4}$ of what she paid for other clothing. How much did all cost?

**36.** A house and lot cost \$8000. The lot cost $\frac{2}{3}$ as much as the house. How much did the lot cost?

**37.** A traveler went 30 miles in two days; the first day he went $1\frac{1}{2}$ times as far as the second. How many miles did he travel the first day?

**38.** A sold a watch to B for $\frac{1}{4}$ more than it cost him; B sold it to C for \$20, thereby losing $\frac{1}{5}$ of what it cost him. How much did A pay for it?

**39.** The difference between two numbers is 36, and the greater is three times the less. What are the numbers?

**40.** If to $\frac{2}{5}$ of Mr. Barnhart's salary you add \$40, the sum will be $\frac{2}{3}$ of his salary. How much is his salary?

**41.** A merchant sold a dry goods store, receiving $\frac{3}{4}$ of the price in cash. He invested $\frac{2}{5}$ of the sum received in a jewelry store bought at \$900. For how much was the dry goods store sold?

**42.** What is the value of $\frac{3}{4}$ of a ship if $\frac{4}{5}$ of it is worth \$48000?

**43.** A man invested $\frac{3}{4}$ of his money in a lot. Had he paid \$100 more he would have invested $\frac{8}{9}$ of his money. Find the cost of the lot.

**44.** Two merchants had a profit of \$9600. After paying $\frac{1}{6}$ of it for rent, they divided the rest so that one received $\frac{3}{5}$ as much as the other. How much did each receive?

**45.** If Wayne can do a piece of work in 6 days, what part of it can he do in 1 day? If Ray can do the same work in 4 days, what part of it can he do in 1 day?

**46.** What part can Ray and Wayne both do in 1 day?

**47.** If Ray and Wayne can do $\frac{5}{12}$ of it in 1 day, in how many days can they do the whole work, working together?

**48.** If 4 men can do a piece of work in 3 days, how long will it take 1 man to do it?

**49.** If one man can do a piece of work in 12 days, how long will it take 2 men to do it?

**50.** If 8 men can do a piece of work in $2\frac{1}{2}$ days, how long will it take 5 men to do it?

**51.** A jeweler sold a watch for \$60, and gained $\frac{1}{4}$ of the cost. What was the cost of the watch?

**52.** A horse, sleigh, and harness cost \$220; the sleigh cost twice as much as the harness, and the horse cost 4 times as much as the sleigh. Find the cost of each.

**53.** Ira can do a piece of work in 12 days; Baxter can do it in 16 days. If Baxter's wages are \$1.50 a day, how much per day should Ira receive?

**54.** A man has three houses which together are worth \$5700. The second house is worth twice as much as the first, and the third is worth $\frac{9}{10}$ as much as the other two. How much is the third house worth?

**55.** If A can do a piece of work in $1\frac{1}{2}$ days, B in 3 days, and C in 4 days, in what time can they do it working together?

Suggestion. — Since A does the whole work in $\frac{3}{2}$ days, he does $\frac{2}{3}$ of it in a day; B does $\frac{1}{3}$ in a day, and C $\frac{1}{4}$ in a day. What part of the work do they do together in a day? How long, then, will it take them to do the whole work together?

**56.** A man bought three automobiles. The first cost \$1500, the second cost $1\frac{2}{5}$ times as much as the third, and the third cost twice as much as the first. Find the cost of the second and the third.

# PERCENTAGE

The term **per cent** means *hundredths* or *by the hundred*. The sign for it is %.

Thus, five hundredths may be written $\frac{5}{100}$, .05, 5 per cent, or 5%. These are called **equivalents**.

**Percentage** is the process of computing by hundredths. It is simply an application of decimal fractions.

Write both as a *decimal* and as a *common fraction* in its lowest terms each of the following per cents; thus, $10\% = .10 = \frac{10}{100} = \frac{1}{10}$.

| | | | |
|---|---|---|---|
| **1.** 5% | **5.** $37\frac{1}{2}$% | **9.** 120% | **13.** 75% |
| **2.** $6\frac{2}{3}$% | **6.** $14\frac{2}{7}$% | **10.** 250% | **14.** 300% |
| **3.** $8\frac{1}{3}$% | **7.** $33\frac{1}{3}$% | **11.** 43% | **15.** $\frac{1}{2}$% |
| **4.** $12\frac{1}{2}$% | **8.** 125% | **12.** 65% | **16.** $\frac{3}{4}$% |

Write the following decimals as fractions and per cents:

| | | | |
|---|---|---|---|
| **17.** .05 | **21.** $.08\frac{1}{3}$ | **25.** 1.50 | **29.** .25 |
| **18.** .20 | **22.** 2.50 | **26.** $.11\frac{1}{9}$ | **30.** $.16\frac{2}{3}$ |
| **19.** $.33\frac{1}{3}$ | **23.** 1.25 | **27.** $.03\frac{1}{3}$ | **31.** $.37\frac{1}{2}$ |
| **20.** .50 | **24.** 1.20 | **28.** $.06\frac{2}{3}$ | **32.** .45 |

Write the following as decimals and as per cents:

| | | | |
|---|---|---|---|
| **33.** $\frac{1}{2}$ | **37.** $\frac{1}{5}$ | **41.** $\frac{1}{9}$ | **45.** $\frac{1}{12}$ |
| **34.** $\frac{1}{3}$ | **38.** $\frac{1}{6}$ | **42.** $\frac{1}{10}$ | **46.** $\frac{5}{8}$ |
| **35.** $\frac{1}{4}$ | **39.** $\frac{1}{7}$ | **43.** $\frac{1}{16}$ | **47.** $\frac{5}{6}$ |
| **36.** $\frac{2}{3}$ | **40.** $\frac{1}{8}$ | **44.** $\frac{3}{8}$ | **48.** $\frac{7}{8}$ |

Memorize the following equivalents:

| | |
|---|---|
| $1\% = \frac{1}{100}$ | $14\frac{2}{7}\% = \frac{1}{7}$ |
| $2\% = \frac{1}{50}$ | $16\frac{2}{3}\% = \frac{1}{6}$ |
| $3\frac{1}{3}\% = \frac{1}{30}$ | $20\% = \frac{1}{5}$ |
| $4\% = \frac{1}{25}$ | $25\% = \frac{1}{4}$ |
| $5\% = \frac{1}{20}$ | $33\frac{1}{3}\% = \frac{1}{3}$ |
| $6\frac{1}{4}\% = \frac{1}{16}$ | $37\frac{1}{2}\% = \frac{3}{8}$ |
| $6\frac{2}{3}\% = \frac{1}{15}$ | $50\% = \frac{1}{2}$ |
| $8\frac{1}{3}\% = \frac{1}{12}$ | $62\frac{1}{2}\% = \frac{5}{8}$ |
| $9\frac{1}{9}\% = \frac{1}{11}$ | $66\frac{2}{3}\% = \frac{2}{3}$ |
| $10\% = \frac{1}{10}$ | $75\% = \frac{3}{4}$ |
| $11\frac{1}{9}\% = \frac{1}{9}$ | $83\frac{1}{3}\% = \frac{5}{6}$ |
| $12\frac{1}{2}\% = \frac{1}{8}$ | $87\frac{1}{2}\% = \frac{7}{8}$ |

**49.** What is 50% of 100? 40? 10? 2? $\frac{1}{2}$? $\frac{1}{4}$?

**50.** What is $16\frac{2}{3}\%$ of 90? 150? 120? 9? 12? 3?

**51.** What is $33\frac{1}{3}\%$ of 3000? 2000? 50? 75? 100?

**52.** What is $12\frac{1}{2}\%$ of 200? 32? 96? 4? 6? 1?

**53.** What is $37\frac{1}{2}\%$ of 72? 56? 800? 2000? 40? 8?

**54.** What is 20% of 400? 600? $\frac{1}{2}$? $\frac{1}{4}$? 16? 20?

**55.** Find $12\frac{1}{2}\%$ of 16; of 48; of 72; of 96; of 168; of 800; of 220; of 404.

**56.** Find 5% of 25; of 50; of 75; of 100; of 125.

In each example in 56 we have two terms, a per cent and a *number*. In each case we are to find 5% of the number. The number of which we take the $\frac{5}{100}$ (viz. 25, 50, etc.) is called the *base*. The number of hundredths (5) to be taken is called the *rate*, and the number of hundredths *actually taken*, that is, the answer, is called the *percentage*.

The **base** is the number on which the percentage is computed.

The **rate** or **rate per cent** is the number of hundredths. We generally express rate as a decimal.

The **percentage** is the product obtained by taking a certain per cent of the base.

The **sum** or **amount** is the base plus the percentage.

The **difference** is the base minus the percentage.

**Finding a given per cent of any number.**

**1.** What is 20% of 300? *Think* of 20% of 300 as $\frac{1}{5}$ of 300, or 60.

Find:

**2.** 2% of 100
**3.** 5% of 400
**4.** 10% of 500
**5.** 20% of 800
**6.** 50% of 1200
**7.** 40% of 1000
**8.** 25% of 360
**9.** 30% of 90
**10.** 3% of 420
**11.** 2% of 500
**12.** 6% of 150
**13.** 7% of 800
**14.** 5% of 440
**15.** 6% of 550
**16.** $3\frac{1}{3}$% of 900
**17.** $8\frac{1}{3}$% of 480
**18.** $9\frac{1}{6}$% of 660
**19.** $11\frac{1}{9}$% of 729
**20.** $12\frac{1}{2}$% of 648
**21.** $16\frac{2}{3}$% of 366
**22.** $33\frac{1}{3}$% of 333
**23.** $62\frac{1}{2}$% of 864
**24.** $66\frac{2}{3}$% of 724
**25.** 75% of 968
**26.** $87\frac{1}{2}$% of 568
**27.** 6% of \$60
**28.** 8% of \$560
**29.** 10% of 350 acres
**30.** 30% of 960 sheep
**31.** $62\frac{1}{2}$% of 856

## Written Work

**1.** What is 7% of 245? $66\frac{2}{3}$% of 300?

(*a*)
$$\begin{array}{rl} 245 & \text{base} \\ .07 & \text{rate} \\ \hline 17.15 & \text{percentage} \end{array}$$

7% of a number equals .07 of it. Therefore, 7% of 245 is .07 times 245, or 17.15.

(*b*) $\frac{2}{\cancel{3}}$ of $\overset{100}{\cancel{300}} = 200$

$66\frac{2}{3}$% of a number equals $\frac{2}{3}$ of it. $\frac{2}{3}$ of $300 = 200$.

***A* given per cent of any number** *is found by multiplying the base by the rate.*

Find:

2. $4\%$ of 328
3. $9\%$ of \$126
4. $11\%$ of 263
5. $15\%$ of 380
6. $24\%$ of 165.5
7. $38\%$ of \$77.50
8. $72\%$ of 328
9. $80\%$ of 6.75
10. $33\frac{1}{3}\%$ of 75
11. $60\%$ of $\frac{5}{6}$
12. $14\frac{2}{7}\%$ of 105
13. $75\%$ of $\frac{4}{5}$
14. $87\frac{1}{2}\%$ of 168
15. $33\frac{1}{3}\%$ of 336

16. In a school of 400 pupils, 45% are girls. How many girls are there? how many boys?

17. A clerk who received \$50 a month had his wages increased 15%. How much were his wages increased?

18. If iron ore yields 63% of pure metal to the ton, how much iron is there in 40 tons of ore?

19. A merchant, failing in business, paid 85% of his debts. How much should a creditor receive whose claim is \$2850?

20. A bill of goods cost \$137.50. How much was gained by selling the goods at a profit of 12%?

21. Compare 48% of \$45 and 45% of \$48.

22. I owe a debt of \$246.50. If I pay 40% of it at one time, and 50% of the remainder at another time, how much do I still owe?

23. An automobile cost \$3500 and the repairs for 2 years were 10% of the cost. If the automobile was sold at 40% reduction from the cost, find the entire loss.

24. The operating expenses of a factory are 45% of the sales. If the sales for a year amount to \$650450, how much are the operating expenses?

**25.** 400 men were employed in a factory at daily wages averaging \$1.95. If 100 of these men received $33\frac{1}{3}\%$ of the entire daily wages, find their average daily wages. Find the average daily wages of the other 300 men.

**26.** Three newsboys, John, James, and Henry earned together \$550 in a year. John earned 40%, James 60% of the remainder, and Henry what remained. Find how much each earned.

**27.** 40% of a Western farm containing 600 acres is in wheat, 30% of the remainder in corn, $66\frac{2}{3}\%$ of the remainder in oats and grass. How many acres are there in each crop, and how much remains not cultivated?

**Finding what per cent one number is of another number.**

**1.** What part of \$10 is \$5? What % of \$10 is \$5? *Think* \$5 is $\frac{1}{2}$ of \$10, or 50% of \$10.

What % of:

| | |
|---|---|
| **2.** 20 is 10? | **8.** $3\frac{3}{4}$ in. is $1\frac{1}{4}$ in.? |
| **3.** 35 ft. is 7 ft.? | **9.** 25 gal. is $6\frac{1}{4}$ gal.? |
| **4.** 100 is $16\frac{2}{3}$? | **10.** 60 rods is 20 rods? |
| **5.** 500 lb. is 100 lb.? | **11.** 1 mile is 80 rods? |
| **6.** $87\frac{1}{2}$ is $12\frac{1}{2}$? | **12.** 1 lb. (av.) is 1 oz. (av.)? |
| **7.** $1\frac{1}{2}$ yd. is $\frac{1}{2}$ yd.? | **13.** 1 dollar is 1 dime? |

## Written Work

**1.** What per cent of 75 is 15?

$15 \div 75 = .20 = 20\%$

The unknown number is the *rate.* Since the percentage equals the base *multiplied* by the rate, the rate must equal the percentage *divided* by the base. 15 divided by 75 is .20, or 20%. Or, 15 is $\frac{15}{75}$, or $\frac{1}{5}$, or 20%, of 75. **Test**: 20% of 75 = 15.

*The* **rate** *equals the percentage divided by the base.*

What per cent of :

2. 25 is 10?
3. 32 is 12?
4. 65 is 39?
5. $96 is $72?
6. $62\frac{1}{2}$ A. is $6\frac{1}{4}$ A.?
7. $\frac{8}{9}$ is $\frac{5}{9}$?
8. 4 is $\frac{3}{5}$?
9. 125 yd. is 75 yd.?
10. 4 bu. is 1 pk.?
11. 40 is $6\frac{2}{3}$?
12. 75 is 3.125?
13. $\frac{7}{8}$ is .125?
14. $1\frac{1}{4}$ is $2\frac{1}{2}$?
15. $18 is $45?
16. 10 qt. is 36 qt.?
17. $3 is 18 cents?

18. Out of 350 words, I spelled 315 correctly. What per cent did I make in spelling?

19. From a farm of 160 acres, 24 acres were sold. What per cent was sold?

20. If a man saves $262.50 out of his salary of $1250, what per cent does he save?

21. A farmer raised 150 bu. of potatoes from 6 bu. of seed. What per cent of the crop was the seed?

22. A merchant owes $8750 and his assets are $3675. What per cent of his debts can he pay?

23. A pupil misspelled 35 words out of 80. What per cent did he spell correctly?

24. .875 is what per cent of .3125?

25. I paid $5.25 for the use of $75. What per cent did I pay?

26. A son, on receiving $5000 from his father, bought a farm for $2750, a store for $1875, and deposited the rest in a bank. What per cent of his inheritance did he deposit?

27. A house rents for $240 a year. The taxes and insurance are $30. If the property is valued at $3500, what per cent does the owner realize on the value of the property?

**Finding the number when a per cent of it is given.**

**1.** If $\frac{1}{5}$ of a number is 10, what is the number? If 20% of a number is 10, what is the number?

**2.** If $33\frac{1}{3}$% of a man's loss is \$300, how much does he lose?

**3.** If $87\frac{1}{2}$% of a man's gain is \$70, how much does he gain?

What is the number of which:

| | | |
|---|---|---|
| **4.** 10 is $66\frac{2}{3}$%? | **11.** 40 is $16\frac{2}{3}$%? | **18.** \$48 is 10%? |
| **5.** \$100 is $33\frac{1}{3}$%? | **12.** 100 is $12\frac{1}{2}$%? | **19.** \$12 is $16\frac{2}{3}$%? |
| **6.** 500 is 50%? | **13.** 300 is $37\frac{1}{2}$%? | **20.** 30 is $33\frac{1}{3}$%? |
| **7.** 60 is 75%? | **14.** 500 is $62\frac{1}{2}$%? | **21.** 60 is $66\frac{2}{3}$%? |
| **8.** \$24 is 40%? | **15.** 700 is $87\frac{1}{2}$%? | **22.** 120 is 20%? |
| **9.** 300 is 60%? | **16.** 500 is 25%? | **23.** \$10 is $83\frac{1}{3}$%? |
| **10.** 500 is 50%? | **17.** 600 is 75%? | **24.** \$50 is $83\frac{1}{3}$%? |

## Written Work

**1.** Find the number if 33% of it is 3135.

(*a*) $3135 \div .33 = 9500$

The unknown number is the *base*. Since the percentage equals the base *multiplied* by the rate, the base equals the percentage *divided* by the rate, $3125 \div .33 = 9500$, the number. **Test**: .33 of 9500 = 3135.

**2.** Find the number if $8\frac{1}{3}$% of it equals 250.

(*b*) $8\frac{1}{3}\% = \frac{1}{12}$
$250 \times \frac{12}{1} = 3000$

$8\frac{1}{3}$% of the number equals $\frac{1}{12}$ of it.
$\frac{12}{12}$ of the number is $12 \times 250$, or 3000.
**Test**: $\frac{1}{12}$ of 3000 = 250.

The base *equals the percentage divided by the rate.*

Find the number if:

3. $8\%$ of it is \$2.40
4. $12\%$ of it is \$3.60
5. $12\frac{1}{2}\%$ of it is \$91
6. $37\frac{1}{2}\%$ of it is \$27
7. $33\frac{1}{3}\%$ of it is \$42.50
8. $6\%$ of it is \$72
9. $32\%$ of it is \$3.60
10. $45\%$ of it is \$3.60
11. $62\frac{1}{2}\%$ of it is \$35.50
12. $87\frac{1}{2}\%$ of it is \$28.28

13. A has \$3612, which is $87\frac{1}{2}\%$ of what B has. How much has B?

$87\frac{1}{2}\%$ of B's = \$3612, A's.

14. If \$14 is $25\%$ of A's salary, find his salary?

15. After a battle $70\%$ of a regiment, or 644 men, were left. How many men were there in the regiment at first?

16. I drew from the bank \$1500, or $83\frac{1}{3}\%$ of my deposit. How much was my deposit?

17. If a man rents a house for \$752 per year, which is $16\%$ of its value, what is the value of the house?

18. A teacher's expenses are \$30 a month, and this amount is $37\frac{1}{2}\%$ of his salary. How much does he save?

19. The number of pupils in attendance at school in a certain town is 576, which is $96\%$ of the enrollment. What is the enrollment?

**Finding a number when the number plus the rate of increase is given.**

### Written Work

1. What number increased by $17\%$ of itself equals 585?

| | |
|---|---|
| $100\%$ of itself | = the number |
| $100\%$ of itself + $17\%$ of itself, or $117\%$ of the number | = 585, amount |
| $1\%$ of the number | = $\frac{1}{117}$ of 585, or 5 |
| $100\%$ of the number | = $100 \times 5$, or 500 |

***Divide the sum by one plus the rate.***

What number increased by:

2. $8\%$ of itself is 324?
3. $30\%$ of itself is 260?
4. $37\frac{1}{2}\%$ of itself is 550?
5. $\frac{3}{4}\%$ of itself is $201\frac{1}{2}$?
6. $50\%$ of itself is 69?
7. $250\%$ of itself is 105?
8. $16\frac{2}{3}\%$ of itself is 1050?
9. $70\%$ of itself is 510?

10. I gained $35\%$ by selling an article for \$4.05. How much did it cost?

11. A laborer had his wages twice increased $10\%$. If he now receives \$2.42 a day, what were his wages before they were increased?

12. A property sold for \$4025, which was an increase of $15\%$ of the cost. How much did the property cost?

13. A receives \$1600 salary, which is $60\%$ more than B receives. What salary does B receive?

14. W. H. Richmond bought a jewelry store for a certain sum and increased the stock $27\%$ of the purchase price. He found that the whole investment amounted to \$5969. What was the purchase price of the store?

15. The land surface of the District of Columbia is 60 square miles, which is $500\%$ more than the water surface. What is the water surface?

**Finding a number when the number minus the rate of decrease is given.**

### Written Work

1. What number diminished by $16\%$ of itself equals 168?

| | |
|---|---|
| $100\%$ of the number | = the number |
| $100\%$ of the number $-$ $16\%$ of the number, | |
| or $84\%$ of the number | = 168 (difference) |
| $1\%$ of the number | $= \frac{1}{84}$ of 168, or 2 |
| $100\%$ of the number, or the number | $= 100 \times 2$, or 200 |

*Divide the difference by one minus the rate.*

What number diminished by:

2. $45\%$ of itself equals 55?

3. $18\%$ of itself equals 246?

4. $62\frac{1}{2}\%$ of itself equals 27?

5. $50\%$ of itself equals 22.5?

6. $18\frac{3}{4}\%$ of itself equals 325?

7. $23\%$ of itself equals 308?

8. $95\%$ of itself equals 25?

9. $10\%$ of itself equals $4\frac{1}{2}$?

**10.** John has $35, which is $12\frac{1}{2}\%$ less than his brother has. How much has his brother?

**11.** Mrs. Lee spent $24 for a coat, which was $33\frac{1}{3}\%$ less than the cost of a suit. Find the cost of both.

**12.** After losing $8\frac{1}{3}\%$ of his money, a man had $352 left. How much had he at first?

**13.** What number decreased by $35\%$ of itself equals $1300? $520? $6500?

**14.** A school enrolls 249 boys, which is $17\%$ less than the number of girls it enrolls. How many pupils are there in the school?

**15.** A lady when shopping spent $15 of her money for a hat, which was $25\%$ less than the amount she spent for a coat. How much did she spend for the coat?

**16.** The fraction $\frac{9}{16}$ is $40\%$ less than what fraction?

**17.** If a certain number is increased $40\%$ of itself, and this sum is diminished by $50\%$ of itself, the result is 700. Find the number.

**18.** I sold two lots for $1200 each; on one I gained $25\%$ and on the other I lost $25\%$. Did I gain or lose and how much?

**19.** The population of a town in 1906 was 14000, which was $12\frac{1}{2}\%$ less than the population in 1907. What was the population in 1907?

## REVIEW OF PERCENTAGE

**1.** What is 50 % of 200?

**2.** .05 is what per cent of .25?

**3.** .25 is 25 % of what number?

**4.** I sold goods for $8.75. My actual loss was $1.25. What was the cost and the per cent of loss?

**5.** A man owns a farm valued at $9600. His annual taxes are $86.40. How much must he make each year to clear 8 % on the cost of his property?

**6.** Ten is what per cent of 20? 15 is 150 % of what number?

**7.** What per cent of 10 days are 30 days? What per cent of 30 days are 10 days?

**8.** $\frac{5}{6}$ of 72 is how many per cent of 120?

**9.** I paid $7200 for a house, $150 for repairs, and $350 for delinquent taxes. I then sold it for $9000. What per cent did I make on my money?

**10.** In a business college 20 % of the students study bookkeeping, 60 % of them study typewriting, and the remaining 68 students study other courses. How many students are there in the school?

**11.** The income from an investment which pays $5\frac{1}{2}$ % is $220. What sum is invested?

**12.** Mr. Wilson buys a house and lot for $6400. The average expenses per year for taxes are $80; for insurance $12; and for repairs $24. What must be the annual rent that he may have an income of 6 % net on the original cost of the property?

**13.** What fraction increased by 35 % of itself equals $\frac{18}{20}$?

**14.** If my property sells for \$7433.25 and I owe \$8745, what per cent of my debts can I pay?

**15.** Ten thousand boxes of fruit were sold for \$9450, which was $33\frac{1}{3}\%$ less than the cost. What was the cost?

**16.** If $12\frac{1}{2}\%$ of $16\frac{2}{3}\%$ of a number is $2\frac{1}{2}$, what is the whole number?

**17.** A section of land was sold for \$4000, which was 25% more than it cost. How much did the land cost per acre?

**18.** Express as a per cent: $\frac{3}{8}$; $\frac{1}{40}$; $\frac{7}{8}$.

**19.** A horse and buggy cost \$300. If the cost of the horse was 200% of the cost of the buggy, what was the cost of each?

**20.** After paying 70% of his debts, a man found that \$3600 would put him out of debt. Find his original indebtedness.

**21.** Thirty-five per cent of 640 pounds is 5.6% of how many tons?

**22.** A bankrupt sold his property for \$4100, which was 18% less than its real value. If the property had sold for \$5250, what per cent above its real value would it have brought?

**23.** Two adjacent properties sold for \$13200. 75% of the sale of one equaled 90% of the sale of the other. Find the selling price of each.

**24.** A certain excavation cost \$120. What would be the cost of an excavation 20% wider? 25% deeper? 50% longer?

**25.** What per cent of 121.92 is 15.24?

**26.** What is the difference between $\frac{1}{4}\%$ of \$7000 and 25% of \$7000?

**27.** I bought 100 bu. apples at 50¢ a bushel, but lost 20 bu. by freezing. At what price per bushel did I sell the remainder, if my entire loss was 4% of the cost of the apples?

**28.** The net profits of a store in two years were \$3483. The profits the second year were 15 % more than the first year. How much were the profits the first year?

**29.** An executor in settling an estate found $7\frac{1}{2}$ % uncollectable, $12\frac{1}{2}$ % invested in city lots, 40 % in cash, 15 % loaned, and the remainder, \$10,000, invested in the home. The estate was equally divided among four sons. How much did each receive?

**30.** A merchant increased his capital the first year $33\frac{1}{3}$%, and the second year 25 % of the capital at the end of the first year. He lost 36 % of his original capital the third year, and had \$11760 left. Was his original capital increased or decreased, and how much?

**31.** An estate was worth \$8400. Had it been sold for that amount, the creditors would have received $87\frac{1}{2}$ % of their claims, but $\frac{1}{3}$ of the estate was sold at $18\frac{1}{2}$ % below its value, and the remainder at $12\frac{1}{4}$ % below its value. What per cent of the debts did the estate pay?

## GAIN AND LOSS

**1.** A dealer bought goods for \$1000, and sold them at a gain of 10 %. What was the selling price? What was the gain?

**2.** Sugar that cost 5¢ per pound was sold at a gain of 20 %. What was the gain per pound?

**3.** A huckster bought fruit for \$20, but found he had to sell it at a loss of 20 %. How much did he lose?

**4.** A grocer sold butter that cost 20¢ per pound, for 25¢. What part of the cost did he gain?

**5.** Books that cost \$1.50 wholesale were sold at a gain of 10 %. Find the selling price.

**Gain** and **loss** are terms used to designate the profits or the losses in business transactions.

The **cost** is the amount paid for an article; the **selling price** is the amount received for it.

The **gross cost** of goods is the original cost increased by what is paid for freight, storage, etc.

The **net proceeds** is the amount received for goods after all charges incident to the sale have been deducted.

*The* **per cent of gain or loss** *is always reckoned on the cost or on the sum invested.*

### Written Work

**1.** A merchant bought goods for \$4500 and sold them at a gain of 12%. How much did he gain?

#### Comparative Study

$$\begin{array}{rl} \$4500, & \text{cost} \\ .12, & \text{rate} \\ \hline \$540.00, & \text{gain} \end{array}$$

The **amount bought or sold** corresponds to what term in *Percentage?*

The **gain or loss** corresponds to what term in *Percentage?*

Find the gain or loss:

**2.** \$75, gain 20%.

**3.** \$96, gain $33\frac{1}{3}$%.

**4.** \$115, loss 15%.

**5.** \$227, loss 19%.

**6.** \$356, gain 21%.

**7.** \$132.50, gain 28%.

**8.** \$485.60, loss 5%.

**9.** \$880.80, gain $12\frac{1}{2}$%.

**10.** How much is gained by selling a property that cost \$3250 at a profit of 8%?

**11.** A grocer bought 120 dozen eggs at 18 cents a dozen, and sold them at a profit of $11\frac{1}{9}$%. What was his gain?

**12.** A real estate dealer bought three lots for \$1500, \$1800, and \$2000 respectively. He sold the first at an advance of 8%, the second at an advance of 10%, and the third at an advance of 12%. What was his gain?

**Finding the gain or loss per cent.**

### Written Work

**1.** I bought a piece of property for \$2500 and sold it for \$2750. Find the gain or loss.

$$\$2750 - \$2500 = \$250, \text{ gain}$$
$$\$250 \div \$2500 = .10, \text{ or } 10\,\%, \text{ gain}$$

Find the gain or loss per cent when the:

**2.** Cost is \$100 and the selling price \$105.

**3.** Cost is \$175 and the selling price \$210.

**4.** Cost is \$240 and the selling price \$230.40.

**5.** Cost is \$476.80 and the selling price \$309.92.

**6.** Cost is \$775.50 and the selling price \$1008.15.

**7.** If flour is bought for \$4.50 a barrel and sold for \$6 a barrel, what per cent is gained?

**8.** A farm was bought for \$3500 and sold for \$4200. What was the gain per cent?

**9.** If hats are bought for \$27 a dozen and retailed at \$2.75 each, what per cent gain is realized?

**10.** A grain dealer bought 500 bushels of wheat at 84 cents a bushel, 600 bushels at 80 cents a bushel, and sold it all at 82 cents a bushel. What per cent did he gain or lose?

**Finding the cost.**

### Written Work

**1.** What was the cost of a house if the owner, by selling it at an advance of 25%, gained \$900?

$$\$900 = \text{gain}$$
$$\$900 \div .25 = \$3600, \text{ cost}$$

Find the cost when:

2. 5 % loss is \$25.
3. $12\frac{1}{2}$ % gain is \$$37\frac{1}{2}$.
4. 150 % gain is \$750.
5. $\frac{1}{2}$ % loss is \$$12\frac{1}{4}$.
6. $\frac{1}{5}$ % gain is \$16.80.
7. 44 % loss is \$1100.
8. 35 % gain is \$12.25.
9. $16\frac{2}{3}$ % loss is \$37.50

10. A dealer sold a buggy at 25 % gain, and received \$90 for it. How much did the buggy cost?

$$\text{Selling price} = \tfrac{5}{4} \text{ of the cost}$$
$$\$90 = \tfrac{5}{4} \text{ of the cost}$$
$$\text{Cost} = \tfrac{4}{5} \text{ of } \$90, \text{ or } \$72$$

Find the cost when selling price at:

11. 10 % gain is \$220.
12. 18 % loss is \$492.
13. $12\frac{1}{2}$ % gain is \$990.
14. $37\frac{1}{2}$ % loss is \$600.
15. $16\frac{2}{3}$ % gain is \$1190.
16. 114 % gain is \$1284.

17. A merchant, after losing 25 % of his goods by fire, had \$8700 remaining. What was the value of his goods at first?

18. An attorney turned over to his client \$1125 after retaining 10 % for his services. What amount of money did the attorney collect?

19. If 8% is lost by selling an article for \$1.15, how much did it cost?

20. If dress goods sold at \$1.50 a yard yielded a profit of 20 %, how much did the goods cost per yard?

21. Mr. Rice sold his farm at a gain of 5 % and received \$4200 for it. What would the gain per cent have been had he sold it for \$4400?

$$\$4200 \div 1.05 = \$4000, \text{ cost}$$
$$\$4400 - \$4000 = \$400, \text{ gain}$$
$$\$400 \div \$4000 = .10, \text{ or } 10\%, \text{ gain}$$

**22.** I sold a horse for \$240 and thereby lost 20%. What selling price would have given me a gain of 20%?

**23.** If a merchant loses 10% by selling goods at 45 cents a yard, for what should they have been sold to gain 20%?

**24.** A piece of cloth was sold for \$32, which was at a loss of 20%. What would have been the loss per cent had it been sold for \$39?

## REVIEW PROBLEMS

**1.** Find the selling price of goods on which there is a loss of $2\frac{1}{2}$% which amounts to \$106.25.

**2.** How many per cent above cost must an article be marked in order to make 25% after a discount of 20% has been given?

**3.** I sold $\frac{2}{3}$ of an acre of land for what $\frac{4}{5}$ of it cost. What per cent did I gain or lose?

**4.** At what price must an article that cost \$60 be marked, so that after deducting 25% from the marked price, a profit of 10% may be realized?

**5.** A sold a property to B and gained 20%; B sold it to C and lost $16\frac{2}{3}$%; C sold it to D for \$2800 and gained 12%. How much did A receive for the property?

**6.** Find the value of a property that increased annually 4% on the previous year's value and after three increases was worth \$11,248.64.

**7.** A note was bought for 5% less than its face and sold for 2% more than its face. If \$40.25 was gained, what was the value of the note?

**8.** I bought a lot for \$1600, which was 20% less than its real value, and sold it for 20% more than its real value. How much did I gain?

**9.** If 80% of a car load of wheat is sold for what the car load costs, what per cent is gained?

**10.** A company sells sewing machines to a wholesaler at $33\frac{1}{3}\%$ profit, the wholesaler sells to the retailer at $12\frac{1}{2}\%$ profit, and the retailer sells to the trade at a profit of 20%. Find the cost of a machine to the company when the retailer's profit on the machine is $9.

**11.** A merchant marked his goods so as to gain 20%. By giving credit, 5% of his sales were uncollectable. His gain was $1050. What was the value of his goods?

**12.** A bankrupt can pay only 80 cents on the dollar. What will be the gain or loss per cent to the retailer who sold him a buggy at 30% profit?

**13.** A merchant marked a lot of goods costing $12,000, at 25% above cost, but sold them at 10% less than the marked price. What per cent did he gain?

**14.** A merchant sold silk at 45 cents a yard above cost, and gained 20%. What was the selling price per yard?

**15.** Mr. McKay bought 150 shares of W. Va. Lumber Co. stock at $140 a share, and 200 shares of telephone stock at $120 a share. He sold the lumber stock at a loss of 10%. For how much did he sell the telephone stock per share, if he gained 10% on the transaction?

**16.** A and B invested an equal amount of money in business; A gained $2000 on his investment, and B lost $1500; B's money was then 65% of A's. How much money did each invest?

**17.** A bought a horse and sold it to B at a gain of 10%; B sold it to C and gained 10%. If C paid $31.50 more for the horse than A, how much did the horse cost A?

## COMMISSION AND BROKERAGE

A person who buys or sells goods or transacts business for another is called an **agent, collector, commission merchant,** or **commission broker,** according to the nature of the business transacted.

A commission merchant *actually* receives the goods which he buys and sells for another. A commission broker simply makes the *contract* between the buyer and the seller for whatever is to be bought or sold, the goods being delivered directly from the seller to the buyer.

The **commission** or **brokerage** is a certain per cent of the amount of money involved in the transaction.

A commission merchant gets a certain per cent on the amount of his sales; a collector gets a certain per cent on the amount *collected;* a broker gets a certain per cent of the *cost* or the *selling price.*

The **net proceeds** is the amount left after commission and all other charges have been paid.

The one who sends the merchandise to be sold is the **principal,** the **shipper,** or the **consignor.**

**Selling or collecting through an agent.**

### Written Work

**1.** A Pittsburg commission house sold 275 barrels of apples at $4 per barrel, on a commission of 10%. The freight from Rochester, New York, was $67.50 and the drayage was $11.25. Find the commission and the net proceeds of the sale.

| | | |
|---|---|---|
| Amount of sale 275 bbl. at $4 | | $1100.00 |
| Commission 10% of $1100 | $110. | |
| Freight . . . . . . . . . . . | 67.50 | |
| Drayage . . . . . . . . . . . | 11.25 | |
| | 188.75 | 188.75 |
| Net Proceeds . . . . . . . . . . . . . . | | $ 911.25 |

### Comparative Study

The **amount bought, sold,** or **collected** in Commission corresponds to what term in *Percentage?*

The **rate** in Commission corresponds to what term in *Percentage?*

The **commission** or **brokerage** in Commission corresponds to what term in *Percentage?*

The **net proceeds** in Commission correspond to what term in *Percentage?*

**2.** A real estate agent sold four lots for \$250, \$325, \$395, and \$405 respectively. How much was his commission at 5%?

**3.** A commission merchant sold 320 barrels of apples at \$3.25 a barrel and 16 barrels of sweet potatoes at \$4.80 a barrel. Find his commission at 7%.

**4.** A lawyer collected an account of \$385 for a client, charging 5%. How much should he remit?

**5.** A cotton broker sold 200 bales of cotton of 225 pounds each, at $12\frac{1}{2}$¢ per pound, charging a commission of $2\frac{1}{2}$%. Find the net proceeds.

**6.** A lawyer collects 80% of an account of \$1125. Find his rate of commission if he charges \$9.

**7.** A real estate agent sold 475 acres of coal at \$85 an acre. His commission was \$1211.25. Find the per cent charged.

**8.** My agent bought 180 barrels of flour at \$4.80 per barrel. He paid \$50 freight and \$6 storage. I sent him \$937.28. What was his rate of commission?

**9.** An attorney succeeded in collecting 90% of the amount of a consignment of cotton sold for \$6700. He remitted \$5874, retaining the balance to pay attorney's fees and \$5.25 freight charges. What per cent did he charge for collecting?

**10.** My Chicago broker sells for me 82560 bushels of wheat at $83\frac{3}{4}$¢ per bushel, charging $\frac{1}{8}$¢ per bushel brokerage. Find the amount remitted to me.

Find the selling price if a commission of:

| | | |
|---|---|---|
| **11.** 2 % = $ 3.05 | **14.** $4\frac{1}{2}$ % = $ 56.25 | **17.** $3\frac{3}{4}$ % = $ 36.25 |
| **12.** $\frac{1}{2}$ % = 3.12 | **15.** $3\frac{1}{3}$ % = 75.60 | **18.** $\frac{1}{5}$ % = 7.59 |
| **13.** $\frac{1}{4}$ % = 1.54 | **16.** $1\frac{7}{8}$ % = 61.50 | **19.** 4 mills = 79.00 |

**20.** An agent for the Diamond Pneumatic Tool Company received 20 % on the sales of 13 pneumatic drills averaging $ 737.50 each. If his expenses for traveling and freight on machines were $ 697.50, find his net profits.

**21.** James Amidon & Co. offered one of their salesmen $ 2400 per year, $ 1800 for traveling expenses, and 2 % on all sales over $ 40000; or 6 % on all sales if he paid his own expenses. He chose the former, and sold $ 72000 worth of goods in the year. Did he gain or lose, and how much, by accepting the first offer?

**22.** For selling a house, a real estate agent received $ 96.25, which included $ 7.65 for advertising, and $ 27.90 for repairs. Find the rate of commission, if the house was sold for $ 3035.

**Buying or investing through an agent.**

### Written Work

**1.** An architect charged $2\frac{1}{2}$ % for plans and specifications and $2\frac{1}{2}$ % for superintending construction. His commission amounted to $ 810. How much did the building cost?

**2.** I telegraphed my agent at Chicago to buy me 10000 bushels of wheat at 79 cents or less. He bought at $77\frac{3}{4}$ cents and charged me $\frac{1}{8}$ cent per bushel brokerage. Find the amount of the check I should send him.

3. An agent's investments in 1905 were $ 80500 and in 1906 $ 87650. His commissions for 1906 were $ 143 more than for 1905. Find his commission for each year.

4. My broker in New Orleans buys 50000 pounds of cotton at $11\frac{7}{8}$¢ per pound. His commission is $\frac{7}{8}$% and freight, storage, and cartage amount to $ 95.80. How much should I remit?

5. A house and lot bought for $8500 was sold afterwards for 80% of the purchase price. If the agent received 2% on each transaction, find his commission.

6. An agent buys a property for his principal for $36790 at 2% commission. The owner puts $674.20 in repairs and afterwards sells the property through the same agent for $43000, at 2% commission. Find the agent's commission, and the principal's rate per cent of gain.

7. An attorney invested for his client $8750 in a mortgage. The owner of the property paid the attorney 2% commission for getting the money and $35.75 for examining title and for docket fees. How much did it cost the mortgager to secure the money?

## REVIEW

Find the values of the missing terms:

| | Gross Sales | Expenses | Commission | Rate of Commission | Net Proceeds |
|---|---|---|---|---|---|
| 1. | $675 | $11.25 | ( ) | 8% | ( ) |
| 2. | $550 | $4.00 | $22.00 | ( ) | ( ) |
| 3. | ( ) | $119.00 | ( ) | 20% | $681 |
| 4. | $560 | 00 | $56.00 | ( ) | ( ) |
| 5. | ( ) | 00 | $48.00 | ( ) | $552 |
| 6. | $1000 | 00 | ( ) | 4% | ( ) |
| 7. | ( ) | ( ) | $134.00 | 2% | $6566 |
| 8. | $2500 | $46.00 | $50.00 | ( ) | ( ) |

**9.** A fruit grower ships to his commission merchant 600 barrels of apples, which are sold at $ 3.50 per barrel. The agent deducts $43.90 freight charges, $27.75 cartage, 12¢ per barrel for cold storage, and 5 % commission. Find the amount remitted.

**10.** My agent sends me a bill for $37800, which includes the cost of some land bought at $100 an acre and his commission of 5 %. Find the number of acres purchased.

**11.** My purchasing agent in Chicago sends me a bill for $6776.60, covering cost of mining machinery, commission of 2 %, and $65 for extra expenses. Find his commission and the cost of the machinery.

**12.** My agent retains $3\frac{1}{2}$ % commission, pays freight charges of $16.80, storage and drayage of $9.75, and remits to me $1594.65. Find the amount of gross sales.

**13.** An investment broker in Denver receives a draft for $ 23935. This includes a commission of 2 % for investing, and an allowance of $175 for traveling expenses. Find the amount invested.

## INSURANCE

**Insurance** is security against loss or damage.

A merchant owns a store, uninsured, valued at $ 5000. If it is burned, who will bear the loss? An insurance company agrees to insure the store for $ 4000 at 1 % annually. In case the store is totally destroyed by fire, how much is the company expected to pay to the merchant? How much must the merchant pay annually to the company to guarantee this loss?

The **policy** is a written contract between the person insured and the insurance company.

The **premium** is the sum paid for the insurance.

The **rate** is a specified number of cents or dollars per $100 of insurance, or a certain per cent of the sum insured.

The **term** is usually a year or a period of years. **Short rates** are rates charged when the term is less than one year.

## Property Insurance

The principal kinds of property insurance are **fire insurance** and **marine insurance.** Other forms are burglar insurance, insurance against bad debts, etc.

**1.** A frame dwelling with a tin roof is insured for \$2800 for one year at 1 %. Find the premium.

### Written Work

#### Comparative Study

| | |
|---|---|
| \$2800, amount insured | The **amount insured** corresponds to what term in *Percentage?* |
| .01, rate | The **rate** corresponds to what term in *Percentage?* |
| \$28.00, premium | The **premium** corresponds to what term in *Percentage?* |

**2.** A brick house is insured for \$4000 at 60¢ on the \$100. Find the rate of premium and the annual premium.

**3.** If the three-year rate is twice the rate for one year, find the cost of insuring a brick dwelling for \$6500 for 3 years when the annual rate is 45¢ per \$100.

**4.** A store building is insured for \$8500 and the annual premium is \$212.50. Find the rate per cent of premium and the annual cost per \$100 of insurance.

**5.** A school board pays annually \$45 for \$6000 of fire protection on a school building. Find the rate of premium.

**6.** The premium on a dwelling insured for \$5500 is \$38.50 for three years. Find the average rate for a year.

**7.** If the premium on a plate-glass policy is \$9.50 and the rate is $\frac{1}{2}$%, find the face of the policy.

**8.** Mr. Lawrence wrote a check for \$31.50 to pay the insurance on his dwelling for 3 years. If the house cost \$2400 and was insured for $\frac{3}{4}$ of its value, find the rate for the term.

**9.** A farmer insured his house for \$2700 at $1\frac{1}{4}\%$, his barn for \$1200 at $\frac{7}{8}\%$, and his furniture for \$900 at $1\%$. What premium did he pay?

**10.** A drug store is insured for $\frac{4}{5}$ of its value at $2\%$. What is the value of the store if the premium is \$192?

**11.** The premium on 8000 bushels of wheat, valued at 90¢ per bushel and insured at $\frac{4}{5}$ of its value, is \$57.60. Find the rate of insurance.

**12.** A jewelry store is insured for \$20000 and its contents for \$27000. The premium is \$705. What is the rate of insurance?

**13.** A clothier insured his stock of goods, valued at \$12000, for 1 year at $1\frac{1}{4}\%$. At the end of 6 months he surrendered his policy. If the "short rate" for 6 months was 90¢ per \$100, how much premium was returned?

**14.** A farmer insured his buildings for \$3500 at $1\frac{1}{2}\%$ for a term of 3 years. After he had paid the premium for 4 terms the buildings were totally destroyed by fire. What was the farmer's loss? the company's loss?

**15.** A vessel worth \$27000 is insured for $\frac{2}{3}$ of its value at $3\frac{1}{4}\%$. In case of shipwreck, what is the company's loss? What is the owner's loss?

**16.** How much insurance, at $\frac{7}{8}\%$, can be placed on a building for \$42?

**17.** A business block valued at \$300000 was insured in 4 different companies, the rate of each being $1\%$. The first company took \$50000; the second, \$60000; the third, \$90000; and the fourth the remainder. After the premiums had been paid four times, the block was damaged by fire to the amount of \$120000. What was the loss of each company?

## Personal Insurance

The principal kinds of personal insurance are **life insurance** and **accident insurance.**

Kinds of life insurance policies:

**1.** A **life policy** is one that guarantees a fixed sum of money on the death of the insured. The premiums on a life policy run for life.

**2.** A **life policy with a twenty-year settlement** is one that guarantees, after twenty annual payments have been made, either a cash surrender value, or a paid-up policy, payable at death.

**3.** An **endowment policy** is one in which the face of the policy and the profits on the premiums are guaranteed to the insured if living at the end of a specified time, or to his estate if his death occurs within the time.

**4.** A **term policy** is one in which the face of the policy is paid, providing the insured dies within the time the policy runs. Otherwise nothing is paid.

Most insurance companies pay dividends on the premiums *already paid*, thus lessening the amount of the annual premium.

The **premium** is always so much on \$1000 of insurance; thus, a premium of \$23.40 means \$23.40 on \$1000.

The **age** of the insured is always reckoned according to the age at his nearest birthday.

This table shows the annual premiums for each \$1000 insurance in a leading insurance company.

| Age | Ordinary Life | 20-Payment Life | 20-Year Endowment | 20-Year Term |
|---|---|---|---|---|
| 20 | \$18.95 | \$27.64 | \$49.35 | \$12.48 |
| 25 | 21.14 | 30.05 | 45.98 | 13.34 |
| 30 | 23.96 | 32.98 | 50.74 | 14.61 |
| 35 | 27.63 | 36.62 | 51.88 | 16.70 |
| 40 | 32.48 | 41.18 | 53.69 | 20.15 |
| 45 | 39.02 | 47.09 | 56.70 | 25.85 |
| 50 | 47.79 | 54.98 | 61.75 | 35.00 |
| 55 | 60.33 | 65.81 | 70.02 | |
| 60 | 77.48 | 81.09 | | |

Dividends vary according to the number of premiums that have been paid. It is fair to estimate that the dividends on an ordinary life policy after 20 annual premiums have been paid, will amount to about 25% of the sum of the annual premiums.

### Written Work

Rates as given in the table on page 262.

**1.** What is the premium on an ordinary life policy of $5000 at the age of 30?

**2.** A man at the age of 25 takes out a $2000 ordinary life policy. If he dies after paying 16 premiums, what per cent of the face of the policy has been paid in premiums?

**3.** If, in example 2, the insured had taken a 20-payment life policy, what per cent of the face of the policy would have been paid in premiums?

**4.** A young man at the age of 20 took out a 20-payment life policy for $2000. The dividends at the end of 20 years amounted to $142 per thousand. What was the net cost of this insurance at the expiration of this policy, if the interest on the premiums is not considered?

**5.** What is the premium on a 20-year endowment policy for $5000 at the age of 40?

**6.** The first annual premium on a 20-year endowment policy for $8000 amounts to $453.60. What is the age of the insured?

**7.** If a man 25 years old takes out a 20-term policy for $3000, how much will he have paid for his insurance at the close of the term?

**8.** A man at the age of 25 took out a 20-payment life policy for $1000. His dividends for the 20 years amounted to $150.45. What was the amount of his premiums, less the dividend?

## COMMERCIAL DISCOUNT

**1.** I owe a bill of \$50 due in 60 days. As the creditor needs the money he offers to take \$40 if I pay at once. What per cent is the reduction? On what is the reduction reckoned?

**2.** A catalogue lists goods at \$1.00, \$.50, \$.25, subject to a discount of 20%. Find the net price of each article.

**3.** In the above examples what numbers correspond to the *base* in *Percentage?*

The *fixed* or *list* price of an article or the *amount* of an obligation is always considered the *base*.

**Commercial discount** is a reduction from the fixed or list price of an article, or from the amount of a bill or obligation.

1. **Trade discounts** are reductions from the fixed or list price of an article at the time of sale.
2. **Time discounts** are reductions from a bill or other obligation for payment within a certain time.
3. **Cash discounts** are reductions made for the immediate payment of a bill of goods sold on time.

The **net price** is the price after trade discounts have been deducted.

### Written Work

**1.** Neckties listed at \$6.00 per dozen are sold at 50% discount. Find the net price. What is the *kind* of discount?

**2.** An agent buys \$100 worth of goods on 60 days' time, or 5% off, if paid immediately. What *kind* of discounts is he offered?

**3.** A merchant offers \$100 worth of goods for \$90. What is the *kind* of discount? What is the per cent of discount?

**4.** I pay cash for a bill of goods sold on 60 days' time, and thereby get 10% discount, or \$12.50. Find the amount of the purchase. Why is this a cash discount?

**5.** A suit marked \$40 is offered for \$28. Find the per cent of discount. What is the *kind* of discount?

**6.** A merchant buys boys' suits at \$60 per dozen list price, less 20 %. What is the *kind* of discount?

**7.** A dealer sells \$75 worth of goods at 5 % discount, if paid within 60 days, or 10 %, if paid in cash. Explain the different kinds of discount, and the amount saved by paying cash.

**8.** Why does a *cash* discount always cut out a *time* discount? What discount is made without reference to *time?*

Business houses print on their billheads their terms of credit. For example:

1. "Terms: 60 days net; 3 % off 10 da."
2. "Terms: 90 days net; 60 days 2 %; 10 days 5 %."
3. "Terms: 30 days net; cash 5 %, etc."

**9.** Johnston Bros., Toledo, O., purchase of Amidon & Co., Chicago, \$300 worth of merchandise. Terms: 60 da. net; 5 % off 10 da. Find the discount if paid within 10 days.

**10.** I buy \$600 worth of goods. Terms: 30 da. net; 5 % off for cash. Find the amount I save by paying cash.

**11.** Explain the meaning of a bill head which reads as follows: "Terms: 60 da. net; 10 % cash."

**12.** \$200 worth of goods are bought Aug. 10, 1905. Terms: 60 da. net; 5 % off 30 da.; 10 % off 10 da. Find the amount saved by payment Sept. 5.

**13.** Jamison & Son, Baltimore, Md., bought \$1500 worth of merchandise from Brown & Co., Philadelphia. Terms: 90 days net; 2 % off 60 da.; 5 % off 30 da.; 10 % off cash. What was the amount of the bill if cash was paid?

**14.** Which is the best discount on a bill of goods for \$200 and how much: 10 % for cash, 5 % for 60 days, or 2 % for 90 days? Explain why.

### Successive Trade Discounts

Wholesale merchants, publishers, and manufacturers usually have fixed price lists for their goods from which the retailer gets a certain **trade discount.** If a reduction from these prices is to be made, an extra discount is taken from the former discount price.

**1.** A music dealer buys an organ on 60 days' time, list price \$100 at 40 %, 20 % off. Find the net price.

.40 × \$100 = \$40, 1st discount
\$100 − \$40 = \$60, 1st discount price
.20 × \$60 = \$12, 2d discount
\$60 − \$12 = \$48, net cost

Observe that, when more than one discount is given, each successive discount is reckoned on the last discount price.

**2.** Providing the music dealer gets a further discount of 10 % for cash, find the price of the organ.

**3.** On what discount price was the last discount reckoned?

NOTE. — The *trade* discount is first deducted; then the *cash* discount is taken from the remainder.

Find the net cost of articles listed at:

**4.** \$90, discount 30 %, 10 %.

**5.** \$120, discount 25 %, 20 %.

**6.** \$240, discount 10 %, $33\frac{1}{3}$ %.

**7.** \$35, discount 20 %, 5 %.

**8.** \$12.50, discount 20 %, 20 %.

**9.** \$348, discount 20 %, 10 %, 5 %.

**10.** \$100, discount 10 %, 5 %, 2 %.

**11.** \$425, discount $37\frac{1}{2}$ %, 16 %.

**12.** \$400, discount 20 %, 10 %, 5 %.

**13.** What is the net price of a bill of goods for \$5700 after discounts of 25 %, 20 %, and 5 % are allowed?

**14.** The amount of discounts at 20 %, 5 % off, is \$42. What is the list price of the bill?

Suggestion.—The total discount expressed in per cent is 20% + (5% of 80%), or 24%. \$42 is 24% of what number?

**15.** What is the cost of a bill of farming implements listed at \$480, discounts 50 %, 5 %, and freight \$3.60?

**16.** Goods marked \$50 were bought at 10 % trade discount, 2 % off for cash. If sold at \$55, what was the rate of gain?

**17.** A dealer bought 50 gross of buttons for 25 %, 10 %, 5 % off and sold them for \$35.91, making a profit of 12 %. What was the list price of the buttons per gross?

**18.** A grocer is offered a discount of 10 % from one firm on a bill of goods of \$1000, and two successive discounts of 5 % by another firm on the same bill. Which is the better offer and how much?

**19.** What is the difference between a discount of 15 %, 5 %, on a bill of \$2000 and a discount of 5 %, 15 %, on the same bill? Show that the same result is obtained from any number of discounts on a bill in whatever order they are taken.

**20.** Three firms bid on the glass for a building as follows: (1) \$2000, 60 %, 20 %, off. (2) \$2100, 70 %, 10 %, off. (3) \$2400, 80 %, 10 %, off. Which offer is the best and how much?

**21.** A jobber buys merchandise listed at \$1500, at 20 %, 15 % off, and sells at 15 %, 10 %, 5 % off the list price. Find his profits.

**22.** I purchased \$2000 worth of goods trade discount 20 %, cash 5 %. I pay cash plus \$5.95 freight. What is the entire cost of the goods?

**23.** Find a single discount equal to successive discounts of 40 %, 20 %, 10 %.

## COMMERCIAL BILLS

**1.** On June 10, 1906, James Boydson, Sandusky, O., bought of J. M. Gordon & Co., Cleveland, O., 35 dozen bronze locks at $5.50 per dozen, less 20 %, 10 %. Terms: 30 days net; 5% off 10 days.

### Form of Bill

CLEVELAND, O., *June 10, 1906.*

*Mr. James Boydson,*
*Sandusky, O.*

Bought of J. M. GORDON & CO.,

HARDWARE MERCHANTS

TERMS: *30 days net; 5 % off 10 days.*

| | | | | | | |
|---|---|---|---|---|---|---|
| | | *35 Doz. Bronze Locks @ $5.50* | *$192* | *50* | | |
| | | *Less $\frac{1}{5}$, $\frac{1}{10}$* | *53* | *90* | | |
| | | | *138* | *60* | | |
| | | *Cash, less 5%* | *6* | *93* | *$131* | *67* |

*Received payment, June 15, 1906,*
*J. M. Gordon & Co.*

In the above problem J. M. Gordon & Co. received payment within 10 days, so a cash discount of 5 % was deducted from the $138.60, making the payment $131.67. Had the bill not been paid for 30 days, the net amount would have been $138.60.

**2.** Harmon & Co., Seattle, Wash., order from Peabody & Sons, Chicago, Ill., 10 doz. Acme lawn mowers, list $10 each less 40 %, 20 %. Terms: 60 days net; 5 % off 10 days. Make out receipted bill if paid in 10 days.

**3.** Howard Johnston, Johnstown, N.Y., orders from the Acme Buggy Co., Cincinnati, O., 12 buggies, list \$100, less 40 %, 20 %. Terms: 30 days net; 2 % off in 10 days. Make out receipted bill if paid in 10 days.

**4.** Fisher Bros., Hagerstown, Md., order from the Fulton Hardware Co., New York, N.Y., the following; discount $33\frac{1}{3}$%:

| | | |
|---|---|---|
| 1 Gas Range | @ | \$32.00 |
| 100 ft. Hose | @ | .15 |
| 24 Garden Rakes | @ | .50 |

Terms: 30 days net; 2 % off in 10 days. Make out receipted bill if paid in 30 days.

## LOCAL AND STATE TAXES

A **tax** is a sum of money levied on a person, his property, or business for public purposes.

Cities, towns, townships, and counties levy and collect taxes annually on all taxable property. States ordinarily levy taxes only on special kinds of property.

A **poll tax** is a tax levied upon male citizens over 21 years of age without regard to the property they own.

**Real property**, or **real estate**, is any fixed property; as land and buildings erected thereon.

**Personal property** is any movable property; as money, stocks, furniture, etc.

An **assessor** is a person elected or appointed to estimate the value of property to be taxed.

The **rate of taxation** is a certain number of mills on each dollar of assessed valuation, or a number of cents on a hundred dollars of assessed valuation. Thus, a tax levy of 2 mills on the dollar, means 2 mills (or $\frac{2}{10}$% ) on each dollar.

A **collector** is a person elected or appointed to collect the tax. He receives either a salary or a commission.

**1.** Find my taxes on property assessed at \$5000 on which I pay 5 mills on the dollar.

Find the tax on property assessed at:

**2.** \$2000, tax levy 3 mills.
**3.** \$3000, tax levy 5 mills.
**4.** \$5000, tax levy 12 mills.
**5.** \$8000, tax levy 11 mills.
**6.** \$3200, tax levy $2\frac{1}{2}$ mills.
**7.** \$5000, tax levy $3\frac{1}{2}$ mills.
**8.** \$12000, tax levy $5\frac{1}{2}$ mills.
**9.** \$20000, tax levy $\frac{1}{2}$ mill.

How many mills on the dollar is paid if the assessment is:

**10.** \$6000, taxes \$60?
**11.** \$1200, taxes \$24?
**12.** \$8000, taxes \$32?
**13.** \$6000, taxes \$3?
**14.** \$600, taxes \$7.20?
**15.** \$1000, taxes \$45?
**16.** \$5000, taxes \$10?
**17.** \$950, taxes \$1.90?

Find the assessed valuation if the taxes are:

**18.** \$4.00, rate 8 mills.
**19.** \$12.00, rate 12 mills.
**20.** \$6.50, rate 13 mills.
**21.** \$20, rate 4 mills.
**22.** \$60, rate 3 mills.
**23.** \$200, rate 10 mills.
**24.** \$180, rate 6 mills.
**25.** \$200, rate 25 mills.

## Written Work

**1.** A town whose property is assessed at \$1750000 needs \$5150 for improvements; there are 620 persons who pay a poll tax of \$1.25 each. What is the rate of taxation, and what is Mr. Randolph's tax, whose property is valued at \$6500 and who pays his own and one other poll tax?

| | |
|---|---|
| $620 \times \$1.25$ | = \$775, the amount of poll tax. |
| $\$5150 - \$775$ | = \$4375, the amount of property tax. |
| $\$4375 \div \$1750000$ | = .0025, the rate of taxation. |
| $.0025 \times \$6500$ | = \$16.25, Mr. Randolph's property tax. |
| $2 \times \$1.25$ | = \$ 2.50, Mr. Randolph's poll tax. |
| $\$16.25 + \$2.50$ | = \$18.75, Mr. Randolph's entire tax. |

### Comparative Study

The **assessed valuation** corresponds to what term in *Percentage?*
**Tax** corresponds to what term in *Percentage?*
The **rate of taxation** corresponds to what term in *Percentage?*

Find the missing terms:

| | Estimated Valuation | Assessed Valuation | Rate | Taxes | Poll Tax |
|---|---|---|---|---|---|
| 2. | \$23000 | \$19000 | .002 | ( ) | 3 polls, \$1 each |
| 3. | \$147500 | 80% | .002 | ( ) | 2 polls, \$2 each |
| 4. | \$120000 | \$95000 | ( ) | \$332.50 | |
| 5. | 125% | ( ) | .015 | 11.25 | |
| 6. | 150% | ( ) | .022 | \$3269.20 | |

**7.** The assessed valuation of a town is \$900000 and the amount of taxes to be raised is \$16200. What is the rate of taxation and what is Mr. Owen's tax who owns property assessed at \$10000 and personal property assessed at \$2500?

**8.** How much tax does a farmer pay who owns 80 acres of land valued at \$100 an acre, assessed at $\frac{2}{3}$ of its value, and personal property assessed at \$1250, if the rate of taxation is three mills?

**9.** Mr. Day's city tax, at the rate of 13 mills on the dollar, is \$84.50. What is the estimated value of his property, if it is assessed at $\frac{4}{5}$ of its value?

**10.** A collector paid to the borough authorities \$21898.50 after deducting his commission of $2\frac{1}{2}\%$ for collecting. What was the amount of taxes collected, and what were the collection fees?

Suggestion. — The proceeds of each dollar returned by the collector was \$.97½.

**11.** The real estate of a town is valued at \$985470 and the personal property at \$195645. A tax of \$7866.69 is to be raised. There are 780 persons, each assessed a poll tax of \$1. How much tax will Mr. Cowperthwait, a non-resident (paying no poll tax), pay whose property is assessed at \$12460?

**12.** Mr. Arbuthnot's property has an actual valuation of \$4800. If he pays 17 mills city tax and $3\frac{1}{2}$ mills county tax on a $\frac{2}{3}$ valuation, in a certain year, find the amount of his tax for that year.

**13.** A borough bridge cost \$4380.48. It was paid by a tax on the assessed valuation of the borough property. If 4% of the tax was uncollectable, and the collector's fee was $2\frac{1}{2}$%, what was the borough assessment?

**14.** A tax collector's report in a certain city, for Oct. 2, 1906, showed by the assessment in the tax book that \$22742.90 was collected during September. If 5% discount was allowed on each one's taxes paid, and the collector's commission was 2%, find the amount paid to the school treasurer.

## DUTIES OR CUSTOMS

The revenues of our national government are of two kinds: *internal revenues* and *custom revenues*, or *duties*.

**Internal revenue** is a tax on the manufacture or sale of malt liquors, tobacco, etc.

**Custom revenues**, or **duties**, are taxes on goods imported from foreign countries. These are collected at the custom-houses, which are maintained by the government at various *ports of entry*.

A **tariff** is a schedule of duties on imports fixed by our government.

All merchandise brought into our country is classified as follows:

(1) Merchandise on the **free list**, that is, free of duty.

(2) Merchandise subject to an **ad valorem duty**, that is, a certain per cent of the cost of the goods, as shown by the invoice.

**Invoices** are statements showing the market price of goods, expressed in the money of the country where the goods are bought.

(3) Merchandise subject to a **specific duty**, that is to a certain amount per yard, pound, bushel, etc., without regard to value.

(4) Merchandise subject to both an **ad valorem** and a **specific** duty.

For example, the duty on Brussels carpet is 28¢ per square yard *specific* and 40 % *ad valorem*.

Duties are not computed on *parts* of a dollar. If the invoice shows a number of cents less than 50, they are rejected in computation of duties. More than 50 cents are counted as another dollar.

**Tare** is an allowance made for the weight of the boxes or bags used in packing. It is deducted before computing duties.

### Comparative Study

The **cost** of imported goods corresponds to what term in *Percentage?*
The **rate** in duties corresponds to what term in *Percentage?*
The **duty** corresponds to what term in *Percentage?*

### Written Work

Find the duty on the following imports:

**1.** 1650 lb. of hops, specific duty 12¢ per pound.

**2.** \$6250 worth of clocks at 40 % ad valorem.

**3.** \$10000 worth of uncut diamonds, ad valorem duty 60 %.

**4.** 2400 pounds of butter at 6¢ per pound.

**5.** 150 boxes plate glass (each 25 plates 16 in. by 24 in.), duty \$.08 per square foot.

**6.** 800 yd. Brussels carpet 27 in. wide, valued at \$1.45 per yard. Duty 40% plus \$.44 per square yard.

Explain the different kinds of duty in this problem.

**7.** 500 boxes cigars (each box weighing 1 lb. and containing 100 cigars) invoiced at \$3.50. Duty 25% plus \$4.50 per pound.

**8.** The specific duty on macaroni is $1\frac{1}{2}$¢ per pound. If the duty is \$57.90, find the number of pounds imported.

**9.** A department store in Buffalo imported from London $3\frac{1}{2}$ tons (l. T.) of woolen blankets invoiced at 40 cents per pound, subject to a specific duty of 22 cents per pound and an ad valorem duty of 30%. How much duty was paid?

**10.** A merchant imported from Sheffield, England, 12 gross of table knives costing 10*s*. per dozen in Sheffield. If the duty was \$1.44 per dozen and 15% ad valorem, and transportation \$9.60, find the cost per dozen delivered.

**11.** A certain painting in Rome is purchased for 50000 lire (\$.193). Find the cost when delivered in New York, if the duty is 20% and freight and insurance \$49.75.

**12.** Find the entire cost of importing 3000 lb. of worsted yarn, invoiced at £360, if the freight charges are \$11.75; duty $27\frac{1}{2}$¢ per pound; and 40% ad valorem.

**13.** Brown & Co. import 1200 sacks of cocoa, each containing 90 lb. If the duty is $2\frac{1}{2}$¢ per pound and tare 1%, find the duty on the goods when imported.

**14.** An importer paid \$480 duty on lace on which he paid a duty of 60%. If the lace cost 50¢ per yard, find the number of yards imported.

## INTEREST

**Interest** is money paid for the use of money.

The **principal** is the sum loaned.

The **amount** is the principal plus the interest.

The **rate** is the *number of hundredths* of the principal paid for its use for *one year*.

**Time** is always a factor in computing interest.

### SIMPLE INTEREST

**Simple interest** is interest allowed on the principal only.

Interest is generally computed on the basis of a year of 12 months of 30 days each, or 360 days. This is called *simple* or *common interest*.

**1.** How much is the interest on \$200 at 6% for 1 year? for 1 month? for 15 days?

**2.** How much is the interest on \$100 at 6% for 1 year? for 1 month? for 15 days?

How much is the interest at 6% on:

**3.** \$100 for $1\frac{1}{2}$ years?

**4.** \$200 for $2\frac{1}{2}$ years?

**5.** \$200 for 6 months?

**6.** \$300 for 9 months?

**7.** \$200 for 60 days?

**8.** \$600 for 90 days?

**9.** \$500 for 4 months?

**10.** \$200 for 120 days?

**11.** \$60 for 1 year 1 month?

**12.** \$600 for 240 days?

**13.** \$300 for 30 days? 1 day?

**14.** \$500 for 11 mo. 12 da.?

**15.** \$700 for 6 mo. 15 da.?

**16.** \$800 for 3 mo. 5 da.?

## Written Work

**1.** Find the interest on \$600 for 2 yr. 9 mo. at 6%.

| | |
|---|---|
| \$600 | = principal |
| .06 | = rate |
| \$36.00 | = int. for 1 yr. |
| $2\frac{3}{4}$ | = time in years |
| \$27 | = int. for 9 mo. |
| 72 | = int. for 2 yr. |
| \$99 | = int. for 2 yr. 9 mo. |

**Comparative Study**

1. The **principal** corresponds to what term in *Percentage?*
2. The **rate** corresponds to what term in *Percentage?*
3. The **interest** corresponds to what term in *Percentage?*
4. What term is found in interest that is not found in *Percentage?*

The **interest** *equals the product of the principal, the rate, and the time expressed in years.*

Find the interest on:

**2.** \$250 for $2\frac{1}{2}$ yr. at 6%.

**3.** \$708 for 4 yr. at 7%.

**4.** \$650 for $1\frac{3}{4}$ yr. at 4%.

**5.** \$800 for 10 mo. at 6%.

**6.** \$260 for $1\frac{2}{3}$ yr. at $4\frac{1}{2}$%.

**7.** \$320 for 5 yr. at $5\frac{1}{2}$%.

**8.** \$600 for $3\frac{7}{12}$ yr. at 6%.

**9.** \$200 for 11 mo. at 6%.

**10.** \$570 for $1\frac{5}{6}$ yr. at 4%.

**11.** \$290 for 3 yr. at $6\frac{1}{2}$%.

### Method by Aliquot Parts for Years, Months, and Days

## Written Work

**1.** Find the amount of \$960 for 3 yr. 7 mo. 18 da. at 5%.

| | | |
|---|---|---|
| Principal | = \$960 | |
| Rate | = .05 | |
| Int. for 1 yr. | = \$48.00 | |
| Int. for 3 yr. | = 3 × \$48, or | \$144.00 |
| Int. for 6 mo. | = $\frac{1}{2}$ of \$48, or | 24.00 |
| Int. for 1 mo. | = $\frac{1}{12}$ of \$48, or | 4.00 |
| Int. for 15 da. | = $\frac{1}{2}$ of \$4.00, or | 2.00 |
| Int. for 3 da. | = $\frac{1}{10}$ of \$4.00, or | .40 |
| Int. for 3 yr. 7 mo. 18 da. | = | \$174.40 |
| Principal | = | 960.00 |
| Amount | = | \$1134.40. |

Find the amount of :

**2.** \$1400 for 3 yr. 3 mo. 12 da. at 6%.

**3.** \$975 for 5 yr. 8 mo. 24 da. at $4\frac{1}{2}$%.

**4.** \$360 for 4 yr. 5 mo. 10 da. at $5\frac{1}{2}$%.

**5.** \$480 for 2 yr. 9 mo. 15 da. at 7%.

**6.** \$2700 for 6 yr. 6 mo. 20 da. at 5%.

**7.** \$1040 for 4 yr. 9 mo. 18 da. at 8%.

**8.** \$176.45 for 3 yr. 7 mo. 25 da. at 6%.

**9.** \$1840 for 5 yr. 4 mo. 27 da. at $4\frac{1}{2}$%.

**10.** \$1875 for 7 yr. 2 mo. 6 da. at 8%.

**11.** \$200 for 2 yr. 1 mo. 15 da. at 6%.

**12.** \$1200 for 1 yr. 8 mo. 10 da. at 7%.

**13.** \$97.30 for 3 yr. 3 mo. 3 da. at 8%.

**14.** \$3500 for 2 yr. 7 mo. 24 da. at 6%.

Find the interest on :

**15.** \$1575 at 5% from Jan. 3, 1907 to Sept. 5, 1909.

**16.** \$1790.80 at $4\frac{1}{2}$% from Sept. 8, 1906 to Dec. 10, 1910.

**17.** \$2005 at $3\frac{1}{2}$% from June 12, 1906 to Oct. 8, 1909.

**18.** \$4000 at $4\frac{3}{4}$% for 5 yr. 8 mo. 21 da.

**19.** \$4670 at $5\frac{1}{2}$% from Oct. 10, 1906 to June 5, 1907.

**20.** \$2890 at $6\frac{1}{2}$% from April 3, 1905 to Oct. 1, 1908.

**21.** \$290.75 at 6% from May 8, 1905 to Sept. 19, 1908.

**22.** \$980.60 at $5\frac{1}{2}$% from Aug. 7, 1906 to June 7, 1912.

**23.** \$759.40 at 7% from May 9, 1907 to June 10, 1909.

**24.** \$800.50 at $4\frac{1}{10}$% for 1 yr. 7 mo. 20 da.

**25.** \$200.80 at $5\frac{1}{4}$% from May 3, 1906 to Sept. 5, 1911.

### The Sixty Day Six Per Cent Method

Since the interest on \$100 at 6% for 1 yr. is \$6, the interest for 60 da. ($\frac{1}{6}$ of a year) = $\frac{1}{6}$ of \$6, or \$1.

How much is the interest at 6% for 60 days on:

| | | | |
|---|---|---|---|
| **1.** \$200 | **3.** \$300 | **5.** \$150 | **7.** \$125 |
| **2.** \$ 50 | **4.** \$ 6 | **6.** \$250 | **8.** \$120 |

Does the interest in each problem equal $\frac{1}{100}$ of the principal?

### Written Work

**1.** Find the interest at 6% on \$650 for 90 days.

Interest for 60 days (2 mo.) = \$6.50
Interest for 30 days (1 mo.) = \$3.25
Interest for 90 days (3 mo.) = \$9.75

*The* **interest of any principal for 60 da. (2 mo.) at 6%** *is found by moving the decimal point in the principal two places to the left.*

Find the interest at 6% on:

**2.** \$275 for 2 mo.

**3.** \$ 95 for 3 mo.

**4.** \$172 for 3 mo.

**5.** \$198 for 3 mo.

**6.** \$475 for 3 mo.

**7.** \$280 for 3 mo.

**8.** Find the interest at 6% on \$345 for 8 mo.

The interest for 2 mo. = \$3.45, or $\frac{1}{100}$ of the principal
The interest for 8 mo. = 4 × \$3.45 = \$13.80

Find the interest at 6% on:

**9.** \$125 for 120 da.

**10.** \$190 for 4 mo.

**11.** \$325.50 for 8 mo.

**12.** \$ 62.50 for 240 da.

**13.** \$805 for 75 da.

**14.** \$425 for $2\frac{1}{2}$ mo.

**15.** \$500 for 30 da.

**16.** \$280 for 45 da.

17. \$600 for 6 mo.

18. \$350 for 9 mo. (8 mo. + 1 mo).

19. \$450 for 19 mo. (18 mo. + 1 mo.).

Find the interest at 6 % on:

20. \$550 for 22 mo.

21. \$640 for 18 mo.

22. \$435 for 180 da.

23. \$552 for 21 mo.

24. \$632 for 120 da.

25. \$562 for 16 mo.

26. \$335 for 8 mo.

27. \$222 for 9 mo.

28. \$375 for 1 yr. 4 mo.

29. \$387.50 for 1 yr. 6 mo.

30. \$345.50 for 1 yr. 6 mo.

31. \$850 for 1 yr. 2 mo.

32. \$392 for 1 yr. 4 mo.

33. \$362 for 1 yr. 2 mo.

34. \$563.25 for 1 yr.

35. \$147.50 for 1 yr.

36. \$150 for 90 da.

37. \$650 for 120 da.

38. \$160 for 100 da.

39. \$60.20 for 10 mo.

40. Find the interest on \$562.50 for 1 yr. 10 mo. 15 da. at 6 %.

| | | |
|---|---|---|
| The interest for 2 mo. | = | \$ 5.625, or $\frac{1}{100}$ of the principal |
| The interest for 20 mo. | = | 56.25, or 10 × 5.625 |
| The interest for 15 da. | = | 1.406, or $\frac{1}{4}$ of \$5.625 |
| Int. for 1 yr. 10 mo. 15 da. | = | \$63.281 |

Find the interest at 6 % on:

41. \$500 for 6 mo. 15 da.

42. \$250 for 8 mo. 20 da.

43. \$360 for 5 mo. 10 da.

44. \$475 for 90 da.

45. \$900 for 6 mo. 25 da.

46. \$125 for 4 mo. 19 da.

47. \$325 for 6 mo. 23 da.

48. \$25.50 for 3 mo. 29 da.

49. \$175.50 for 105 da.

50. \$150 for 9 mo. 12 da.

51. \$387.50 for 6 mo. 25 da.

52. \$125.50 for 10 mo. 21 da.

53. \$345.50 for 6 mo. 15 da.

54. \$755 for 1 yr. 9 mo. 6 da.

55. \$544 for 5 yr. 3 mo. 5 da.

56. \$80.80 for 2 yr. 15 da.

57. \$5175 for 4 yr. 10 mo. (60 mo. − 2 mo.).

58. \$640 for 3 yr. 2 mo. (40 mo. − 2 mo.).

59. \$1240.60 for 2 yr. 9 mo. 15 da.

From the interest on any principal at 6 %, the interest at other rates may be found by adding or subtracting aliquot parts of the interest at 6 % ; thus,

| | |
|---|---|
| $4\,\% = 6\,\% - \frac{1}{3}$ of $6\,\%$ | $7\frac{1}{2}\,\% = 6\,\% + \frac{1}{4}$ of $6\,\%$ |
| $4\frac{1}{2}\,\% = 6\,\% - \frac{1}{4}$ of $6\,\%$ | $8\,\% = 6\,\% + \frac{1}{3}$ of $6\,\%$ |
| $5\,\% = 6\,\% - \frac{1}{6}$ of $6\,\%$ | $9\,\% = 6\,\% + \frac{1}{2}$ of $6\,\%$ |
| $7\,\% = 6\,\% + \frac{1}{6}$ of $6\,\%$ | $10\,\% = \frac{1}{6}$ of $6\,\% \times 10$ |

60. Find the interest at 5 % on \$360 from May 1, 1905 to March 16, 1907.

Time, 1 yr. 10 mo. and 15 da. = $22\frac{1}{2}$ mo.

| | | |
|---|---|---|
| | Principal | = \$360.00 |
| Interest of \$360 for 20 mo. at 6 % | = | \$36.00 |
| Interest of \$360 for 2 mo. at 6 % | = | 3.60 |
| Interest of \$360 for $\frac{1}{2}$ mo. at 6 % | = | .90 |
| Interest of \$360 for $22\frac{1}{2}$ mo. at 6 % | = | \$40.50 |
| *Less* interest of \$360 for $22\frac{1}{2}$ mo. at 1 % | = | 6.75 |
| Interest of \$360 for $22\frac{1}{2}$ mo. at 5 % | = | \$33.75 |

Find the interest on :

61. \$216 from July 25, 1891 to Sept. 10, 1893, at 6 %.

62. \$348 from Jan. 16, 1893 to Feb. 4, 1896, at 7 %.

63. \$650.40 from April 19, 1903 to March 4, 1906, at 4 %.

64. \$1200 from Nov. 2, 1900 to Oct. 19, 1903, at $4\frac{1}{2}$ %.

65. \$1800 from Aug. 25, 1902 to June 1, 1906, at $4\frac{1}{2}$ %.

66. \$476.25 from Aug. 19, 1902 to June 1, 1906, at 8 %.

67. \$1600 from Sept. 25, 1902 to May 18, 1906, at $4\frac{1}{2}$ %.

68. \$164.88 from July 3, 1903 to Dec. 31, 1905, at 5 %.

### The One Dollar Six Per Cent Method

The interest on \$1 for 30 da. (1 mo.) = \$.005.
The interest on \$1 for 1 da. ($\frac{1}{30}$ of \$.005) = \$.000$\frac{1}{6}$.

*Change the time to months and days. Since the interest on \$1 for 1 mo. is $\frac{1}{2}$ of a cent and for 1 da. $\frac{1}{6}$ of a mill, the interest on one dollar will be $\frac{1}{2}$ as many cents as there are months and $\frac{1}{6}$ as many mills as there are days. Multiply the result by a number equal to the number of dollars in the principal.*

### Written Work

**1.** What is the interest on \$240.60 for 2 yr. 3 mo. 13 da. at 6%?

2 yr. 3 mo. = 27 mo.
Interest on \$1 at 6% for 27 mo. = \$.135
Interest on \$1 at 6% for 13 da. = \$.002$\frac{1}{6}$

Interest on \$1 at 6% for 2 yr. 3 mo. 13 da. = \$.137$\frac{1}{6}$
Interest on \$240.60 = 240.60 × \$.137$\frac{1}{6}$, or \$33.00

Find the interest at 6% on:

**2.** \$7450 for 93 da.
**3.** \$8400 for 65 da.
**4.** \$9800 for 40 da.
**5.** \$8440 for 72 da.
**6.** \$5500 for 5 da.
**7.** \$6750 for 8 da.
**8.** \$4765 for 25 da.
**9.** \$6245 for 110 da.
**10.** \$8425 for 52 da.
**11.** \$5150 for 3 mo.
**12.** \$8465 for 4 mo.
**13.** \$9640 for 7 mo.
**14.** \$8790 for 5 mo.
**15.** \$8250 for 6 mo.
**16.** \$150 for 3 mo. 6 da.
**17.** \$180 for 5 mo. 9 da.
**18.** \$195 for 6 mo. 10 da.
**19.** \$250 for 8 mo. 12 da.
**20.** \$340 for 2 yr. 4 mo.
**21.** \$275 for 3 yr. 8 mo.
**22.** \$450 for 1 yr. 5 mo.
**23.** \$675 for 3 yr. 7 mo.
**24.** \$64.60 for 2 yr. 9 mo.
**25.** \$78.40 for 4 yr. 3 mo.

Find the interest:

**26.** At 5% on \$237.50 from Jan. 3, 1906 to Sept. 11, 1908.

**27.** At 7% on \$309.75 from May 5, 1905 to Jan. 12, 1907.

**28.** At $7\frac{1}{2}$% on \$7500 from June 12, 1907 to Nov. 1, 1909.

**29.** At $6\frac{1}{2}$% on \$6225 from Oct. 11, 1907 to Mch. 1, 1910.

**30.** At $4\frac{1}{2}$% on \$750 from Feb. 12, 1907 to Aug. 9, 1911.

**31.** At 8% on \$2900 from July 1, 1906 to May 10, 1910.

**32.** At 4% on \$3675 from June 4, 1907 to Apr. 1, 1910.

**33.** At 3% on \$290.80 from Nov. 12, 1907 to Apr. 10, 1912.

**34.** At 5% on \$875 from June 11, 1907 to Mch. 11, 1912.

**35.** At $5\frac{1}{2}$% on \$8000 from Apr. 1, 1907 to May 9, 1912.

Solve the following problems by both the *six per cent methods* and compare results as to time and accuracy.

Find the interest from the following conditions:

| | Prin. | Time | Rate | | Prin. | Time | Rate |
|---|---|---|---|---|---|---|---|
| **36.** | \$350 | 105 da. | 7% | **39.** | \$129 | 23 mo. | 9% |
| **37.** | \$685 | 4 mo. | 8% | **40.** | \$750 | 13 mo. | $9\frac{1}{2}$% |
| **38.** | \$850 | 87 da. | 6% | **41.** | \$492 | 97 da. | $7\frac{1}{4}$% |

**42.** Find the amount of \$1275.80 for 1 yr. 7 mo. 24 da. at 7%.

**43.** Find the interest of \$4780 from April 1, 1907 to Sept. 18, 1909, at $6\frac{1}{2}$%.

**44.** On the 16th of September, 1907, I borrowed \$3600 at 8%. How much will settle the loan April 1, 1909?

**45.** July 28, 1907, a broker borrows \$3200 at 5% interest and on the same day loans it at $7\frac{1}{2}$% interest. If full settlement is made April 1, 1909, how much will the broker make by reloaning the money?

## PROBLEMS IN SIMPLE INTEREST

### Finding the principal.

#### Written Work

**1.** What principal invested at 4% per annum will yield an annual income of $200?

Since the interest on $1 for 1 year at 4% is $.04, as many dollars must be invested to yield $200 per year as $.04 is contained times in $200. $200 ÷ $.04 = 5000. Hence, $5000 must be invested.

*The* **principal** *equals the given interest divided by the interest on* $1 *for the given time at the given rate.*

**2.** What principal at $4\frac{1}{2}$% will gain $213.75 interest in 4 yr. 9 mo.?

**3.** What principal at 5% will gain $120.70 interest in 3 yr. 6 mo. 18 da.?

**4.** What principal at 8% will gain an interest of $163.20 from Sept. 30, 1903 to June 12, 1905?

**5.** A man gave his note April 1, 1901, at 6%. When he settled the note Aug. 13, 1903, he paid $195.25 interest. What was the principal, or face of the note?

### Finding the rate.

#### Written Work

**1.** At what rate must $500 be invested to yield $75 interest in 2 yr. 6 mo.?

Since the interest on $500 for $2\frac{1}{2}$ yr. at *1* % is $12.50, it will require a rate of as many per cent to yield $75, as $12.50 is contained times in $75, or 6%.

*The* **rate** *equals the given interest divided by the interest for the given time at* 1%.

**2.** The interest on $1125 for 3 yr. 4 mo. 24 da. is $229.50. What is the rate per cent?

**3.** The interest on \$1800 for 4 yr. 8 mo. 16 da. is \$424. What is the rate?

**4.** At what rate will \$2460 give \$682.65 interest in 5 yr. 6 mo. 18 da.?

**5.** A note for \$880 was given July 5, 1900, and settled May 2, 1904, for \$1081.96. What rate per cent was charged?

**Finding the time.**

### Written Work

**1.** In what time will \$450 at 6% yield \$90 interest?

Since the interest on \$450 for *1 yr.* at 6% is \$27, the required time is as many years as \$27 is contained times in \$90, or $3\frac{1}{3}$ yr.

*The* **time** *in years equals the given interest divided by the interest at the given rate for* 1 *year.*

**2.** In what time will \$275 gain \$55 interest at 6%?

**3.** In what time will any principal double itself (that is, gain 100% of itself) at 5%? at 6%? at 8%?

**4.** In what time will any principal treble itself (that is, gain 200% of itself) at 5%? at 6%? In what time will it quadruple itself at 8%? at 10%?

**5.** A note of \$500 at 5% interest was paid May 1, 1905, the interest amounting to \$63.75. When was the note given?

### REVIEW PROBLEMS

**1.** Find the interest on \$80 for 6 yr. 6 mo. 18 da. at 6%.

**2.** In what time will \$180 at 5% yield \$22.50 interest?

**3.** In what time will \$600 amount to \$715.50, at $5\frac{1}{2}$% interest?

4. At what rate will \$1650 gain \$326.70, in 4 yr. 4 mo. 24 da.?

5. What sum of money loaned at 6% will give a semi-annual income of \$13.02?

6. Interest \$31.80; time 3 yr. 6 mo. 12 da.; rate 5%. Find the principal.

7. The interest of $\frac{1}{3}$ of a principal for 3 yr. 6 mo. at 6% is \$19.25. Find the principal.

8. If \$186 pays a debt of \$150 which has been due for 4 years, what is the rate of interest?

9. I borrowed \$450 at 5% and kept it until it amounted to \$525. When did I settle the note?

10. Dec. 1, 1901, I loaned \$300 at $5\frac{1}{2}$%. Find the amount due March 16, 1905.

11. Find interest at 6% on \$350 from June 1, 1898 to Aug. 13, 1902.

12. If \$300 was borrowed April 1, 1902, at 5%, when should the principal and interest be paid that their sum may be \$357?

13. The amount of a certain principal at 6% for a given time is \$780, and at 10% for the same time it is \$900. Find the principal.

14. If \$148 is loaned April 1, 1902, at 5%, when will it amount to \$179.45?

15. What principal for 3 mo. at 8% will yield the same interest as \$5100 for 5 yr. 8 mo. at 6%?

16. A farmer bought 75 acres of land at \$50 an acre, paying $\frac{1}{3}$ cash, and giving his note for the balance, due in 3 yr. 6 mo., with interest at 6%. What was the amount of the note at maturity?

## ANNUAL INTEREST, OR SIMPLE INTEREST ON UNPAID INTEREST

In some states when a note reads with interest "payable annually," simple interest may be collected upon the principal and upon each year's interest from the time it was due until paid.

In most states annual interest is not collectable by law.

Interest payable annually is simple interest; but interest collected on the principal and on the overdue payments of simple interest is **annual interest**.

### Written Work

**1.** James Brown borrows $1200 at 6 % interest, "payable annually." In case no interest is paid for 3 years, 6 months, and 15 days, how much money is necessary to pay the debt?

| | | |
|---|---|---|
| Simple interest on $1200, at 6%, for 3 yr. 6 mo. 15 da. | 255 | |
| The interest for each year is $72. . . . . . | | |
| The 1st annual int., $72, remains unpaid for 2 yr. 6 mo. 15 da. | | |
| The 2d annual int., $72, remains unpaid for 1 yr. 6 mo. 15 da. | | |
| The 3d annual int., $72, remains unpaid for 6 mo. 15 da. | | |
| Interest on $72 at 6% for . . . . 4 yr. 7 mo. 15 da. | 19 | 98 |
| The **annual interest** on $1200 for 3 yr. 6 mo. 15 da. . | 274 | 98 |
| The principal . . . . . . . . . . | 1200 | 00 |
| The **amount** of $1200 at annual interest for 3 yr. 6 mo. 15 da. | 1474 | 98 |

**Annual interest** *is the simple interest on the principal for the given time plus the simple interest on each year's interest for the time it remains unpaid.*

**2.** Find the total interest due on a note of $675 for 2 years, 8 months, and 20 days at 6 %, with interest payable annually, if no interest has been paid.

**3.** Find the amount of $6400 for 4 years, 5 months, and 15 days, with interest payable annually at 6 %.

**4.** An attorney collects a note of \$3750 with annual interest on it at 6% for 4 yr. 9 mo. 18 da. Find the amount collected and his commission on it at 10%.

## EXACT INTEREST

**Exact interest** is simple interest on the principal reckoned on the basis of 365 days to a common year and 366 days to a leap year.

It is used in computing interest on all obligations by the United States government; on all foreign securities; and to some extent by city controllers and bankers.

Since common interest is computed on the basis of 12 months of 30 days each, or 360 days; and *exact interest* is reckoned on the basis of 365 days to a common year, or 366 to a leap year, 1 day's exact interest is $\frac{1}{365}$ of a year's common interest.

It is evident that the *common* and *exact* interest for 1 year are the same. Thus, $\frac{365}{365}$ of one year's common interest equals one year's exact interest. They differ only for parts of a year.

### Written Work

**1.** Find the exact interest on \$2400 for 95 days at 6%.

Exact interest for 1 year = 6% of \$2400, or \$144
Exact interest for 1 day = $\frac{1}{365}$ of \$144, or \$.3945
Exact interest for 95 days = 95 × \$.3945, or \$37.48

**Exact interest** *is found by dividing the common interest at the given rate for one year by* 365 *and multiplying the quotient by the exact number of days.*

Find the exact interest of:

**2.** \$800 for 78 days at 6%.

**3.** \$2000 for 92 days at 7%.

**4.** \$2400 for 115 days at 8%.

**5.** \$1775 for 100 days at $8\frac{1}{2}$%.

**6.** \$500 for 90 days at 9%.

**7.** \$1020 for 74 days at 10%.

**8.** \$6500 for 280 days at 6%.

**9.** \$10000 for 61 days at 7%.

**10.** Find the exact interest on \$1020 from Oct. 19, 1905 to April 1, 1907, at 6%.

NOTE. — Why do we find exact interest for a fraction of a year only? The exact number of days from Oct. 19, 1905 to April 1, 1907, is found as follows: Oct., 12 da.; Nov., 30 da.; Dec., 31 da.; Jan., 31 da.; Feb. 28 da.; March, 31 da.; April, 1 da. Total, 164 days.

**11.** Find the exact interest on $1795.80 from July 7, 1904 to Sept. 1, 1907 at 7 %.

**12.** The United States government paid exact interest at 4 % on a warrant of $650000, 83 days past due. Compute the amount paid.

## COMPOUND INTEREST

Mr. Reed Colburn loans Robert Patterson $200 for 2 years at 6 %.

Suppose Mr. Patterson says to Mr. Colburn, at the end of the first year: "I cannot pay you the $12 interest due, but will pay you interest at 6 % on the $12 for a year." How much interest should Mr. Patterson pay Mr. Colburn at the end of the 2 years? How does the $24.72 interest differ from simple interest?

**Compound interest** is interest on both the principal and the unpaid interest added to the principal when due.

Interest may be added to the principal annually, semiannually, or quarterly, according to agreement.

### Written Work

**1.** Find the compound interest on $200 for 2 years 6 months at 6 %.

| | |
|---|---|
| Principal | $ 200.00 |
| Interest for 1st yr. at 6 % | 12.00 |
| Principal for 2d year | 212.00 |
| Interest for 2d yr. at 6 % | 12.72 |
| Principal for 3d yr. | 224.72 |
| Interest for 6 mo. at 6 % | 6.74 |
| Amount for 2 yr. 6 mo. at 6 % | 231.46 |
| Original principal | 200.00 |
| Compound interest for 2 yr. 6 mo. at 6 % | 31.46 |

NOTE. — 1. Unless otherwise stated in the agreement, interest is compounded annually.

2. When interest is compounded semiannually, consider the rate as $\frac{1}{2}$ the annual rate, or if quarterly, $\frac{1}{4}$, etc.

**2.** Find the compound interest on $1000 for 2 years at 5%, with interest compounded semiannually.

**3.** Find the compound interest at 6% on $800 for 1 yr. 5 mo., interest payable quarterly.

**4.** Find the compound interest on $600 for 9 mo. at 6%, interest payable quarterly.

## SAVINGS ACCOUNTS

Compound interest is no longer allowed on *notes*. Its only practical application for elementary schools is found in computing interest on savings accounts.

Many banks to-day have a savings department. The amounts thus deposited are not subject to check, but draw from 2% to 4% interest which is usually compounded semiannually.

The interest periods are generally *January* 1 and *July* 1 of each year, although sometimes the interest is compounded quarterly. Thirty days are reckoned to a month.

Interest on savings accounts is sometimes calculated from the 1*st and* 15*th of each month succeeding the several deposits.* Thus, $10 deposited on the 1*st of any month* would draw interest from date; but $10 deposited on the 2*d of any month* would draw interest from *the* 15*th;* or money deposited on the 16th would draw interest from the 1st of the next month. There is no fixed rule, however, as each bank determines for itself when the interest date begins. No interest is allowed on a fractional part of a dollar, and parts of a cent are omitted on all interest credits.

Most banks require *notice* from a depositor before a savings account may be withdrawn. Amounts withdrawn before the end of an interest period draw no interest for that period.

### Written Work

**1.** On July 1, 1905, Raymond Wilkinson makes a savings deposit of $400 at 4 % interest, payable semiannually. If the interest at each period is added to the deposit, what is the total amount in bank January 1, 1907?

| | |
|---|---|
| Deposit July 1, 1905 | $400.00 |
| Interest on $400 at 4 %, July 1, 1905 to Jan. 1, 1906 | 8.00 |
| Amount in bank Jan. 1, 1906 | 408.00 |
| Interest at 4 % on $408 from Jan. 1, 1906 to July 1, 1906 | 8.16 |
| Amount in bank July 1, 1906 | 416.16 |
| Int. at 4 % on $416 (why?) from July 1, 1906 to Jan. 1, 1907 | 8.32 |
| Amount in bank Jan. 1, 1907 | 424.48 |

**2.** Find the difference between the simple interest on a note of $200 dated July 1, 1906, due in two years at $4\frac{1}{2}$ %, and the interest on $200 deposited in a savings bank at 4 %, compounded semiannually, for the same period.

**3.** A savings account of $150 deposited April 1, 1906, at 3 % interest, payable January 1 and July 1, is withdrawn April 12, 1908. Find the amount withdrawn.

**4.** A savings bank pays 4 % interest, calculated from the 1st and the 15th of each month succeeding the several deposits. The deposits are Sept. 1, $20; Oct. 10, $15; Nov. 15, $20; Dec. 10, $25. Find the amount in bank the following January 1, if the interest periods are January 1 and July 1.

**5.** The Lincoln School had on deposit in the Holmes Savings Bank Jan. 1, 1907, $495.80. The deposits were as follows: Feb. 1, $76.90; March 1, $105.05; April 1, $114.29; May 1, $129.70; June 1, $98.75. Find the amount in the bank Jan. 1, 1908, at 4 % interest, compounded the first of January and July.

Find the amount in bank from the following deposits:

| | Deposit | Date | Rate | Int. Payable | Amount in Bank |
|---|---|---|---|---|---|
| **6.** | $200 | Jan. 1, 1905 | 3 % | Jan. 1 and July 1 | July 1, 1906 |
| **7.** | $150 | Mar. 16, 1906 | 4 % | Jan. 1 | Jan. 1, 1908 |
| **8.** | $875 | May 29, 1906 | 2½ % | Jan. 1 | Jan. 1, 1908 |
| **9.** | $1200 | Aug. 10, 1906 | 2 % | Jan. 1, Apr. 1, July 1, Oct. 1 | Jan. 1, 1908 |

## INVESTMENTS

**Compound interest tables** are frequently used by insurance companies, building and loan associations, and trust companies, to calculate the income from investments where the interest is added each interest period to the amount invested.

The following table shows the amount of $1 at compound interest at the given rates for 10 years.

### Compound Interest Table

| Yr. | 1½ % | 2 % | 2½ % | 3 % | 3½ % | 4 % |
|---|---|---|---|---|---|---|
| 1 | 1.0150 000 | 1.0200 0000 | 1.0250 0000 | 1.0300 0000 | 1.0350 0000 | 1.0400 0000 |
| 2 | 1.0302 250 | 1.0404 0000 | 1.0506 2500 | 1.0609 0000 | 1.0712 2500 | 1.0816 0000 |
| 3 | 1.0456 784 | 1.0612 0800 | 1.0768 9062 | 1.0927 2700 | 1.1087 1787 | 1.1248 6400 |
| 4 | 1.0613 636 | 1.0824 3216 | 1.1038 1289 | 1.1255 0881 | 1.1475 2300 | 1.1698 5856 |
| 5 | 1.0772 840 | 1.1040 8080 | 1.1314 0821 | 1.1592 7407 | 1.1876 8631 | 1.2166 5290 |
| 6 | 1.0934 433 | 1.1261 6242 | 1.1596 9342 | 1.1940 5230 | 1.2292 5533 | 1.2653 1902 |
| 7 | 1.1098 450 | 1.1486 8567 | 1.1886 8575 | 1.2298 7387 | 1.2722 7926 | 1.3159 3178 |
| 8 | 1.1264 926 | 1.1716 5938 | 1.2184 0290 | 1.2667 7008 | 1.3168 0904 | 1.3685 6905 |
| 9 | 1.1433 900 | 1.1950 9257 | 1.2488 6297 | 1.3047 7318 | 1.3628 9735 | 1.4233 1181 |
| 10 | 1.1605 408 | 1.2189 9442 | 1.2800 8454 | 1.3439 1638 | 1.4105 9876 | 1.4802 4428 |

The compound interest on any amount for 4 years at 8 % payable semiannually is evidently the same as upon the same amount for 8 years at 4 % payable annually.

*The* **amount of any given principal for any given number of years** *is found by multiplying the principal by the amount of* **$1** *at the given rate for the time as given in the table.*

### Written Work

**1.** Find the amount of \$1200 invested for 7 years at $3\frac{1}{2}$ %, interest compounded annually.

**2.** Find the compound interest at 4 % on \$10000 invested for 9 years.

**3.** The amount of \$12000, invested for 10 years at $3\frac{1}{2}$ %, interest compounded annually, is divided equally among 3 sons. Find each one's share.

**4.** Find the amount of \$1200 for 2 years and 6 months at 4 %, compound interest payable semiannually.

## PROMISSORY NOTES

Mr. James H. Ames, a grocer, Erie St., Buffalo, N.Y., has an account of \$52.00 against Robert Patterson for groceries, and Mr. Ames asks Mr. Patterson to give him a note at 6 % interest for the amount of the bill.

The note reads as follows:

> *\$52.00* *Buffalo, N.Y., Nov. 21, 1905.*
>
> *Six months after date I promise to pay to the*
>
> *order of James H. Ames*
>
> *Fifty-two Dollars.*
>
> *Value received, with interest at 6%.*
>
> *Robert Patterson.*

*A* **promissory note** is a *written* promise to *pay* to a *certain person* named in the note, or his *order*, a *specified sum* of *money* at a specified *time*.

**The Essentials of a Promissory Note:**

1. It should state the **place** where and the **time** when given.

2. It should promise to pay to a **certain person** or to his **order**.

3. It should promise to pay a **certain sum of money**, expressed both in **figures** and in **writing**.

4. It should state **when** the money is to be paid.

5. It should state **by whom** the money is to be paid.

6. It should state for **value received**.

(Not absolutely necessary, but usually written in a note.)

7. It should state **with interest** and the **rate**, if it is an interest-bearing note.

The promissor is called the **maker** of the note.

The person who is to receive the money is called the **payee** of the note.

1. Who is the *maker* of the note on page 292?

2. Who is the *payee* of the note on page 292?

3. Find the *amount* to be paid when due.

4. The **face** of the note is the sum written in the note. What is the *face* of the note on page 292?

5. This note reads "pay to the *order of* James H. Ames," and means that Mr. Ames has the right to sell this note to any one by simply writing his name across the back of the note and delivering it to the purchaser. What words in the above note give Mr. Ames the right to sell it?

When the owner of a promissory note writes his name across the back of it, he is said to **indorse** the note.

If Mr. Ames indorses the note and then sells it to Mr. B., and Mr. B. indorses it and sells it to Mr. C., to whom does the note belong?

A promissory note, therefore, like any other property may be bought and sold; hence it is called **negotiable paper.**

When a note is made payable to a definite person, it cannot be transferred, and is therefore *not* negotiable.

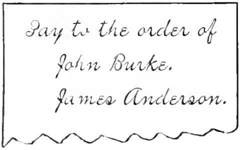
Promissory notes may be indorsed as follows:

(1) **In blank**: In this form the indorser simply writes his name across the back of the note, thus making the note payable to the holder.

INDORSEMENT IN BLANK

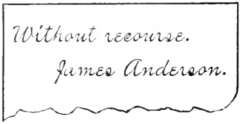

(2) **In full**: In this form the indorser designates that the note is to be paid to the order of a definite person.

INDORSEMENT IN FULL




(3) **In limited form**: In this form the indorser writes "without recourse" above his name. This means that the holder cannot compel him to pay if the maker fails to do so.

LIMITED INDORSEMENT




Every indorser in blank, or in full, makes himself *liable* for the amount of the note if the maker and the previous indorsers *fail* to pay. Banks are required to notify the indorsers in a manner prescribed by law in case the note is not paid when due. This is called **protesting** the note. If the note is not protested, the indorsers are released from the liability of payment.

## Forms of promissory notes.

### I. As to time:

1. If the words "on demand" are substituted for the words "six months" in the note of Mr. Ames, page 292, it will then be a "demand note"; that is, the maker may be called upon to pay it at any time after date.

2. The note of Mr. Ames is a "time note" because it is not to be paid until a certain time named in the note.

The *time* of payment in a note must be definite. A promise to pay "when able" is too indefinite, and not binding.

### II. As to payees:

1. When a note is payable to the *order of* some particular person, he alone can collect it, or sell the note by indorsement.

2. When a note is payable to some particular person, or bearer, the holder can collect it, or sell it by indorsement.

### III. As to the number of makers:

1. An individual note is a promise made by one person.

2. "A joint and several note" is a promise made by more than one person. It contains the words "we, or either of us," and is signed by the makers.

### Maturity of Promissory Notes.

A note is said to *mature* on the last day of the time named in the note. Some states allow 3 days, called "days of grace," from the time a note matures before the payee can proceed to collect the note. In this case three days are added to the time on which the interest is computed. Days of grace are now abolished in most states. The note on page 292 matures May 21, 1906.

If a note falls due on Saturday, Sunday, or a legal holiday, it is usually payable on the next *succeeding* business day. Some states require such notes to be paid on the *preceding* business day.

### Interest on Promissory Notes.

If either a time or a demand note contains the words "with interest," the note bears interest from *date* at the legal rate in that state.

If the words "with interest" are omitted from a time note, it bears interest from the date of *maturity* until paid.

If the words "with interest" are omitted from a demand note, it bears interest from the time payment is demanded until paid.

## Written Work

**1.** Is the promissory note given by Mr. Patterson (p. 292) a *demand* or a time note? an individual or a joint and several note? payable to order or bearer?

**2.** Write a promissory note, in which you are the maker, for $125 due in 6 months, payable to the order of Ellsworth Slater, with the legal rate of interest in your state.

**3.** Mr. Slater sells this note to Herman Gross, and indorses it in full. Write the indorsement on the note.

**4.** Write a joint and several note for $250, dated Sept. 24, 1905, due on demand, with interest at 6%, payable to the order of James Harbison. Your teacher and yourself may sign this note as makers.

**5.** May the two names to the above note be written by the same person? Why not?

**6.** In case James Harbison sells this note to James Brown, but says to Mr. Brown, "I shall not be responsible for the collection of this note," write the indorsement and explain why you use that form.

**7.** Find the amount paid to the holder of Mr. Harbison's note (Ex. 4), if settled Jan. 4, 1907.

NOTE. — Time from Sept. 24, 1905 to Jan. 4, 1907, 1 yr., 4 mo., 11 da.

**8.** Name the different kinds of negotiable notes:

(1) as to *time*; (2) as to *payee*; (3) as to *number of makers*.

**9.** Find the amount to be paid on the following note, if settled March 1, 1907:

The note is legally due June 1, 1906; therefore it bears interest at the legal rate from that date.

*$300.00* *Chicago, Ill., March 1, 1906.*

*Three months after date I promise to pay to the order of James Bryce*

*Three hundred Dollars.*

*Value received.*

*James Anderson.*

**10.** Find the interest to be paid on the following note, if settled Aug. 10, 1906, with interest at 6 %:

| | |
|---|---|
| $175.50 | *Boston, Mass., Jan. 20, 1906.* |
| *On demand after date* I *promise to pay to the order of* Theodore Noel | |
| One hundred seventy-five $\frac{50}{100}$ *Dollars.* | |
| *Value received, with interest.* | Arthur Mason. |

**11.** If the words "with interest" are not mentioned in a time note, from what date does the note bear interest?

**12.** If a note reads "with interest," and no rate is mentioned, what rate per cent is to be taken?

**13.** If the words "with interest" are not written in a demand note, does it ever bear interest?

**14.** If days of grace are allowed in your state, how do you find the time on which the interest is to be reckoned?

**15.** How do you compute time on a promissory note?

**16.** Which is the safer form:

Pay to the order of Henry James, or

Pay to the order of Henry James or bearer? Why?

NOTE. — The note of Mr. Ames (p. 292) could have been made payable to James H. Ames or bearer. Why is the form as written in the note better?

It is safest to indorse a note *in full*, for the reason that if it is lost in its delivery by mail or messenger, it can be collected only by the party named on the back of it.

Write negotiable notes, observing the essential conditions as given on p. 293, and adding days of grace if allowed in your state. Find the amount due at date of settlement. On overdue, non-interest bearing notes, compute interest at 6%.

| | Date | Face | Time | Payee | Maker | Int. Rate | Settlement |
|---|---|---|---|---|---|---|---|
| **1.** | 2/5/05 | $100 | 6 months | George Kimes | Yourself | 6% | Maturity |
| **2.** | 4/21/06 | 250 | On demand | A. J. Edwards | James Clyde | 8% | 7/29/06 |
| **3.** | 6/10/05 | 500 | 1 year | John Dunn | John Grant | ( ) | 6/15/07 |
| **4.** | 8/16/04 | 350 | 6 months | N. J. Noel | James Palm | ( ) | 12/15/05 |
| **5.** | 7/12/05 | 125 | On demand | James Bryce | Yourself | 7% | 1/2/06 |
| **6.** | 5/15/05 | 1200 | 3 months | M. J. Boyce | B. J. Morrow | $6\frac{1}{2}$% | Maturity |
| **7.** | 10/10/05 | 300 | 6 months | Ralph George | Ben Jarrett | 6% | 9/12/06 |

## PARTIAL PAYMENTS OF PROMISSORY NOTES

### United States Rule

It is frequently inconvenient for the borrower to pay the face of the note all at one time. He is sometimes permitted, by special contract, to make payments at any time or at interest bearing periods, until the note is paid. These amounts are called **partial payments** and are credited on the back of the note, together with the date of payment.

1. For example, a borrower gives the following note:

*$200.* *Braddock, Pa., Nov. 26, 1906.*

*On demand, for value received, I promise to pay*
*James Jones* ~~~~~~~~~~~~~~~~ *or order,*
*Two Hundred and* $\frac{no}{100}$ ~~~~~~~~~~~~~~~~ *Dollars.*

*With interest at 6%.* *Henry Brown.*

The following payments are indorsed on this note:

| |
|---|
| Nov. 26, 1907, $50.00. |
| Jan. 2, 1908, $ 25.00. |

What amount is due March 2, 1908?

SOLUTION:

| | |
|---|---|
| Principal . . . . . . . . . | $200.00 |
| Interest on $200 for 1 yr. . . . . . | 12.00 |
| The amount of the note Nov. 26, 1907 . . | 212.00 |
| Payment Nov. 26, 1907 . . . . . | 50.00 |
| **Balance = new principal** due Nov. 26, 1907 . | 162.00 |

| yr. | mo. | da. |
|---|---|---|
| 1908 | 1 | 2 |
| 1907 | 11 | 26 |
| | 1 | 6 |

| | |
|---|---|
| Interest on $162 for 1 mo. 6 da. . . . . . . | $ .97 |
| Amount due Jan. 2, 1908 . . . . . . . | 162.97 |
| Payment Jan. 2, 1908 . . . . . . . . | 25.00 |
| **Balance = new principal** due Jan. 2, 1908 . . . | 137.97 |

| | | |
|---|---|---|
| 1908 | 3 | 2 |
| 1908 | 1 | 2 |
| | 2 | 0 |

| | |
|---|---|
| Interest on $ 137.97 for 2 mo. . . . . . . | 1.38 |
| The **amount** due March 2, 1908 . . . . . . | $ 139.35 |

**2.** What was the amount due on the note on Nov. 26, 1907?

**3.** How much interest was due Nov. 26, 1907? What payment was made? How much greater was the payment than the interest?

**4.** How much was the new principal due Nov. 26, 1907, after the payment of $ 50.00?

**5.** How much interest was due Jan. 2, 1908? How much greater was the payment than the interest?

Observe: 1. That the interest was computed on the principal to the time of the first payment; then on the balance, as a *new principal*, to the time of the second payment; then on the balance, as a *new principal*, until March 2, 1908.

2. That the *interest* at each payment was first paid and the *balance* of the payment was credited on the principal.

3. As the interest must *first* be paid, in case the payment does not *equal* the interest, the interest must be computed until such time as the sum of the payments equals or exceeds the interest.

This is the **United States rule** of partial payments, and is the legal one in most states.

*Find the amount of the principal to the time of the first payment, and from the amount subtract the first payment. Consider the remainder as a new principal and proceed as before until the time of final settlement. If any payment does not equal or exceed the interest, then find the interest to the time when two or more payments equal or exceed the interest.*

The Supreme Court of the United States has decreed:

(1) That the payment on a note must first be applied to cancel the interest then due, before the principal may be diminished.

(2) That interest must not be charged upon interest.

## Written Work

**1.** A note for \$1800, bearing 6% interest, was given April 1, 1903, and settled Oct. 22, 1906. On the back of the note were these indorsements: May 10, 1904, \$225; June 16, 1905, \$50; Sept. 28, 1905, \$340; March 10, 1906, \$475. Find the balance due on the note at date of settlement.

| | | |
|---|---|---|
| Principal . . . . . . . . . . . . . . | | $1800.00 |
| Interest from April 1, 1903 to May 10, 1904 . . . . . | | 119.70 |
| Amount due May 10, 1904 . . . . . . . . . | | 1919.70 |
| First payment made May 10, 1904 . . . . . . . | | 225.00 |
| Balance = **new principal** due May 10, 1904 . . . . . | | 1694.70 |
| Interest from May 10, 1904 to June 16, 1905 . . | $111.85 | |

The interest exceeds the payment and a new principal is not formed.

| | | |
|---|---|---|
| Interest from June 16, 1905 to Sept. 28, 1905, . . | 28.81 | |
| Interest from May 10, 1904 to Sept. 28, 1905 . . . . | | 140.66 |
| Amount due Sept. 28, 1905 . . . . . . . . | | 1835.36 |
| Sum of second payment June 16, 1905, and third payment Sept. 28, 1905, $50.00 + $340.00 . . . . . = | | 390.00 |
| Balance = **new principal** due Sept. 28, 1905 . . . . . | | 1445.36 |
| Interest from Sept. 28, 1905 to March 10, 1906 . . . . | | 39.024 |
| Amount due March 10, 1906 . . . . . . . . . . | | 1484.384 |
| Fourth payment made March 10, 1906 . . . . . . | | 475.00 |
| Balance = **new principal** due March 10, 1906 . . . . . | | 1009.384 |
| Interest from March 10, 1906 to Oct. 22, 1906 . . . . | | 37.347 |
| Balance due Oct. 22, 1906 . . . . . . . . . . | | $1046.731 |

**2.** A mortgage for $960, bearing 6 % interest, was given June 20, 1900, and settled Dec. 26, 1904. On the mortgage were these indorsements: Nov. 2, 1901, $140; Jan. 14, 1903, $200; June 1, 1904, $30; June 20, 1904, $150. Find the balance due on date of settlement.

**3.** On a claim of $850, dated May 2, 1901, interest 5 %, the following payments were made: Aug. 8, 1901, $200; Dec. 14, 1901, $200; April 26, 1902, $200. How much was due at final settlement April 26, 1903?

**4.** A note of $1000, dated Aug. 1, 1903, bearing interest at 6 %, had the following payments indorsed upon it: Dec. 20, 1903, $250.50; May 12, 1904, $300; Nov. 20, 1904, $400. Find the amount due June 26, 1905.

**5.** June 1, 1900, a note was given for $1700, with interest at 6%. The following payments were indorsed on this note:

| | | |
|---|---|---|
| Dec. 1, 1900 | . . . | $300. |
| June 1, 1901 | . . . | 300. |
| Nov. 1, 1901 | . . . | 300. |
| April 1, 1902 | . . . | 15. |
| May 1, 1902 | . . . | 585. |

Find the amount due July 1, 1902.

**6.** A note for $240 was made June 1, 1903, with interest at 6%. The following indorsements were made on the note: Oct. 13, 1903, $120; Jan. 19, 1904, $60; June 1, 1904, $60. Find the amount due on the note Dec. 16, 1905.

### Merchants' Rule

When partial payments are made on mercantile accounts, *overdue*, or on notes running a *year or less*, the interest is often computed by the merchants' rule.

*Find the amount of the principal from the time it begins to bear interest to the date of settlement.*

*Find the amount of each payment from the time it was made to the date of settlement.*

*From the amount of the principal, subtract the sum of the amounts of the payments. The result will be the balance due.*

### Written Work

**1.** A note for $1200, dated July 15, 1905, has the following indorsements: Sept. 25, 1905, $450; Jan. 1, 1906, $200; March 9, 1906, $150. How much is due July 1, 1906, at 6% interest?

**2.** A note for $2500, dated Jan. 1, 1907, has the following indorsements: Feb. 1, 1907, $50; March 1, 1907, $75; May 1, 1907, $100. How much is due Oct. 1, 1907?

# PART III — EIGHTH YEAR

## BANKS AND BANKING

A **bank** is an institution that receives and lends money. A national bank may also issue notes that circulate as money.

Among the various forms of banks in the United States may be mentioned **national banks**, which are under control of the Federal government; **state banks**, which are under the control of the state; **private banks**; and **savings banks**.

A **trust company** is an institution empowered by its charter to accept and execute all kinds of trusts, to act as executor, administrator, assignee, and receiver. In most states it is also empowered to do a general banking business.

**Savings accounts** have been treated under the head of Compound Interest, as the computations involved are a direct application of it.

The chief business of banks is to receive deposits for safe keeping; to lend money on approved security; and to collect drafts and bills of exchange.

**Discounting notes** is simply lending money on approved security.

**Opening an account with a bank.**

When a person opens an account with a bank, he first fills out a **deposit slip**, as indicated on page 304, and gives it, together with the deposit, to the "cashier" or "receiving teller."

| DEPOSITED WITH<br>American National Bank<br>PITTSBURG, PA. | | |
|---|---|---|
| By *James Anderson* | | |
| *Oct. 10,* 1907 | | |
| | Dollars | Cents |
| Bills | *50* | |
| Gold | *60* | |
| Silver | *20* | |
| Checks | | |
| ENTER CHECKS SEPARATELY | | |
| *1st Nat. Bank* | *65* | *80* |
| *Union Trust Co.* | *130* | *10* |
| | | |
| *Total* | *325* | *90* |

Some banks require the name of the bank on which the check is issued to be written on the deposit slip.

The depositor then writes his name and address in a book kept by the bank, so that the bank may have his signature for identification.

He then receives a **bank book**, which should always be presented to the teller when a deposit is made, in order that the dates and amounts of all the deposits may be entered. He also receives a **check book**, each page of which has one or more blank checks and stubs. When he wishes to pay a bill by check, he fills out a form from the check book, similar to the following:

| Stub | Check |
|---|---|
| No. *1401* | Pittsburg, Pa., *June 26, 1907.* No. *1401* |
| Date *June 26, '07* | American National Bank |
| Payable to *John R. Thompson* | Pay to the order of *John R. Thompson* $*900.25* |
| For *Rent to date* | *Nine hundred and* $\frac{25}{100}$ Dollars. |
| Am't. $*900.25* | *T. A. Smith.* |

A **check** is a written order by a depositor in a bank, directing the payment of money.

The **stubs** remaining in a check book, after the checks are torn out, give a complete record of the checks issued.

**1.** Who is the maker of the check on p. 304?

**2.** To whose order is this check written?

Observe: 1. The maker of a check is the one who **signs** the check.

2. The payee is the one to whose **order** the check is made payable.

3. This check is made payable to the **order** of John R. Thompson, which means that in order to receive the money from a bank, or transfer the check to another person, he must write across the back of the check the name "John R. Thompson." This is called a **blank** indorsement, because it does not state to whom the check is made payable. If John R. Thompson should write across the back of the check the following:

Pay to the order of
Marshall Field & Co.,
Chicago, Ill.
John R. Thompson,

this would be known as a **full** indorsement, for no one but Marshall Field & Co. could collect or indorse the check.

Checks, like promissory notes, may be written in different ways, as follows:

1. Pay to *bearer*,
2. Pay to *cash*,
3. Pay to *James Ogden, or bearer*,

} collectable by bearer.

4. Pay to *self* (collectable by maker only).
5. Pay to the *order of self* (collectable by indorsement of the maker).
6. Pay to the *order of James Ogden* (collectable by indorsement of James Odgen only).

Note. — The last form of check is the one in general use.

**3.** How may the check on p. 304 be indorsed in blank?

**4.** How may it be indorsed in full?

**5.** Suppose Mr. Thompson wishes to send this check to Sage, Allen & Co., Hartford, Conn., in payment of an

account: first, write the check as indorsed by Mr. Thompson in blank; second, write the check as indorsed by Mr. Thompson in full.

**6.** Give reasons why it will be better for Mr. Thompson to indorse the check in full.

**7.** When a check is indorsed and sent by mail, what form of indorsement should always be used? Why?

**8.** Give the essentials of a check.

**Balancing Accounts; Depositing; Checking on Accounts:**

**1.** Your deposits in a bank for the month of September are as follows:

Sept. 1, currency, \$50; silver, \$10; check, \$15.
Sept. 6, currency, \$20; silver, \$10; check, \$100; gold, \$20.
Sept. 10, currency, \$45; silver, \$4.75.
Sept. 16, currency, \$20; silver, \$3.40; check, \$80.
Sept. 25, gold, \$40; check, \$40; silver, \$10.
Sept. 29, check, \$80; currency, \$80; silver, \$35.

Make out deposit slips and find amount of deposits for September.

**2.** Your check book shows the following:

Balance in bank Sept. 1, \$847.10.
No. 1, Sept. 4, Keller Bros., for coal, \$15.50.
No. 2, Sept. 4, Geo. K. Stevenson & Co., for groceries for August, \$49.50.
No. 3, Sept. 4, Dr. S. N. Pool, for services to date, \$90.
No. 4, Sept. 5, cash, \$55.
No. 5, Sept. 7, Jos. Horne Co., for merchandise, \$65.30.
No. 6, Sept. 11, Midland Lumber Co., for lumber, \$93.75.
No. 7, Sept. 15, Johnson & Co., for repairs on automobile, \$29.35.
No. 8, Sept. 19, cash, \$25.

No. 9, Sept. 24, J. H. McFarland, for interest due on note, $24.

Write the checks for the bills paid for September, and find balance in bank.

**3.** Arriving at Chicago, I find in my mail a check from the Keystone Lumber Co., Pittsburg, Pa., for $415.40, in payment of my salary and expenses for September. I wish to deposit the same to my account in the Colonial Trust Co., Pittsburg, Pa. How should I indorse the check before sending it through the mail?

## BORROWING FROM BANKS AND COMPUTING BANK DISCOUNT

Banks usually lend money on promissory notes drawn in one of three forms:

1. The note is made payable to the *indorser*, who signs his name across the *back* of it.
2. A joint and several note is made payable to the *order of the bank* and signed by *both* parties as makers.
3. The note is made payable to the *order* of the bank. The security in the form of stocks, bonds, mortgages, etc., is deposited as collateral.

*$200.00* *Pittsburg, Pa., Sept. 8, 1905*

*Three months after date I promise to pay to the order of R. D. Watson at the*

**Lincoln National Bank of Pittsburg**

*Two Hundred and 00/100 Dollars without defalcation, for value received.*

*J. B. Chandler.*

This note **matures** three months after Sept. 8, or Dec. 8. If the time in the note were "ninety days" instead of "three months," the note would mature ninety days after Sept. 8, or Dec. 7.

If Mr. Chandler wishes to borrow money at the bank, he may make out a note as on p. 307 and get Mr. Watson to *indorse* it.

If both men are responsible from a financial point of view, the bank will *buy* the note and give Mr. Chandler the difference between the value of the note at its maturity and the interest on that value at the legal rate for the exact number of days the bank is without the use of its money.

The **value** of Mr. Chandler's note is the amount the Lincoln National Bank will receive from Mr. Chandler at its maturity. If Mr. Chandler fails to pay, Mr. Watson will be held responsible.

The buying of notes by a bank is called **discounting notes,** and the interest deducted is called **bank discount.**

The **proceeds** of a note discounted by a banker or a broker is the value of the note at its maturity less the discount.

The **term of discount** is the exact number of days that the borrower has the use of the money.

There are two methods, however, of reckoning this *term:* the first method counts the day of *maturity*, but not the day of *discount;* the second counts both: thus, by the first method Mr. Chandler had the use of the money 22 days in Sept., 31 days in Oct., 30 days in Nov., and 8 days in Dec., or 91 days in all; by the second method he had the use of the money 23 days in Sept., 31 days in Oct., 30 days in Nov., and 8 days in Dec., or 92 days in all.

NOTE. — Pupils should solve the problems according to the practice in their vicinity. Answers are given for both methods.

When *days of grace* are allowed they are included in the term of discount, but in this book days of grace are not reckoned.

Computing bank discount on note on p. 307:
**Date of maturity,** December 8.
**Term of discount.** 91 days (not including day of discount).
**Bank discount** on \$200 for 91 days at 6% = \$3.03.

**1.** How much does the bank pay to Mr. Chandler? How much does Mr. Chandler pay to the bank at maturity?

**2.** If Mr. Chandler had borrowed $200 from Mr. Watson for 3 months at 6 %, how much would he have paid Mr. Watson when the note became due?

**3.** How much would Mr. Chandler have received from Mr. Watson at the time he borrowed the money?

**4.** Find the difference between the discount paid to the bank and the interest he would have paid to Mr. Watson.

### Comparative Study

Banks differ from individuals in lending money, as follows:

1. *Banks* require the interest on a note to be paid in advance; *individuals* demand interest when the note is due, or annually, if for a longer period than a year.

2. *Banks* compute *interest* for the *exact number of days*: *individuals* compute interest by *months and years.*

3. *Banks* lend money for *short periods*, usually not exceeding four months; *individuals* lend for *longer periods*, not exceeding five years in most states.

4. *Banks* require the maker to give additional security; *individuals* may or may not demand security.

5. *Interest* is computed on the face value of a note; *bank discount* is computed on the value of a note at its maturity.

**Bank discount** is the simple interest paid in advance on the value of a note at its maturity for the exact number of days the banker is without his money.

**Given the dates and time of notes, to find the date of maturity.**

Find the date of maturity of the following:

| Date | Time | Date | Time |
|---|---|---|---|
| **1.** June 1 | 2 months | **5.** Jan. 2 | 3 months |
| **2.** July 3 | 50 days | **6.** March 3 | 75 days |
| **3.** Aug. 5 | 100 days | **7.** April 1 | 70 days |
| **4.** Sept. 10 | 1 month | **8.** May 5 | 4 months |

Find the date of maturity and the term of discount.

| Date of Note | Time of Note | Date of Discount | Date of Note | Time of Note | Date of Discount |
|---|---|---|---|---|---|
| **9.** March 1 | 60 da. | April 1 | **14.** Jan. 2, '08 | 90 da. | March 1 |
| **10.** April 10 | 3 mo. | June 15 | **15.** March 23 | 4 mo. | June 2 |
| **11.** July 10 | 4 mo. | Sept. 30 | **16.** Oct. 8 | 60 da. | Nov. 1 |
| **12.** May 24 | 30 da. | June 1 | **17.** June 5 | 90 da. | July 10 |
| **13.** August 5 | 70 da. | Sept. 1 | **18.** Sept. 24 | 30 da. | Oct. 1 |

### Written Work

**1.** Write a promissory note for $300 payable to John Jackson, dated Aug. 3, 1907, due in four months, with interest at 6%, and signed by Glenn Campbell.

**2.** Mr. Jackson indorses the note in example 1, and Mr. Campbell borrows the money from the Park National Bank. Indorse the note in full and find the bank discount and proceeds.

The following notes are each discounted on the day of issue. Find the date of maturity and the bank discount.

| Date of Note | Time | Face | Rate of Discount |
|---|---|---|---|
| **3.** Aug. 10, 1906 | 90 da. | $ 150 | 6% |
| **4.** June 12, 1906 | 2 mo. | 515 | $6\frac{1}{2}$% |
| **5.** July 2, 1906 | 3 mo. | 1000 | $4\frac{1}{2}$% |
| **6.** Jan. 8, 1906 | 4 mo. | 625 | $7\frac{1}{2}$% |
| **7.** Mar. 5, 1906 | 50 da. | 570 | 6% |
| **8.** May 8, 1906 | 70 da. | 423.25 | $5\frac{1}{2}$% |
| **9.** Sept. 1, 1906 | 1 mo. | 1200 | $8\frac{1}{2}$% |
| **10.** Dec. 1, 1906 | 100 da. | 387.75 | 6% |
| **11.** Mar. 5, 1906 | 72 da. | 1125 | 7% |

**Discounting Interest and Non-interest bearing Notes.**

Business men frequently take notes from their customers due at a future date, and in case they need the money before the notes become due, they sell them to a bank. The bank deducts from the value of each note at maturity the interest (bank discount) for the term of discount. These notes may or may not bear interest.

Mr. James Edwards has two notes that read as follows:

| | |
|---|---|
| *\$90.$\frac{00}{100}$* | *Columbus, Ohio, March 2, 1907* |
| *Three months after date I promise to pay to the order of James Edwards Ninety and $\frac{no}{100}$ Dollars.* | |
| *Value received.* | |
| | *Henry Austin.* |

| | |
|---|---|
| *\$150$\frac{00}{100}$* | *Columbus, Ohio, March 10, 1907* |
| *Four months after date I promise to pay to the order of James Edwards One Hundred Fifty and $\frac{no}{100}$ Dollars.* | |
| *Value received, with interest at 6%.* | |
| | *Frank Woods.* |

**1.** What is the value of the first note at maturity?

**2.** What is the value of the second note at maturity?

**3.** Mr. Edwards gets both notes discounted April 20, 1907, at 6 %. Why is the discount on the first note computed on \$90? on the second note, on \$153?

Banks always discount notes on the amount they are to receive at maturity.

Discounting the first note on page 311:

| | |
|---|---|
| **Maturity** of note, June 2. | |
| **Value** of the note at maturity . . . | = \$ 90.00, or *face* |
| **Bank Discount** for 43 da. at 6 % . . | = .65 |
| **Proceeds** April 20 . . . . . . . . . | = \$ 89.35 |

Discounting the second note on page 311:

| | |
|---|---|
| **Maturity** of note, July 10. | |
| **Value** of the note at maturity | = \$ 153.00 or *face + interest for* 4 *months.* |
| **Bank Discount** for 81 da. at 6% | = 2.07 |
| **Proceeds** April 20 . . . . | = \$ 150.93 |

## Written Work

**1.** A 60-day note for \$ 2500, without interest, dated Jan. 12, 1907, was discounted Feb. 12, 1907. Find the proceeds of this note.

**2.** Find the proceeds of a 90-day note for \$ 1560, with interest at 6 %, dated March 8, and discounted April 12.

**3.** Find the proceeds of the note in example 2 without interest.

**4.** A 90-day note for \$ 4500, with interest at 6 %, is discounted 30 days after date. Find the proceeds.

Suggestion. — Note the difference between a note for 90 days and a note discounted for 90 days.

**5.** Find the proceeds of the note in example 4 without interest.

**6.** A 120-day note for $3500, without interest, dated June 5, is discounted Aug. 10. Find the proceeds.

**7.** Find the proceeds in the note of example 6, if the note bears interest at 6 %.

**8.** The proceeds of a 90-day note, without interest, discounted 30 days after date, is $990. Find the face.

**9.** A note for $1200, bearing interest for 3 mo. at 6 %, was dated Jan. 15 and discounted Feb. 20. Find the proceeds.

**10.** Mr. Boyd gives his note Jan. 10, 1905, to William Sayers for $200, payable in 9 months, with interest at 6 %. What is Mr. Boyd's note worth on the day of issue? on the day of maturity?

**11.** Should Mr. Boyd get the note discounted July 5, on how much money would the bank reckon the discount?

**12.** Find the amount Mr. Boyd would receive July 5. What is the money received by Mr. Boyd called with reference to the note?

**13.** Write the note given by Mr. Sanders and transfer it by indorsement in *full* to one of your local banks.

**14.** Face, $223.50; time, 90 days; rate of interest, 6 %; term of discount, 50 days; rate of discount, 8 %. Find proceeds.

**15.** A business man's bank account is overdrawn $381.50, and he presents to the bank, May 1, two notes to be discounted, at 6 %, and the proceeds to be placed to his credit.

| FACE | DATE | TIME | RATE OF INTEREST |
|---|---|---|---|
| $290 | Mar. 10 | 5 mo. | Without interest |
| $355 | Apr. 20 | 90 da. | 6 % |

Find his balance.

**16.** The discount on a note for \$400 for 60 days, exact time, is \$6.00. Find the rate of discount.

**17.** A broker buys a \$300 note, thirty days before maturity, for \$297. Find the rate of discount.

**18.** Mr. James sold a horse for \$155, and took the purchaser's note, dated Jan. 20, 1905, due in one year, with interest at $4\frac{1}{2}$%. Mr. James sold the note to the Farmers' Bank, Oct. 10, at 7% discount. How much did he realize?

**19.** A note for \$1800, at 8%, dated August 1, due in 3 months, was discounted October 6, at 6%. Find the proceeds.

**20.** What should the Merchants' National Bank pay for a note of \$1200, bearing 8% interest, dated April 12, due in 4 months, if purchased June 1, at 6% discount?

**21.** What are the proceeds of a note for \$2500, dated February 10, 1907, and due in 4 months, without interest, if discounted March 24, at 6%?

**22.** A merchant's bank book shows a balance of \$1375.50, and he presents at the bank four notes, which are discounted June 1, at 6%, and the proceeds placed to his credit:

| Face | Date | Time | Rate of Interest |
|---|---|---|---|
| \$600 | March 4 | 90 days | 6% |
| \$1375 | April 2 | 4 months | No interest |
| \$1050 | March 19 | 90 days | 5% |
| \$2000 | May 29 | 3 mo. | No interest |

Find his balance in bank then.

**23.** For how much must I give my note, discounted at a bank for 60 days, at 6%, to realize \$990?

Note. — The proceeds of \$1, discounted for 60 days, at 6% = \$.99.

**24.** For what sum must I draw my note so that when discounted at 6% for 90 days I may realize \$2758?

# EXCHANGE

## Paying Bills at a Distance

What is meant by a *debtor?* by a *creditor?* How may a debtor pay a bill in a distant city without the actual transfer of cash? How may a creditor collect a bill in a distant city without the actual transfer of cash?

**Exchange** is a method of paying or collecting bills at a distance without the actual transfer of money.

There are several different ways in which bills may be paid at a distance without the transmission of money:

(1) By a postal money order.
(2) By an express money order.
(3) By a telegraphic money order.
(4) By a personal check.
(5) By a bank draft (banker's check).

### Paying by postal or express money order.

If you wish to order from Siegel, Cooper & Co., Chicago, $15 worth of merchandise, unless you have credit there, you will probably send them (1) either a *postal money order*, or (2) an *express money order.* The first will direct the postmaster at Chicago, the second some express agent at Chicago, to pay to the order of Siegel, Cooper & Co. $15.

The *cost* of either of the above orders is the same; the only difference being that a postal money order is payable to the order of the party or firm upon identification at the place named in the order, while an express money order is payable to the party or firm upon identification at *any* office of the same company where orders are sold.

Money orders may be purchased for any amount up to \$100, payable to any person or firm in the United States, or foreign countries where such orders are sold.

The rates charged in the United States are as follows:

| | |
|---|---|
| \$2.50 and under | 3¢ |
| Over \$2.50 and not exceeding \$5.00 | 5¢ |
| Over \$5.00 and not exceeding \$10.00 | 8¢ |
| Over \$10.00 and not exceeding \$20.00 | 10¢ |
| Over \$20.00 and not exceeding \$30.00 | 12¢ |
| Over \$30.00 and not exceeding \$40.00 | 15¢ |
| Over \$40.00 and not exceeding \$50.00 | 18¢ |
| Over \$50.00 and not exceeding \$60.00 | 20¢ |
| Over \$60.00 and not exceeding \$75.00 | 25¢ |
| Over \$75.00 and not exceeding \$100.00 | 30¢ |

The rates to *foreign* countries are from 10¢ to \$1 for the same amounts as domestic orders.

This fee of from 3¢ to 30¢ for domestic orders, and from 10¢ to \$1 for foreign orders, to cover the cost of paying the bills at a distance, is called the exchange for issuing the orders.

### Paying by telegraphic money order.

Such orders are drawn by agents of the telegraph company, and direct the agent at some designated office to pay to the person named in the telegraphic message, upon identification, the sum specified.

The present rates for sending money by telegraphic order are as follows:

For \$25 or less, double the cost of a ten-word message, plus 25¢.

Above \$25, double the cost of a ten-word message, plus 1% of the amount of the order.

### Paying by checks.

Business men find it necessary to pay bills in their vicinity or at a distance, almost daily, and if their financial standing is good, their checks are generally accepted in payment. In fact, most bills to-day are paid by checks.

Sometimes the seller does not know the financial standing of the purchaser, and therefore requires the check accompanying the order to be *certified;* that is, the cashier of the bank on which the check is drawn stamps the word "certified," with the date and his signature, across the face of the check. The check is thereafter the check of the bank, and is good as long as the bank is solvent.

A **certified check** is a notice to the payee of the check that the amount named on the face has been taken from the maker's deposit and placed with the bank's funds for the payment of the check when presented.

Certified checks are frequently demanded in payment of notes and collections at banks, and in payments where the payee does not wish to take a personal check. Like other checks, they are mailed daily in payment of bills in all parts of the country.

**1.** What is a check? What are the essentials of a check?

**2.** In buying a lot from James Carothers for $800, you are asked to give your certified check for the amount. Write the check on your local bank, yourself being the maker.

**3.** Moore & Co., Youngstown, Ohio, purchase $825 worth of furniture at 20% and 10% off from James Boydson & Co., Detroit, Mich., 3% off for cash in 10 days. They send a certified check within ten days on the Diamond Trust Co., of which James Patterson is secretary and treasurer. Write the certified check in payment of the bill.

NOTE. — The *secretary and treasurer* of a trust company corresponds to the cashier of a bank.

**4.** William Anderson, 7531 Hermitage Ave., Chicago, Ill., receives a check on the First National Bank of Wilkinsburg, Pa., from Freeman Lewis for $730.80 in settlement of an estate. The Commercial National Bank of Chicago charges Mr. Anderson $1.50 for collecting the check. This fee is called the **exchange** for collecting.

A person who cashes a check at a bank in which he is not a depositor is frequently charged an *exchange* of 10¢ and upward, according to the amount of the check.

**Paying by bank draft.**

A **draft** is a check drawn by one bank on another.

As New York City and Chicago banks collect exchange on outside checks, nearly all banks keep deposits there, as

well as in most of the other large commercial centers, to accommodate their depositors and others, who have occasion to remit payment for bills in any part of the country.

Banks usually charge exchange on drafts to cover the cost of keeping funds on deposit at these commercial centers. This fee varies from $\frac{1}{10}\%$ to $\frac{1}{4}\%$ of the face. When the draft is less than \$100, a fixed charge is frequently made, varying from 10¢ to 50¢.

The custom of banks is not to charge depositors for drafts.

In issuing or collecting a draft, the **exchange** is either a fee or a certain per cent of the face of the draft.

New York and Chicago drafts are usually cashed at any point in the United States without exchange. Drafts on other large cities are cashed without exchange in the territory contiguous to those cities.

Brown & Foster, Cleveland, O., buy \$2500 worth of merchandise from John Wanamaker & Co., New York; and \$650 worth of machines from the Wheeler Wilson Co., Bridgeport, Conn. The business method of paying these bills is either by a check or by a bank draft. The draft is made payable to the order of the purchaser, who indorses it in full to the payee. For example:

**Cleveland National Bank**

*Cleveland, Ohio. June 2, 1097, No. 1040*

*Pay to the order of Brown & Foster...... \$2500 $\frac{00}{100}$.*

*Twenty-five Hundred & $\frac{no}{100}$ ~~~~~~~~~~ Dollars.*

*To* The Mercantile National Bank,

*New York, N. Y.*

*A. M. Holmer,*
*Cashier.*

This draft simply means that Brown & Foster purchased at the bank where they kept their deposit a draft (banker's check) for the above amount. The Cleveland National Bank had money on deposit at the Mercantile National Bank, and simply checked on its deposit. Had Brown & Foster not been depositors in the Cleveland National Bank, they would probably have been charged $\frac{1}{10}$% exchange. The draft would then have cost them \$2502.50.

The party who signs a draft is called the **drawer** of a draft.

The party to whose order the draft is drawn is called the **payee** of the draft.

The party who is to pay the money is called the **drawee** of the draft.

Thus, in the draft on p. 318, the cashier of the Cleveland National Bank is the **drawer**; Brown & Foster the **payee**; and The Mercantile National Bank the **drawee**.

## Written Work

**1.** Find the cost of a New York draft for \$550.25 at $\frac{1}{10}$% exchange.

**2.** Mr. Amidon buys \$2500 worth of farm implements at 30% and 10% off, and pays by a Chicago draft at $\frac{1}{6}$% exchange. Find the face of the draft and the exchange.

**3.** Find the cost of sending \$80 from Pittsburg to Chicago by telegraphic money order, the rate being 25¢ for 10 words.

**4.** I sent \$75.80 to Chicago by express money order. How much could I have saved by purchasing a bank draft at 15 cents exchange?

**5.** A draft costs \$1080, including the exchange at $\frac{1}{10}$%. Find the face.

Suggestion. — \$1080 is $100\frac{1}{10}$% of the face.

**6.** Write a draft for \$2600, making one of your local banks the drawer, the First National Bank of Buffalo, N.Y., the payee, and the Niagara Falls Power Co. the drawee. Indorse the draft in full to James Osborne & Co., Syracuse, N.Y.

**7.** Mr. Madison had a note for \$1000 discounted for 60 days at 6%, and with the proceeds bought a Chicago draft at $\frac{1}{10}$% exchange, which he mailed to Mandel Bros., Chicago, to apply on account. Find the face of the draft and the cost of exchange.

**8.** My settlement of an account in New Orleans gives me \$26785.50. After investing \$13750 of this amount in a land deal on which I pay my agent 2% commission, I purchase a New York draft with the balance at $\frac{1}{10}$% exchange. Find the exchange, the commission, and the face of the draft.

**9.** James Anderson & Son, Helena, Montana, order \$790 worth of goods from a Boston firm, and send in payment a New York draft at $\frac{1}{4}$% exchange. Find the cost of the draft.

**10.** A dealer in San Francisco buys \$2000 worth of goods at 30% and 10% off and sells them at an advance of 25% on the cash price. After paying for these goods with a Chicago draft at $\frac{1}{4}$% exchange, find his profit.

### Collecting Bills at a Distance

Bills are collected at a distance in *two* ways:

(1) By a sight commercial draft of a creditor on a debtor.

(2) By a time commercial draft of a creditor on a debtor.

**Bills collected by a sight commercial draft.**

If Letche & Co., grain dealers, Pittsburg, Pa., order from Harris Bros., Chicago, Ill., 1 car load of No. 1 oats, Harris Bros. will ship

the car load of oats to Pittsburg to the order of themselves and draw a sight draft on Letche & Co., payable to the order of some Chicago bank, and deposit it with the bill of lading for collection. The Chicago bank will then mail the draft, together with the bill of lading, to some bank in Pittsburg. The Pittsburg bank will notify Letche & Co. If the car load of oats is accepted by Letche & Co., they will pay the draft and receive the bill of lading which will entitle them to the oats.

This form of draft is commonly known as a **commercial sight draft** and reads as follows:

> *$450 $\frac{00}{100}$.* *Chicago, Ill., June 27, 1906*
>
> *At sight* *Pay to*
>
> *the order of* *1st National Bank, Chicago,*
>
> *Four Hundred Fifty & $\frac{no}{100}$* *Dollars.*
>
> *Value received, and charge to account of*
>
> *To Letche & Co.* 
> *No. 14 Pittsburg, Pa.*
>
> *Harris Bros.,*
> *Chicago, Ill.*

A **bill of lading** is a receipt given by the carrier to the shipper. The goods shipped and their value are described on its face, and on the back of the receipt is stated the contract of shipment.

In case Letche & Co. refused to accept the oats, the draft would be returned to the Chicago bank, which in turn would notify Harris Bros.

Creditors use sight drafts in the collection of debts due or past due.

**Bills collected by a time commercial draft.**

Often a draft reads "30 to 90 days after sight." Such a draft is called a **time commercial draft.**

The method of collecting by a **time commercial draft** is as follows:

| | |
|---|---|
| *$1200 $\frac{00}{000}$* | *Pittsburg, Pa., June 27, 1906* |
| *Sixty days after sight ........................ Pay to* | |
| *the order of ............ Liberty National Bank ............* | |
| *Twelve Hundred and $\frac{no}{100}$ ~~~~~~~~~~~~ Dollars.* | |
| *Value received, and charge to account of* | |
| *To The Acme Buggy Co.,* <br> *No. 132. Cincinnati. O.* | *Jordan & Co., ..........* <br> *Pittsburg, Pa.* |

The Cincinnati bank to which this draft is mailed immediately notifies the Acme Buggy Co. and if the Company agrees to pay the draft, when due, the following is written across the face of it:

"Accepted
Date.............
Acme Buggy Co."

If the Company refuses to accept the draft, the Cincinnati bank will return it to the Liberty National Bank, and Jordan & Co. will be notified that the goods are at Cincinnati at their risk.

The **term of discount** in a draft payable "after sight" begins to run from the *date of acceptance;* in a draft payable after date, from the *date of the draft.*

In collecting by draft, the exchange is always collected on the face, not on the proceeds, of the draft.

## Written Work

**1.** If the draft of Jordan & Co. was accepted and discounted June 29, 1906, find the term of discount.

**2.** Find the proceeds remitted to Jordan & Co. if this draft was discounted June 29 at 7 %, with $\frac{1}{4}$ % exchange for collecting.

**3**. The Jareki Mfg. Co., Sandusky, O., draw at sight on James Howard, Canonsburg, Pa., for \$159.70 through the Erie National Bank, Sandusky, O. Write the draft.

**4**. Freeman Bros., Fargo, N.D., Jan. 2, 1907, sell on 60 days' time to the Standard Milling Co., Minneapolis, Minn., 12000 bu. No. 2 wheat at $92\frac{1}{2}$¢ per bu., delivered at Minneapolis, and draw a time draft which is accepted by the Standard Milling Co. The Merchants National Bank of Minneapolis buys this draft Feb. 1, 1907, at 7% discount. If exchange is $\frac{1}{6}$%, find the proceeds from the sale of the wheat.

**5**. Charles Boyd, Fremont, O., owes Samuel Johnson, Jacksonville Ill., \$600 due June 10, 1907. Mr. Johnson desires the money immediately, and draws on Mr. Boyd March 24, 1907, through the Illinois National Bank, Jacksonville, a time draft due June 10, which Mr. Boyd accepts March 30. The Fremont bank discounts the draft at 6% on the day of acceptance. If the exchange is $\frac{1}{6}$%, how much is remitted to Mr. Johnson?

**6**. T. F. Bowman & Co., Chicago, sell to Speer Bros., Seattle, Wash., \$4000 worth of merchandise. Terms: 60 days net; 5% off 30 days. Write the banker's check given by the Seattle National Bank and indorsed in full by T. F. Bowman & Co. Find the cost of the banker's check at $\frac{1}{4}$% exchange, if paid within 30 days.

**7**. James Brown, Lansing, Mich., sells \$2500 worth of celery on March 1, 1908, to Grimm Bros., Boston, Mass., and draws a draft for 90 days after sight. The draft is accepted March 18, and discounted the same day at 6%. If the cost of collection is $\frac{1}{4}$% exchange, find the proceeds from the sale of the celery.

# STOCKS AND BONDS

## STOCKS

When an individual or a few persons do not wish to furnish all the *capital* or *money* required for a business, or to assume all the responsibility, they may secure a charter from the state government to form a corporation or stock company, and choose a board of directors to transact the business in the name of the firm designated in their charter.

A **corporation** is a company authorized by a charter to transact business as an individual.

The **capital stock** of a company is the amount of stock for which shares are issued. Thus, 1000 shares at $10 each make a capitalization of $10000.

The par value of the shares in different corporations varies from $1 to $100. The persons who form the corporation determine the number and par value of the shares. Observe that the certificate on p. 325 gives the number and value of the shares.

The **par value** of a share of stock is the amount written on the face of the stock certificate.

What is the par value of the stock certificate on p. 325?

The **market value** of a stock is the price at which it is selling.

A stock is selling at a *discount* when purchased for less than its par value, and at a *premium* when selling for more than its par value.

**STOCK CERTIFICATE**

***Incorporated under the Laws of the State of Pennsylvania***

| | |
|---|---|
| No. 25 | 20 Shares |

**Independent Iron Company of Pittsburg**

*This certifies that* James Wood *is the owner of* Twenty *full paid shares of the Capital Stock of One Hundred Dollars each of the Independent Iron Company.*

*Transferable only on the books of the Company by the holder in person or by an attorney upon the surrender of this certificate.*

J. G. Meyer, *President.*
B. S. Smith, *Secretary.*

*Pittsburg June 1, 1906.*

A **stockholder** is one who holds stock in a corporation.

An **assessment** is a sum levied on the par value of each share of stock to defray expenses and losses when the earnings are not sufficient.

A **dividend** is a part of the net profits divided among the stockholders, in proportion to the par value of their stock. These dividends are paid yearly, half-yearly, or quarterly, as the board of directors may determine.

A **stock broker** is a person who buys and sells stocks for another. The charge, called **brokerage**, is usually $\frac{1}{8}\%$ to $\frac{1}{4}\%$ on a par value of a hundred dollars. Most brokers belong to some stock exchange.

**Par Value and Brokerage.**

**1.** Mr. James buys 10 shares of railroad stock, par value \$100 per share, at \$89 per share, brokerage $\frac{1}{8}$%.

1. What is the *par value* of each share?
2. What is the *market value* of each share?
3. Show that each share costs Mr. James \$89.12½.

In the study of this subject the following should always be observed:

1. When the par value of a stock is not stated it is always regarded as \$100.

2. When a stock is quoted at 90, 110, 78, etc., it always means so many % of the par value. A stock quoted above 100 is said to be *above par*, or *at a premium*, and one quoted below 100, *below par*, or *at a discount*.

3. Brokerage is always reckoned on the *par value* and the broker collects brokerage from both the *buyer* and the *seller*. $\frac{1}{8}$% brokerage means \$.12½ on a par value of \$100, or \$.06¼ on a par value of \$50.

To find the cost we add the brokerage to the selling price or purchase price and then multiply that amount by the number of shares bought or sold. Brokerage is not to be computed, unless stated in the problem.

**Written Work**

**1.** Find the cost of 48 shares of railroad stock bought at 95, brokerage $\frac{1}{8}$%.

$$\$95 + \$\tfrac{1}{8} = \$95\tfrac{1}{8} = \text{cost of 1 share.}$$
$$48 \times \$95\tfrac{1}{8} = \$4566, \text{ cost of 48 shares.}$$

Find the cost of:

**2.** 60 shares of stock at 101, brokerage $\frac{1}{8}$%.

**3.** 88 shares of stock at 102, brokerage $\frac{1}{8}$%.

**4.** 104 shares of bank stock at $116\frac{1}{2}$, brokerage $\frac{1}{4}$%.

**5.** 120 shares railroad stock at $94\frac{1}{2}$, brokerage $\frac{1}{8}$%.

**6.** My broker sold for me 128 shares of mining stock at 156, brokerage $\frac{1}{4}\%$. What sum should I receive?

Find the net amount received from the sale of the following, including brokerage at $\frac{1}{8}\%$:

**7.** 125 shares at $69\frac{3}{8}$.

**8.** 145 shares at $142\frac{3}{4}$.

**9.** 500 shares at $132\frac{1}{2}$.

**10.** 1000 shares at $37\frac{3}{8}$.

**11.** I bought 125 shares of Penn. R. R. stock at $123\frac{1}{2}$ and sold it at $129\frac{3}{8}$; brokerage $\frac{1}{8}\%$. Find the net gain.

$$\$129\tfrac{3}{8} - \$\tfrac{1}{8} = \$129.25, \text{ amount realized from each share}$$
$$\$123\tfrac{1}{2} + \$\tfrac{1}{8} = \underline{\$123.625}, \text{ cost of each share}$$
$$\$5.625, \text{ gain on each share}$$
$$125 \times \$5.625 = \$703.13, \text{ net gain}$$

Why do we subtract the brokerage when selling stock?
Why do we add the brokerage when buying stock?

**12.** How many shares of railroad stock at $108\frac{5}{8}$, brokerage $\frac{1}{8}\%$, can be purchased for \$26100?

**13.** How many shares of stock must be sold at $99\frac{1}{4}$, brokerage $\frac{1}{8}\%$, to pay a debt of \$793?

**14.** I receive \$1375 net profits on stock bought at 58 and sold at 72, brokerage $\frac{1}{8}\%$ in each case. Find the number of shares.

**15.** If I realized \$1595 net from the sale of stock, brokerage $\frac{1}{8}\%$, find the number of shares sold at \$40 per share.

**16.** How many shares of stock selling at \$45 per share, brokerage $\frac{1}{8}\%$, can be purchased for \$2000? (Parts of a share are not sold.)

**17.** A note for \$5000, with interest at 6%, was paid 1 year 4 months and 18 days after date and the amount invested in stock at $87\frac{7}{8}$. Find the number of shares purchased.

**Premium and Discount.**

**1.** What is the cost of 1 share of stock, par value \$100, selling at 10 % discount? at 40 % premium? at 30 % discount?

**2.** What is the cost of 60 shares of bank stock, par value \$50, at 5 % premium, brokerage $\frac{1}{4}$ %?

Par value of 1 share = \$50.
Premium of 1 share = $.05 \times \$50 = \$2.50$.
Brokerage on 1 share = $.00\frac{1}{4} \times \$50 = \$.12\frac{1}{2}$.
Cost of 1 share = \$50. + \$2.50 + \$.12½, or \$52.62½.
Cost of 60 shares = $60 \times \$52.62\frac{1}{2} = \$3157.50$.

NOTE. — All premiums are *reckoned on*, and *added to*, the par value; and all discounts are *reckoned on*, and *subtracted from*, the par value.

Find the cost of the following stock, brokerage $\frac{1}{8}$ %:

**3.** 16 shares, par value \$50, at 3 % premium.

**4.** 42 shares, par value \$50, at 10 % discount.

**5.** 100 shares, par value \$25, at 14 % discount.

**6.** 220 shares, par value \$100, at $\frac{1}{2}$ % premium.

**7.** 150 shares, par value \$50, at $\frac{3}{4}$ % discount.

**8.** My broker purchased for me 256 shares of milling stock, par value 100, at $4\frac{1}{2}$ % premium, brokerage $\frac{1}{8}$ %. How much did the stock cost me?

**9.** How much will 120 shares of railroad stock cost at $114\frac{3}{4}$, brokerage $\frac{1}{8}$ %?

**10.** How much is realized from the sale of 480 shares of gas stock, par value \$50, at 2 % discount, brokerage $\frac{1}{8}$ %?

**11.** How many shares of stock can be bought for \$2475, at 3 % premium, brokerage $\frac{1}{8}$ %?

Par value of 1 share = \$ 100
Premium on 1 share = \$ 3.
Brokerage on 1 share = \$ .125
Entire cost of 1 share = \$ 103.125
\$ 24750 ÷ \$ 103.125 = 240, number of shares

Find the number of shares bought for:

**12.** \$5827.50, par value \$50, at 3% discount, brokerage $\frac{1}{8}$%.

**13.** \$11970, at $106\frac{3}{4}$, brokerage $\frac{1}{8}$%.

**14.** \$2165, par value \$50, at 8% premium, brokerage $\frac{1}{4}$%.

**15.** \$19677, at $116\frac{7}{8}$, brokerage $\frac{1}{4}$%.

**16.** \$10025, par value \$50, brokerage $\frac{1}{4}$%.

**Dividends and Investments.**

**1.** What is the income from \$1000 loaned for 1 year at 6%?

**2.** What is the income from \$1000 invested in a manufacturing plant that pays 8% in dividends each year?

**3.** Why is a \$100 share of stock that pays \$12 in dividends each year worth more than \$100?

**4.** Why is a \$100 share of stock that pays only \$2 in dividends each year worth less than \$100?

**5.** $1\frac{3}{4}$% dividend payable quarterly is equivalent to what per cent payable annually?

**6.** \$60 per year is the dividend on \$1200 worth of stock par value. Find the rate of dividend.

**7.** I receive \$120 from a dividend of 6%. What is the par value of my stock?

**8.** Mr. Johnston owns 100 shares of stock in a company whose capital is \$200000. If a dividend of 8% is declared, find the amount of the check that will pay the whole dividend; the amount of Mr. Johnston's share.

Observe: 1. Dividends are always declared on the *par value* of a stock.

2. Incomes are always reckoned on how much a stock *costs*.

**9.** A dividend of 6 % is declared on a stock, par value \$100, purchased at \$120. What % is received on the investment?

$$6\% \text{ of } \$100 = \$6, \text{ income on one share}$$
$$\$120 = \text{cost of one share}$$
$$\$6 \div \$120 = .05, \text{ or } 5\%$$

**10.** A share of stock, par value \$100, is sold at \$250. What is the per cent of income if an annual dividend of 10% is declared?

Find the rate of income when:

**11.** \$150 is paid for 9% stock, par value \$100.

**12.** $\$133\frac{1}{3}$ is paid for 6% stock, par value \$100.

**13.** \$75 is paid for 8% stock, par value \$50.

**14.** \$80 is paid for 5% stock, par value \$100.

**15.** \$50 is paid for 4% stock, par value \$100.

**16.** Would a stock yielding 6% have to be purchased for more or less than par value to yield 8% on the investment? Explain why.

**17.** Explain why a stock, par value \$100, dividend 12%, yields only 6% on the money invested when purchased at \$200.

**18.** A man buys 120 shares of stock at $137\frac{1}{2}$, and receives a 6% dividend. He sells it at $141\frac{5}{8}$. Find his net profits on the investment after paying brokerage at $\frac{1}{8}$% each for buying and selling.

**19.** Which yields the better income and how much, 6% stock at \$120 or 4% stock at \$85?

**20.** The Amidon Asbestos Co. is capitalized at \$80000. The gross receipts for a year are \$170000. The expenses, material, and repairs amount to \$146000. If \$14000 is put in the surplus fund, what dividend can be declared from the balance?

**21.** A gas company declares a dividend of 8% which amounts to \$64000. What is its capitalization?

**22.** A bank is capitalized at \$100000, and pays 8% dividend. How much is the dividend on 36 shares?

**23.** How much must be invested in 6% stock, at 108, brokerage $\frac{1}{8}$%, to yield an annual income of \$366?

$\$366 \div \$.06 = 6100$; \$6100, par value
$1.08\frac{1}{8} \times \$6100 = \$6595.63$, sum invested

Since \$1 of stock yields \$.06 income, the par value to yield \$366 must be as many dollars as \$.06 is contained times in \$366, or \$6100. Adding brokerage, the sum invested must be $1.08\frac{1}{8}$ times \$6100, or \$6595.63.

**24.** If a stock paying $3\frac{1}{2}$% semiannual dividend is quoted at 120, how much must be invested in it to produce an annual income of \$1400, brokerage $\frac{1}{8}$%?

What sum must be invested, at $\frac{1}{8}$% brokerage, in:

**25.** $3\frac{1}{2}$% stock, at 104, to yield an annual income of \$245?

**26.** 5% stock, at 109, to yield an annual income of \$1675?

**27.** $4\frac{1}{2}$% stock, at $116\frac{1}{2}$, to yield an annual income of \$364.50?

## BONDS

When corporations need large sums of money to carry on their business, instead of issuing more stock, they frequently issue a series of bonds payable at some future date with interest.

**Bonds** are written obligations, under seal, by which corporations or governments bind themselves to pay specified sums, at a fixed rate of interest, at or before the time specified in the bonds.

The bonds of a **business corporation** are secured by a mortgage on its property. This mortgage authorizes the

sale of the property in case the conditions of the bonds are not fulfilled. The bonds of governments are without mortgage.

Bonds and stocks are at a **premium** when they sell for more than the face, or par value; at a **discount** when they sell for less than the face, or par value.

**Coupon**

| $30. | ACME GLASS COMPANY | $30. |
|---|---|---|

will pay to Bearer at the

**Colonial Trust Co. of Pittsburg, Pa.**

*on the first day of June, A.D., 1907,*

Thirty Dollars

*in United States Gold Coin, being six months' Interest on Bond No. 501*

James Grine, *Treasurer.*

A **coupon bond** is a bond with interest coupons attached. These coupons are detached when the interest is due, and the amount may be collected personally or through a bank. Coupon bonds are payable to the bearer.

A **registered bond** is a bond registered on the books of the corporation issuing it. The interest when due is sent by check to the owner. Registered bonds are payable to the owner or to his assignee.

The name of a bond often indicates its *rate of interest* and the time when the bond is *due*. Thus, "U. S. 4's, 1907," are United States 4% bonds, due in 1907; "Western Union 7's, 1920," are Western Union bonds, due in 1920, and

bearing 7% interest; "U. S. Steel 7's," are United States Steel bonds, bearing 7% interest.

**Stock or bond quotations** are the prices at which stocks and bonds are selling. Thus, B. & O. 4's quoted at 96 means that Baltimore & Ohio bonds are selling at 96% of their par value. The *buyer* pays $96 + \frac{1}{8}$ brokerage $= 96\frac{1}{8}$, or \$96.125, per share. The *seller* receives $96 - \frac{1}{8}$ brokerage $= 95\frac{7}{8}$, or \$95.875, per share.

### Comparative Study

**Stockholders** are the *owners* of corporate property; **bondholders** are *creditors* who have loaned money to the corporation or government. **Bonds** bear *interest* at a *fixed rate* and mature at a time specified in the bond; **stocks** continue while the corporation exists and pay *dividends* according to the earnings of the company.

**Commission** or **brokerage** in *commission* is reckoned on the *actual amount* of goods sold, or the amount of money involved in a transaction; **brokerage** in *stocks and bonds* is always reckoned on the *par value* of the stocks or bonds bought or sold.

### Problems in Stocks and Bonds

1. Find the cost of 15 \$1000 United States bonds at $103\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

2. How many Mound City water bonds, at $4\frac{1}{2}\%$ premium, can be purchased for \$5225, brokerage not included?

3. Find the par value of 4% government bonds that yield \$1200 annually.

4. Find the face of a 3% bond, when an interest coupon brings \$60 annually.

5. A man desires to invest in 4% bonds sufficient to yield an income of \$1200 per year. If the bonds are selling at 118, find the amount that must be invested, including brokerage.

6. A \$2000 4% bond, interest payable semiannually, is sold after one interest period at 10% premium. Find the per cent of gain on the investment if the bond was bought at par.

**7.** \$5 is the dividend on a \$100 stock bought at \$80. Find the rate of income on the investment.

**8.** Copper stock, par value \$10 and paying 24% dividend, is bought at \$40. No allowance being made for brokerage, find the rate of income from the investment.

**9.** I purchase 100 shares of stock, par value \$50, at \$65 and, after receiving 2 dividends of 2% each, sell at \$78. Find my gain, brokerage $\frac{1}{8}$% in each transaction.

**10.** Compute the rate of income from 5% stock bought at 80; at 90; at 100; at 120; at 125.

**11.** My income from 6% bonds is \$240 per year. How much have I invested?

**12.** I bought 24 shares of mining stock at 89 and, after keeping it 3 years, sold it at 39. As no dividends were paid, find my loss, money being worth 6% simple interest.

**13.** A \$1000 5% bond, bought at par, after paying 6 annual dividends, is sold for \$.60 on the dollar. Find the loss, money being worth 5% simple interest.

**14.** A certain stock bought in 1904 at 40¢ per share was sold in 1906 at \$2.40 per share. Find the gain per cent on the investment.

**15.** A \$1000 5% bond due in 10 years was purchased for \$1100. Find the average rate of interest on the investment, if the bond is held to maturity.

The total income on the bond for 10 years at 5% = \$500. Loss by redemption of bond \$1100 − \$1000 = \$100. Total income in 10 yr. = \$500 − \$100 = \$400. Average annual income = $\frac{1}{10}$ of \$400 = \$40. Average annual rate = \$40 ÷ \$1100 = $3\frac{7}{11}$%.

**16.** A \$2000 4% bond due in 5 yr. was bought for \$1900. Find average rate of interest, if bond is held to maturity.

NOTE. The purchaser gains \$100 when the bond is redeemed.

## TEST PROBLEMS IN PERCENTAGE

**1.** Mr. Byers's farm is valued at \$18000. He pays $4\frac{2}{10}$ mills taxes on an 80% valuation of it. Find his tax.

**2.** An agent buys 30 tons of fertilizer, at \$1.50 per hundred, 20%, 10% off. Terms: 30 days net, or 2% for cash. If he pays cash, find the cost.

**3.** A western farmer sells 10000 bu. of wheat, $\frac{1}{8}$¢ per bushel brokerage, at $89\frac{1}{2}$¢. After deducting freight and drayage of \$$67\frac{1}{2}$, find the net amount of the sale.

**4.** An Iowa farmer buys cattle for \$1500, and sells them for \$2350. If the grazing and feeding are 20% of the cost, find the per cent of profit on the sale.

**5.** A merchant has a note against Mr. Johnston for \$500, bearing 6% interest, dated June 1, 1906, due in one year. If he discounts the note at his bank March 1, 1907, at 7%, find the proceeds.

**6.** A farmer has the following annual insurance on his property: house valued at \$2000, insured at $\frac{3}{4}$%; barn valued at \$2500, insured at $\frac{9}{10}$%; grain valued at \$1000, insured at $\frac{3}{4}$%; live stock \$1200, insured at $\frac{4}{5}$%. If twice the annual premium covers the cost of the insurance for 3 years, find the yearly cost if his property was insured for 3 years.

**7.** Mr. Ames, who keeps a general store, sold goods in one year amounting to \$24000. If he has an average of 20% profit on the cost of the goods, find his profits for the year.

**8.** A manufacturer, owing to a depression in business, offers goods at $12\frac{1}{2}\%$ discount, but finally sells at a further discount of $8\%$. Find the entire per cent of discount.

**9.** What per cent is gained by buying stocks at $15\%$ discount and selling at $5\%$ premium, brokerage $\frac{1}{8}\%$?

**10.** The tax rate in a certain city is 17 mills on the dollar on a valuation of \$66390. Find the tax on a property valued at \$12500.

**11.** A salesman who received a salary of \$2400 and \$1500 expenses, sold \$75000 worth of goods. In addition he received $2\%$ on all sales over \$60000. What per cent of the selling price of the goods did it cost the firm to sell them?

**12.** A man buys through his broker 10 shares of railroad stock (par value \$100) at \$125 per share. After receiving 5 semiannual dividends of $3\%$ each, he sells the stock at $\$131\frac{1}{2}$. Find the rate per cent of gain on the investment.

**13.** By selling a piano at $40\%$ above cost, a profit of \$150 is realized. For how much must the piano be sold to realize a profit of $56\%$?

**14.** A collector is given a bill of \$1500 to collect at $5\%$. He succeeds in collecting 90 cents on the dollar. Find how much is due his client and how much is the collector's commission.

**15.** A house and lot cost \$6000. The insurance averages \$14, taxes \$50, and repairs \$56 annually. For how much must the house rent per year to realize $6\%$ net on the investment?

**16.** A house rents for \$40 per month, and it costs the owner on an average \$125 per year for insurance, taxes, and repairs. If the property yields him $5\%$ net on the investment, find the cost of the house.

# RATIO AND PROPORTION

## RATIO

**1.** The quotient of $30 \div 10$ is 3. Compare 30 with 3 in such a way as to show how many times 3 is contained in 30. What, then, is the relation of 30 to 3?

**Ratio** is the relation of two similar numbers as expressed by the quotient of the first divided by the second.

**2.** What is the ratio of 10 to 8? of 12 to 4? of 3 to 6? of 2 yd. to 8 yd.? of \$12 to \$3.

Since the division of two similar numbers gives an *abstract quotient*, all ratios are *abstract*.

The **sign** of ratio is a colon : placed between the numbers. Thus, the ratio of 12 to 3 is written 12 : 3. It is read, "the ratio of 12 to 3." It may be written also $12 \div 3$, or $\frac{12}{3}$.

The **terms** of a ratio are the numbers compared. The first is the **antecedent**; the second the **consequent**.

$$12 : 6 = \frac{\text{antecedent}}{\text{consequent}} = 12 \div 6 = \frac{\text{dividend}}{\text{divisor}} = \frac{12}{6} = \frac{\text{numerator}}{\text{denominator}}$$

Since the antecedent of a ratio may be regarded as the numerator, and the consequent as the denominator of a fraction, *both terms of a ratio may be multiplied or divided by the same number without changing the value of the ratio.*

Find the ratio of :

| | | | |
|---|---|---|---|
| **3.** 10 to 5 | **7.** 18 to 9 | **11.** 40 to 10 | **15.** 50 to 25 |
| **4.** 5 to 15 | **8.** 27 to 9 | **12.** 8 to 24 | **16.** 24 to 8 |
| **5.** 8 to 2 | **9.** 35 to 5 | **13.** 2 to $\frac{1}{2}$ | **17.** 4 to $\frac{1}{4}$ |
| **6.** $\frac{1}{2}$ to $\frac{1}{4}$ | **10.** $\frac{2}{3}$ to $\frac{1}{3}$ | **14.** $\frac{5}{8}$ to $\frac{1}{8}$ | **18.** $\frac{3}{4}$ to $\frac{1}{4}$ |

### Written Work

Find the value of the following ratios:

| | | |
|---|---|---|
| **1.** 125 : 25 | **5.** $\frac{2}{3}$ to $\frac{1}{12}$ | **9.** \$225 to \$2.25 |
| **2.** 6.25 : 25 | **6.** 6.4 to 16 | **10.** 3 yd. to 3 ft. |
| **3.** $\frac{3}{4}$ to $\frac{1}{4}$ | **7.** $37\frac{1}{2}$ to 200 | **11.** 75% to $12\frac{1}{2}$% |
| **4.** $\frac{1}{8}$ to $1\frac{1}{2}$ | **8.** $62\frac{1}{2}$ to 500 | **12.** 1 mi. to 1 rd. |

## SIMPLE PROPORTION

**Proportion** is an equality of ratios; thus, $12:6=8:4$ or $\frac{12}{6}=\frac{8}{4}$ is a proportion.

Proportion is generally indicated by the equality sign or by a double colon : : between the ratios. Thus, 12 : 6 as 8 : 4, is written, $12:6=8:4$, or $12:6::8:4$.

A proportion may be read in two ways; thus, $12:6=8:4$ is read, "The ratio of 12 to 6 is equal to the ratio of 8 to 4," or, "12 is to 6 as 8 is to 4."

The **extremes** are the *first* and the *fourth* terms of a proportion; the **means** are the *second* and the *third terms*.

In $15:5=12:4$, the extremes are 15 and 4; the means, 5 and 12.

Find the product of the means; then the product of the extremes:

| | | |
|---|---|---|
| **1.** $8:4=10:5$ | **3.** $24:4=36:6$ | **5.** $\frac{2}{3}=\frac{6}{9}$ |
| **2.** $15:3=30:6$ | **4.** $\frac{3}{8}=\frac{6}{16}$ | **6.** $\frac{1}{5}=\frac{3}{15}$ |

Observe how the product of the extremes in each proportion compares with the product of the means.

*In every proportion the product of the means is equal to the product of the extremes.*

## Written Work

Find the value of $x$, the unknown term:

**1.** $36 : 6 = 24 : x$

Then, 36 times $x$, or $36\,x = 144$, and once $x$, or $x = 4$.

**2.** $15 : 25 = x : 40$

Then, $25\,x = 15 \times 40$, or 600, and $x = 24$.

**3.** $60 : 15 = 75 : x$

**4.** $75 : x = 90 : 18$

**5.** $40 : x = 72 : 18$

**6.** $x : 30 = 8 : 48$

**7.** $x : 45 = 7 : 63$

**8.** $\frac{3}{5} : \frac{2}{3} = 9 : x$

**9.** $\frac{5}{8} : x = 25 : 8$

**10.** $7.5 : 1.5 = 2.5 : x$

**11.** $6.25 : 2.5 = x : 1$

**12.** $60 : 150 = 36 : x$

**13.** If 8 sheep cost \$48, how much will 20 sheep cost?

8 sheep cost \$48
20 sheep cost \$$x$

Since *ratio* is the relation of two similar numbers, 8 sheep and 20 sheep form one ratio, and \$48 and \$$x$, the other ratio.

Write as the second ratio \$48 : \$$x$. Since 20 sheep cost more than 8 sheep, \$$x$ represents a larger sum than \$48. Therefore, as the larger number is the consequent of the second ratio, the larger number must be made the consequent of the first ratio. The proportion, therefore, is,

$$8 \text{ sheep} : 20 \text{ sheep} = \$48 : \$x$$
$$8\,x = \$960$$
$$x = \$120$$

**14.** It is estimated that 25 men can build a bridge in 18 days. How long at the same rate will it take 15 men to build it?

**15.** How much will 30 bushels of potatoes cost, if 70 bushels cost \$42?

**16.** It is estimated that 90 men are necessary to grade a certain street in 45 days. If only 81 men are hired to do the work, how long will it take them?

**17.** The ratio is $\frac{3}{2}$. The first term is $\frac{1}{2}$ of $(6 \times 4)$. What is the second term?

**18.** A bankrupt's debts are \$32000, and his assets \$10000. Counting nothing for court costs, how much will be paid on a claim of \$6150?

**19.** If \$2.25 is paid to clean $35\frac{1}{2}$ square yards of paper, how much at the same rate will it cost to clean $65\frac{3}{4}$ square yards?

**20.** A bakery sells 5¢ loaves weighing 6 oz. when flour is \$4. What size loaves should they sell at 5¢ when flour is \$6 per barrel?

**21.** A map is drawn on a scale of 100 miles to $\frac{3}{4}$ of an inch. What distance is represented on the map by $\frac{5}{16}$ of an inch?

**22.** If the interest on \$500 for 6 months is \$15, how much is the interest on the same sum for 1 year 4 months?

**23.** If 6.5 tons of coal cost \$55.90, how much will 9.25 tons cost at the same rate?

**24.** A estimates that he can do a piece of work in 20 days, working 8 hours per day. How long will it take him to do $\frac{1}{4}$ of it, working 10 hours per day?

**25.** Sound travels 1120 feet per second. How long will it take the sound of a cannon to travel 8 miles?

**26.** In a stamp canceling machine 1000 letters were canceled in one minute and 20 seconds. If the machine was in operation for 5 minutes and 10 seconds, how many letters at the same rate were canceled?

**27.** A monument casts a shadow 150 feet long. At the same time a post 3 feet in height casts a shadow 2 feet and 6 inches long. Find the height of the monument.

**28.** A machine for making pressed brick turns out 7500 brick in 6 days. How large an order for pressed brick can be filled in 25 days?

**29.** An automobile passed 5 mile-posts in 8 minutes 10 seconds. How many miles per hour was it moving?

**30.** It is estimated that 24 men working 18 days can repair a certain street. The contract calls for the work to be completed in 8 days. How many extra men must be employed?

**31.** It is estimated that 60 men can dig a sewer on Main Street in 24 days. The contract time is 40 days. How many men may be discharged and yet have the work completed within the contract time?

**32.** A's property is assessed at \$2750, on which \$37.90 taxes are paid each year. How much tax should B pay on his property assessed at \$4375?

**33.** A train of 30 cars of ore contains 1200 tons. How many cars must be added so that the train may carry 2700 tons?

**34.** A city's assessed valuation is \$5675000. There must be raised in taxes on this valuation \$85125. How much is Mr. Templeton's tax on a property assessed at \$15750?

## PARTITIVE PROPORTION AND PARTNERSHIP

**Partitive proportion** is the process of separating a number into parts proportional to two or more numbers.

### Written Work

**1.** Separate 180 into parts proportional to 1, 2, and 3.

Since the parts are in the ratio of 1, 2, and 3, then $1+2+3$, or 6 parts $= 180$.

The 1st number $= 180 \div 6$, or 30
The 2d number $= 2 \times 30$, or 60
The 3d number $= 3 \times 30$, or 90

**Test:** $30 + 60 + 90 = 180$; $30 : 60 : 90 = 1 : 2 : 3$.

**2.** A man and two boys earn \$162 and agree to divide it as follows: 3 parts to the man, 2 parts to the first boy, and 1 part to the second boy. How much should each receive?

**3.** The receipts of a street railway in one month are \$15600, and the expenses are to the profits as 1 to 2. Find the expenses and the net savings.

**4.** Four men own a gold mine valued at \$805000. The parts owned by each are in the ratio of 6, $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{9}{16}$. Find each man's share of the mine.

**5.** The cost of shipping a train load of 2000 tons of iron ore from Duluth, Minn., to Bessemer, Pa., is \$2800. If the lake freight is to the railroad freight as 50 to 90, find each one's share of the freight charges.

**6.** Two railroads, valued at \$6900000 and \$23000000, share charges of \$299000 for freight carried over both roads in proportion to the valuation of each road. Find the earnings apportioned to each road.

**Partnership** is the associating of two or more persons who agree to combine their money, labor, goods, skill, or "good will" in some enterprise, and to share the profits or losses of the business in proportion to the interest each partner owns.

The partnership is frequently called a **firm**, or a **house**, and derives its name from the persons that compose it; as, "Brown & Hamilton."

The **capital** of a partnership is the sum of the investments of the partners. This capital may be money or anything that has a money value, as skill, good will, experience, labor, etc.

*Gains and losses in a common partnership are usually apportioned in proportion to the amount of capital each partner invests and the length of time such capital is invested; but in case any partner cannot pay his proportionate share of the loss, the remaining partners are liable for the whole loss.*

## Written Work

**1.** A and B engage in business; A furnishes \$800 and B, \$1200; they gain \$500. What is each man's share?

$\$800 + \$1200 = \$2000$, entire capital

$\frac{\$800}{\$2000} = \frac{2}{5}$, A's share of the capital

$\frac{\$1200}{\$2000} = \frac{3}{5}$, B's share of the capital

$\frac{2}{5}$ of $\$500 = \$200$, A's share of the gain

$\frac{3}{5}$ of $\$500 = \$300$, B's share of the gain

Or, $\$2000 : \$800 = \$500 : \$200$

$\$2000 : \$1200 = \$500 : \$300$

The ratio of the whole capital to each partner's investment is equal to the ratio of whole gain or loss to each partner's share of the gain or loss.

**2.** A, B, and C engaged in manufacturing iron. A invested \$42000, B \$96000, and C his skill, valued at \$72000. Their profits the first year were \$12600. How much was each man's gain?

**3.** M, N, and R formed a partnership; M furnished $\frac{1}{3}$ of the capital, N $\frac{2}{5}$, and R the remainder. They gained \$7560. What was each man's share of the gain?

**4.** E, F, and G engaged in merchandizing with a capital stock of \$28000. E furnished \$7000, F \$6000, and G the remainder. They gained $14\frac{2}{7}\%$ on the investment. What was each man's share of the gain?

**5.** The assets of a firm that failed in business were \$3750; their liabilities \$22000. How much will two creditors, to whom they owe \$7800 and \$5400 respectively, receive?

**6.** A storeroom belonging to Smith, Jones, & Brown was entirely destroyed by fire. They received \$9675 insurance. What was each man's share, if Smith owned $\frac{1}{3}$, Jones $\frac{4}{9}$, and Brown the remainder of the stock?

**7.** A and B formed a partnership January 1, and each invested \$2500; May 1 A added \$500, and B withdrew \$500. At the end of a year their gain was \$1800. How much should each one receive?

| | |
|---|---|
| A's capital, \$2500 for 4 mo. | = \$10000 for 1 mo. |
| A's capital, \$3000 for 8 mo. | = \$24000 for 1 mo. |
| A's total capital | = \$34000 for 1 mo. |
| B's capital, \$2500 for 4 mo. | = \$10000 for 1 mo. |
| B's capital, \$2000 for 8 mo. | = \$16000 for 1 mo. |
| B's total capital | = \$26000 for 1 mo. |
| Total capital of both | = \$60000 for 1 mo. |

$$\text{A's gain} = \frac{34000}{60000}, \text{ or } \tfrac{17}{30}, \text{ of } \$1800 = \$1020.$$

$$\text{B's gain} = \frac{26000}{60000}, \text{ or } \tfrac{13}{30}, \text{ of } \$1800 = \$780.$$

**Test**: \$1020 + \$780 = \$1800, total gain.

**8.** M and N formed a partnership for 2 years. M put in \$6400; N put in \$3600 and at the end of 6 months added \$1400. Their settlement at the end of 2 years showed \$7956 profits. How should it be divided?

**9.** R and S began business as partners April 1, 1904, each investing \$5000. On July 1, 1904, R added \$3000 and S, \$2000. They dissolved partnership January 1, 1905, sharing a profit of \$3150. Find each one's share.

**10.** A, B, and C formed a partnership for 3 years. A put in \$10000, B \$8000, and C \$6000. A withdrew \$2000 at the end of 18 months. They dissolved partnership at the end of 2 years with a loss of \$4750. As nothing could be collected from C, what proportionate share of the loss should A and B pay?

## PROBLEMS FOR ORAL AND WRITTEN ANALYSIS

**1.** Two properties are valued at \$1000; $\frac{1}{2}$ of the value of the first equals $\frac{3}{4}$ of the value of the second. Find the value of each.

**2.** At a certain election 1080 votes were cast for A and B; $\frac{3}{4}$ of the votes cast for A equaled $\frac{3}{5}$ of those cast for B. How many votes were cast for each candidate?

SOLUTION. — $\frac{3}{4}$ of A's vote = $\frac{3}{5}$ of B's vote.
$\frac{1}{4}$ of A's vote = $\frac{1}{3}$ of $\frac{3}{5}$ of B's vote, or $\frac{1}{5}$ of B's vote.
$\frac{4}{4}$ or A's vote = $4 \times \frac{1}{5}$ of B's vote, or $\frac{4}{5}$ of B's vote.

$\frac{5}{5}$ of B's vote = B's vote.
$\frac{4}{5}$ of B's vote = A's vote.

$\frac{9}{5}$ of B's vote = vote of both, or 1080 votes.
B's vote = 600.
A's vote = 480.

**3.** A real estate dealer paid \$7200 for two city lots; $\frac{3}{8}$ of the cost of the first lot equaled $\frac{3}{10}$ of the cost of the second. How much did each cost?

**4.** A mill and machinery cost \$27000; $\frac{2}{3}$ of the cost of the mill equaled $\frac{5}{6}$ of the cost of the machinery. How much did the machinery cost?

**5.** What per cent of a day are 12 hours? 6 hours? 36 hours?

**6.** $\frac{2}{3}$ of Frank's money equals $\frac{4}{5}$ of Henry's, and Frank has \$3 more than Henry. How much has each?

**7.** Walter and Philip bought sleds; $\frac{3}{4}$ of the cost of Walter's sled equaled $\frac{3}{5}$ of the cost of Philip's; both sleds cost $2.70. How much did each cost?

**8.** A pair of shoes that cost a dealer $2.50 were sold for $3.50. What was his gain per cent?

**9.** An estate was so divided between two sons that the share of the elder was to that of the younger as $\frac{1}{2}$ to $\frac{1}{3}$. If the elder son received $1000 more than the younger, what was the value of the estate?

**10.** Brown and Long were partners in business; Brown furnished $\frac{7}{8}$ as much capital as Long, and their profits for the first year were $2250, which was divided in the ratio of the capital invested. What was the share of each?

**11.** In a partnership A invested $\frac{5}{8}$ as much as B, and C invested $\frac{3}{8}$ as much as B; they shared a loss of $2000. How much should C pay?

**12.** A piano sold for $360, which was at a loss of 20 %. What was the cost?

**13.** Moore, Silvens, and Rogers were partners in business and made a profit of $4500. Moore owned $\frac{4}{15}$ of the stock, Silvens $\frac{2}{5}$, and Rogers $\frac{1}{3}$. What was each partner's share of the total profit?

**14.** A clerk's expenses are $30 a month, which is $66\frac{2}{3}$ % of his salary. How much is his salary?

**15.** A stone cutter received $4 a day for his labor and paid $6 a week for his board. At the end of 16 weeks he had saved $212. How many days did he work?

**16.** If in an investment $\frac{1}{3}$ of A's capital equaled $\frac{1}{4}$ of B's, and A received $900 for managing the business, how should profits of $5100, including cost of management, be divided?

**17.** A wagon was sold for \$42, which was $12\frac{1}{2}$ % less than the price paid. What was the cost of the wagon?

**18.** A carpenter's wages were \$3.50 a day, and he paid 50 cents a day for his board. If in 40 days he saved \$99, how many days was he idle?

**19.** A real estate dealer bought some lots at \$150 each, and twice as many at \$175 each. He sold them at \$200 each, thereby gaining \$300. How many did he buy in all?

**20.** A manufacturer pays boys \$1, women \$1.25, and men \$1.75 a day, and his weekly pay roll is \$348. He employs three times as many boys as men, and twice as many women as men. How many persons does he employ?

**21.** A farmer paid \$7200 for two farms of equal size, paying \$50 an acre for one and \$40 an acre for another. How many acres were there in both farms?

**22.** A dealer bought 20 dozen glasses at 50¢ per dozen. At what price per dozen must he sell them to make a profit of 20 % on the transaction?

**23.** It is estimated that 80 men can make an excavation for a public building in 30 days. After working 12 days, $\frac{1}{4}$ of the men were discharged. In how many days could the remainder finish the work?

**24.** A father desires that the amount of \$5000 for 6 years at 6 % shall be divided between his son and daughter in the ratio of 8 to 9. Find the share of each.

**25.** An architect gets a commission of 5 % for drawing plans and superintending the construction of a building costing \$25000. How much is his commission?

**26.** The interest on $\frac{3}{5}$ of Robert's money and $\frac{3}{4}$ of Samuel's money for $4\frac{1}{2}$ years, at 6%, is \$81 and \$121.50 respectively. How much money has each?

**27.** The sum of E's and F's money being on interest for five years, at 5 %, amounts to \$3000. How much money has each, if E's is $\frac{2}{3}$ of F's?

**28.** The amount of a certain principal for 4 years at a certain per cent is \$620; and for 7 years, \$710. Find the principal and the rate per cent.

**29.** 30 is 6 % of what number? 25 is $\frac{1}{2}$ % of what number?

**30.** It is estimated that 15 men can build an embankment of earth in 20 days. If 5 additional men are employed, in how many days can it be built?

**31.** Harley spent $\frac{1}{2}$ of his money and \$5 more for a suit of clothes, and had \$11 remaining. How much money had he at first?

**32.** After paying 25 % of his debts, a merchant found that \$240 would pay the remainder. How much did he owe at first?

**33.** How shall I mark goods that cost \$750, so that I can deduct 10 % from the marked price, and yet make 20 % on the cost?

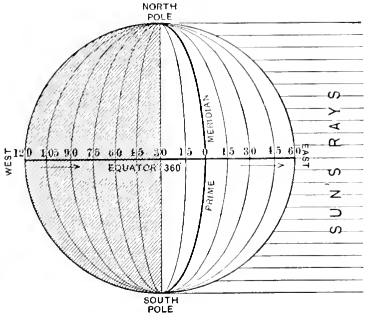
**34.** A speculator bought wheat at 80¢ per bushel and sold it at 90¢ per bushel. How many bushels did he buy if his gain was \$2000?

**35.** An agent remitted \$95 as the proceeds of an account he collected. How much did he retain if his rate of commission was 5 %?

**36.** A bankrupt, who owed \$12000, paid 60¢ on the dollar. Find A's claim, and his loss if he received \$600.

**37.** 8 men take equal shares in an oil lease, agreeing to give the owner of the land $\frac{1}{8}$ royalty on all oil produced. How much greater interest in the oil has the owner than any of the other men?

# LONGITUDE AND TIME




**Meridians** are imaginary lines passing north and south from one pole of the earth to the other.

The **equator** is an imaginary line passing around the earth midway between the poles.

These imaginary lines aid in locating places on the earth and in determining differences in time.

Observe that the equator is a circumference of a circle; therefore distances along it are measured in degrees.

The **prime meridian** is a meridian from which time and place, east and west, are reckoned.

The meridian passing through the Royal Observatory at Greenwich, England, is the prime meridian in common use.

**Longitude** is the distance east or west of this prime meridian measured in degrees.

Places *east* of this prime meridian have *east longitude;* places *west* of this prime meridian have *west longitude.*

From the time the sun's rays are vertical over any meridian until they are vertical again it is 24 hours. Therefore, any

point passes through 360° in one rotation of the earth on its axis.

Since 360° of longitude pass under the sun's vertical rays during 24 hours, how many degrees pass during 12 hours? 1 hour?

Since $\frac{1}{24}$ of 360° or 15° pass under the sun's rays in 1 hour, then 1 hour of time corresponds to 15° of longitude.

Since 15° of longitude correspond to 1 hour of time, $\frac{1}{60}$ of 15° or $\frac{1}{4}$°, or 15′ of longitude, correspond to 1 minute of time, and 15″ of longitude to 1 second of time.

Table of relation between *longitude* and *time:*

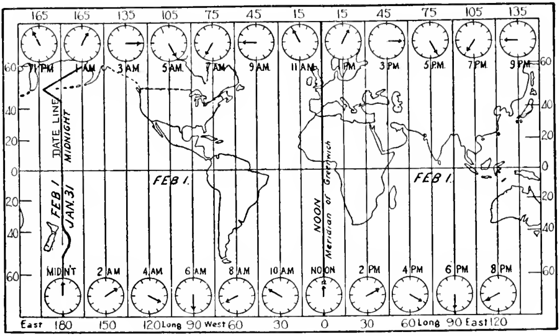
| | | |
|---|---|---|
| 360° of longitude correspond | to | 24 hours of time |
| 15° of longitude correspond | to | 1 hour of time |
| 15′ of longitude correspond | to | 1 minute of time |
| 15″ of longitude correspond | to | 1 second of time |
| 1° of longitude corresponds | to | 4 minutes of time |
| 1′ of longitude corresponds | to | 4 seconds of time |

When the sun's rays are vertical on the 90th meridian, all places on that meridian have noon.

The rotation of the earth from west to east makes the sun appear to move from east to west. The Mercator's map on p. 351 shows that when it is noon at Greenwich, it is *before* noon or **earlier** at all places **west**, because the sun's rays are not yet vertical on any meridian west of the prime meridian. It is *after* noon or **later** at all places **east**, because the sun's rays have already been vertical on all meridians east of the prime meridian.

Examine the map. What time is it on the meridian of Greenwich? 45° east of Greenwich? 45° west of Greenwich? In traveling from London to New York would a watch be set *forward* or *backward?* about how much? About how much change in time must be made in traveling from Cal-

cutta westward to San Francisco? from Honolulu eastward to Cape Town?




MAP SHOWING NOON, FEBRUARY 1, AT GREENWICH

What is the difference in degrees between a place 30° east longitude and a place 45° east longitude? What is the difference in time and which has the earlier time?

### Table of Longitude of Some Important Places

| | | | |
|---|---|---|---|
| London | 0° 5′ 48″ W. | Cape Town | 18° 28′ 45″ E. |
| New York | 74° 0′ 3″ W. | Honolulu | 157° 50′ 36″ W. |
| Pittsburg | 80° 2′ 0″ W. | Tokyo | 139° 44′ 30″ E. |
| Washington | 77° 3′ 00″ W. | Manila | 120° 58′ 6″ E. |
| Chicago | 87° 36′ 42″ W. | Canton | 113° 16′ 30″ E. |
| San Francisco | 122° 25′ 42″ W. | Berlin | 13° 23′ 44″ E. |
| Boston | 71° 3′ 50″ W. | Rome | 12° 27′ 14″ E. |
| Denver | 104° 58′ 0″ W. | Paris | 2° 20′ 15″ E. |

Longitudes are given to the nearest seconds.

## Written Work

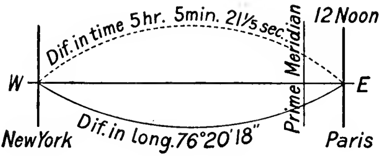
**1.** When it is noon, solar time, at Paris, what is the solar time at New York?

$$\begin{array}{rrr}
2^\circ & 20' & 15''\text{ E.} \\
74^\circ & 0' & 03''\text{ W.} \\ \hline
15)\overline{76^\circ} & 20' & 18'' \\ \hline
5^\circ & 5' & 21\frac{1}{5}''
\end{array}$$

5 hr. 5 min. $21\frac{1}{5}$ sec.

Since the earth rotates 15° in 1 hr., 15′ in 1 min., and 15″ in 1 sec., the difference in time is as many hours, minutes, and seconds as there are degrees, minutes, and seconds in $\frac{1}{15}$ of the difference in longitude.

The difference in time is 5 hours, 5 minutes, $21\frac{1}{5}$ seconds. Since New York is *west* of Paris, the time in New York is *earlier;* that is, when it is noon at Paris, it is 6 o'clock, 54 min., and $38\frac{4}{5}$ sec. A.M. at New York.




What is the difference in longitude between the two places? the difference in time? In going west from Paris to New York would a traveler set his watch forward or backward? how much?

NOTE. — Study this diagram and make a similar one for each problem.

Find difference in degrees, difference in time, and which place has earlier time:

| PLACES | PLACES |
|---|---|
| **2.** 60° W. and 45° W. | **7.** 120° W. and 30° E. |
| **3.** 120° W. and 75° W. | **8.** 90° W. and 30° E. |
| **4.** 15° W. and 45° E. | **9.** 135° E. and 30° E. |
| **5.** 30° E. and 60° E. | **10.** 45° W. and 60° E. |
| **6.** 75° W. and 30° E. | **11.** 45° E. and 15° E. |

When the sun's rays are vertical on the meridian of Washington, find the solar time in the following places:

**12.** Denver **15.** Paris **18.** Berlin

**13.** Chicago **16.** Rome **19.** New York

**14.** San Francisco **17.** Honolulu **20.** Pittsburg

**21.** When it is midnight (solar time) on the last day of the year in Boston, how much of the year (solar time) remains to the people of Honolulu?

**22.** The first shock of the earthquake at Kingston, Jamaica (long. 76° 47′ W.), Jan. 14, 1907, occurred at 3:25 P.M. What was the solar time at New York? at Cape Town?

**23.** A ship sets sail from Liverpool for New York, Jan. 10, 1907. When in longitude 34° 6′ 10″ W. its chronometer reads 2:30 P.M. Jan. 15. Find the difference in the readings between the ship's time and the meridian time of New York.

**24.** Berlin meridian time is 6 hr. 44 min. and $1\frac{11}{15}$ sec. later than Chicago meridian time. Find the longitude of Berlin.

| hr. | min. | sec. |
|---|---|---|
| 6 | 44 | $1\frac{11}{15}$ |
| | | 15 |
| 101 | 0 | 26 |

| ° | ′ | ″ | |
|---|---|---|---|
| 101 | 0 | 26 | difference in longitude |
| 87 | 36 | 42 | W. (Chicago) |
| 13 | 23 | 44 | E. (Berlin) |

15 times the difference in time expressed in hours, minutes, and seconds corresponds to the difference in longitude expressed in degrees, minutes, and seconds.

Therefore, 15 × 6 hr. 44 min. $1\frac{11}{15}$ sec. corresponds to 101° 0′ 26″ of longitude.

This difference in longitude would not tell us whether Berlin is east or west of Chicago, but as Berlin has *faster* time than Chicago, it must be east of it. Chicago is 87° 36′ 42″ west of the prime meridian. Therefore, Berlin must be 13° 23′ 44″ east of the prime meridian.

**25.** The "Treaty of Portsmouth" between Japan and Russia was signed at Portsmouth, N.H., Sept. 5, 1905, at 47 minutes past 3 P.M., 75th meridian time. What was the corresponding solar time at St. Petersburg, Russia, 30° 17′ 51″ E. and at Tokyo, Japan, 139° 44′ 30″ E.?

Tokyo is east of the 75th meridian 214° 44′ 30″, therefore its time is 14 hr. 18 min. 58 sec. faster than Portsmouth, which has 75th meridian time. Counting this time forward from 3:47 P.M. Sept. 5, gives 5 minutes and 58 seconds past 6 o'clock A.M. Sept. 6, Tokyo solar time.

**26.** The President of the United States takes the oath of office at 12 noon, 75th meridian time. Find the solar time and date in each of the following places for the inauguration of March 4, 1909: Honolulu; Berlin; San Francisco; London.

## International Date Line

The nations have agreed upon the 180th meridian, with slight changes as shown on page 355, as the place where the new day always begins. The calendar is set forward one day on ships crossing this line sailing *westward*: the calendar is set back one day on ships crossing this line sailing *eastward*.

**1.** A ship sets sail from San Francisco for Manila, Oct. 9, 1906, at 9 A.M., 120th meridian time; it arrives at Manila, Oct. 27, at 9 A.M., meridian time. How long is the voyage?

**2.** The same ship sets sail from Manila, Nov. 3, 1906, at 3 P.M., meridian time, and arrives at San Francisco, Nov. 23, at 3 P.M., 120th meridian time. How long is the voyage?

## Standard Time

The railroads of the United States in 1883 agreed upon a system of **standard time** and divided our country into four time belts, as shown on the map. In the **eastern** time belt all trains keep the time of the 75th meridian, known as

eastern standard time. In the **central** belt all trains keep the time of the 90th meridian, known as central standard time. In the **mountain** time belt all trains keep the time of the 105th meridian, known as mountain standard time. In the **Pacific** time belt all trains keep the time of the 120th meridian, known as Pacific standard time.

Each railway has selected the most convenient towns on its route, as is shown on the map, to change from the standard time of one belt to the standard time of another belt. The time in any belt is 1 hour *faster* than the time in the belt west of it, or 1 hour *slower* than the time in the belt east of it. Correct time is telegraphed each day to all parts of the United States from the Naval Observatory at Washington.


**STANDARD TIME BELTS**

1. What is the differenee in time between closing of the election polls at 7 P.M. in New York and 7 P.M. Seattle, on the day for choosing presidential electors?

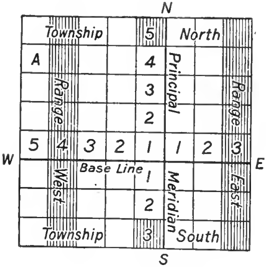
2. A telegraph message was sent from Philadelphia at 11 A.M. Oct. 12, 1906, to San Francisco and delivered at 7:45 A.M. San Francisco time. Why could this be true?

# GOVERNMENT LAND MEASURES

**Surveyor's square measure** is used by surveyors in measuring and computing land areas.

| | | |
|---|---|---|
| **16 square rods** | = | **1 square chain** |
| **10 square chains** | = | **1 acre** |
| **640 acres** | = | **1 square mile** |
| **36 square miles** | = | **1 township** |

The Gunter's chain for measuring land is gradually going out of use. In its place surveyors use a steel tape 100 ft. long, divided into feet and decimal parts of a foot. They find the number of square feet in the plot to be measured, and change the result to acres by dividing by 43560, the number of square feet in an acre.




A Group of Townships

The public lands of the United States are surveyed by selecting a north and south line, called a **principal meridian**, and intersecting this by an east and west line, called a **base line.**

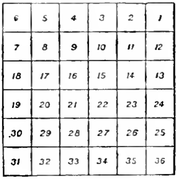
**Range lines** are lines running north and south on each side of the principal meridian, at distances of 6 miles. They divide the land into strips 6 miles wide, called *ranges.*

East and west lines parallel to the base line, and at distances of 6 miles, divide the ranges into *townships.* A **range** is, therefore, a row of townships running north and south.

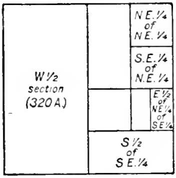
The *townships* in each range are numbered north and south

from the base line, and the *ranges* are numbered east and west from the principal meridian.

A township is designated by its number and direction from the base line, the number and position of its range, and the name or number of the principal meridian. Thus, Township A is 4 North, Range 5 West of Principal Meridian.




A Township




A Section

A township is 6 miles square and is divided into 36 **sections** each *one mile* square. Each section contains 640 acres.

**1.** W. $\frac{1}{2}$ Sec. 31, T. 22 N., 4 E. 3d P. M. is read west $\frac{1}{2}$ section 31, township 22 North, Range 4 east of third principal meridian.

**2.** Read : S. $\frac{1}{2}$ of S.E. $\frac{1}{4}$, Sec. 31.

**3.** Read : N.W. $\frac{1}{4}$ of S.E. $\frac{1}{4}$, Sec. 31.

**4.** Read : N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, Sec. 31.

Locate the tract of land in sections and give number of acres:

**5.** S.W. $\frac{1}{4}$, Sec. 5, T. 4 S., R. 3 W.

**6.** S.$\frac{1}{2}$ of N.E. $\frac{1}{4}$, Sec. 4, T. 15 N., R. 5 E.

**7.** S.E. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec.8, T. 125 S., R. 4 W.

**8.** Find the number of rods of fence required for the tract in problem 7.

**9.** How many rods of fence are required for the tract mentioned in problem 6?

## POWERS AND ROOTS

**1.** $3 \times 3 = 9$. $4 \times 4 = 16$. $5 \times 5 \times 5 = 125$.

**2.** Name the two equal factors that produce 9. 16.

**3.** Name the three equal factors that produce 125.

**4.** What two equal factors produce 25? 36? 49? 81?

**5.** What three equal factors produce 8? 27? 64?

**6.** How many times is 5 used as a factor in 5? 25? 125?

**7.** How many times is 3 used as a factor in 3? 9? 27?

A **power** of a number is the product obtained by taking the number one or more times as a factor. Thus, 9 is a power of 3, and 8 is a power of 2.

The *first* power of a number is the number itself. The *second* power of a number is called the **square** of the number. Thus, 16 is the square of 4. The *third* power of a number is called the **cube** of the number. Thus, 64 is the cube of 4.

**8.** What is the square of 4? 5? 6? 7? 8? 9?

**9.** What is the cube of 2? 3? 5? 6? 7?

**10.** $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$. What, then, is the square of $\frac{2}{3}$?

**11.** $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$. What, then, is the cube of $\frac{3}{4}$?

**12.** $.3 \times .3 = .09$. $.5 \times .5 = .25$. What, then, is the square of .3? of .5?

*The square of a fraction is found by squaring both terms, the cube of a fraction by cubing both terms.*

**13.** $3 \times 3 = 9$. The square of 3 is indicated thus, $3^2$; $4 \times 4 \times 4 = 64$. The cube of 4 is indicated thus, $4^3$.

**14.** How much is $5^2$? $6^2$? $7^2$? $5^3$? $6^3$? What is the value of $(\frac{3}{4})^2$? $(\frac{2}{3})^3$? $.4^2$? $.4^3$? $(2\frac{1}{2})^2$?

An **exponent** is a small figure placed at the right of, and a little above the number, to indicate the number of times it is to be taken as a factor; thus, $4^3 = 4 \times 4 \times 4 = 64$. Exponent, 3.

## Written Work

**1.** Square 6, 7, 8, 9, 10, 12, 15.

**2.** Cube 3, 4, 5, 6, 7, 8, 9, 10, 12.

**3.** Square 30, 50, 60, 80, 120.

**4.** Cube 20, 30, 40, 50, 100.

**5.** Find the value of $6^2$, $8^2$, $9^2$, $5^3$, $6^3$, $7^3$.

**6.** Find the value of $(\frac{3}{5})^2$, $(\frac{2}{5})^3$, $(\frac{3}{4})^2$, $(\frac{4}{5})^3$, $(\frac{3}{7})^2$, $(1\frac{1}{2})^2$.

**7.** Square .3, .04, .05, .6, .06. Cube .4, .04, .6.

Find the value of:

| | | |
|---|---|---|
| **8.** $15^2$ | **11.** $22^2$ | **14.** $.75^2$ |
| **9.** $16^2$ | **12.** $25^2$ | **15.** $(1\frac{3}{5})^2$ |
| **10.** $18^2$ | **13.** $6.5^2$ | **16.** $(16\frac{1}{2})^2$ |

Find the number of square units in a surface whose side is:

| | | |
|---|---|---|
| **17.** 15 in. | **20.** 8 ft. 6 in. | **23.** 10 yd. |
| **18.** 25 ft. | **21.** 5 in. | **24.** 5 yd. 2 ft. |
| **19.** 16 yd. | **22.** 8.5 in. | **25.** 6 mi. |

Find the number of cubic units in a volume whose edge is:

| | | |
|---|---|---|
| **26.** 8 in. | **28.** 3 ft. 3 in. | **30.** 1 yd. 10 in. |
| **27.** 2 ft. | **29.** 42 in. | **31.** 12 ft. 4 in. |

## EXTRACTING ROOTS

A **root** of a number is one of the equal factors that produce that number. Thus, 3 is a root of 9.

The **square root** of a number is one of its *two* equal factors. Thus, 4 is the square root of 16.

The **cube root** of a number is one of its *three* equal factors. Thus, 4 is the cube root of 64.

**1.** What is the square root of 25? 36? 49? 81? 100?

**2.** What is the cube root of 8? 64? 125? 216?

The root of a number is generally indicated by writing the number under the *radical or root sign* $\sqrt{\phantom{x}}$, and placing a figure called the *index* in the angle of the sign; thus, $\sqrt[3]{27}$ denotes the cube root of 27. The square root is indicated by $\sqrt{\phantom{x}}$ without the index.

*The root of a fraction equals the root of the numerator divided by the same root of the denominator.*

Find the required root: Thus, $\sqrt{64} = \sqrt{8 \times 8} = 8$.

| | | | |
|---|---|---|---|
| **3.** $\sqrt{16}$ | **8.** $\sqrt{\frac{64}{100}}$ | **13.** $\sqrt[3]{216}$ | **18.** $\sqrt[3]{512}$ |
| **4.** $\sqrt[3]{27}$ | **9.** $\sqrt{49}$ | **14.** $\sqrt[3]{1000}$ | **19.** $\sqrt{144}$ |
| **5.** $\sqrt{\frac{49}{81}}$ | **10.** $\sqrt[3]{125}$ | **15.** $\sqrt{121}$ | **20.** $\sqrt{169}$ |
| **6.** $\sqrt{25}$ | **11.** $\sqrt[3]{\frac{27}{125}}$ | **16.** $\sqrt[3]{343}$ | **21.** $\sqrt{2500}$ |
| **7.** $\sqrt[3]{64}$ | **12.** $\sqrt{81}$ | **17.** $\sqrt[3]{.064}$ | **22.** $\sqrt{8100}$ |

**Some perfect powers and their roots.**

Memorize:

| | | | |
|---|---|---|---|
| $\sqrt{1} = 1$ | $\sqrt{36} = 6$ | $\sqrt[3]{1} = 1$ | $\sqrt[3]{216} = 6$ |
| $\sqrt{4} = 2$ | $\sqrt{49} = 7$ | $\sqrt[3]{8} = 2$ | $\sqrt[3]{343} = 7$ |
| $\sqrt{9} = 3$ | $\sqrt{64} = 8$ | $\sqrt[3]{27} = 3$ | $\sqrt[3]{512} = 8$ |
| $\sqrt{16} = 4$ | $\sqrt{81} = 9$ | $\sqrt[3]{64} = 4$ | $\sqrt[3]{729} = 9$ |
| $\sqrt{25} = 5$ | $\sqrt{100} = 10$ | $\sqrt[3]{125} = 5$ | $\sqrt[3]{1000} = 10$ |

**Finding the root of a perfect square or cube by factoring.**

**Written Work**

**1.** Find the square root of 1225.

When factored $1225 = 5 \times 5 \times 7 \times 7$.

Arranged into *two* like groups $1225 = (5 \times 7) \times (5 \times 7)$.

$\sqrt{1225} = 5 \times 7$, or 35.

Find the root of each number, as indicated, by factoring:

| | | | |
|---|---|---|---|
| **2.** $\sqrt{225}$ | **7.** $\sqrt{784}$ | **12.** $\sqrt{1764}$ | **17.** $\sqrt{5184}$ |
| **3.** $\sqrt{576}$ | **8.** $\sqrt{1296}$ | **13.** $\sqrt[3]{13824}$ | **18.** $\sqrt[3]{15625}$ |
| **4.** $\sqrt{441}$ | **9.** $\sqrt[3]{4096}$ | **14.** $\sqrt{3136}$ | **19.** $\sqrt[3]{32768}$ |
| **5.** $\sqrt{196}$ | **10.** $\sqrt{2304}$ | **15.** $\sqrt[3]{5832}$ | **20.** $\sqrt{4096}$ |
| **6.** $\sqrt{1600}$ | **11.** $\sqrt[3]{42875}$ | **16.** $\sqrt[3]{8000}$ | **21.** $\sqrt[3]{19683}$ |

## SQUARE ROOT

**Comparing roots and periods.**

The **squares** of the smallest and the largest integers composed of one, two, and three figures are as follows:

| | | |
|---|---|---|
| $1^2 = 1$ | $10^2 = 100$ | $100^2 = 10000$ |
| $9^2 = 81$ | $99^2 = 9801$ | $999^2 = 998001$ |

**1.** Separate each of these squares into periods of two figures each, beginning at the right; thus, 99′ 80′ 01.

**2.** How does the *number of periods* in each square compare with the *number of figures* in the corresponding roots?

*The number of periods of two figures each, beginning at units, into which a number can be divided equals the number of figures in the root.*

Note. — The left-hand period may contain only one figure.

**3.** How many figures are there in the square root of 4225? of 12544? of 133225? of 810000?

### Written Work

**1.** Square 25. $25 = 20 + 5$, hence it may be squared in two ways, thus:

$$\begin{array}{rr}
25 = & 20 + 5 \\
25 = & 20 + 5 \\ \hline
125 = & 20 \times 5 + 5^2 \\
500 = & 20^2 + 20 \times 5 \\ \hline
625 = & 20^2 + 2\,(20 \times 5) + 5^2
\end{array}$$

The square of 25 has three partial products:

$$25^2 = \left\{\begin{array}{ll} (1) & 20^2 = 400 \\ (2) & 2\,(5 \times 20) = 200 \\ (3) & 5^2 = \ \ 25 \end{array}\right\} = 625$$

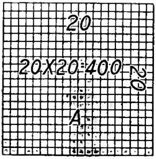
**2.** Find the square root of 625.

$$\begin{array}{rr|l}
 & 6'25 & 20 \\
20^2 = & 4\ 00 & \\ \hline
\text{Trial divisor, } 2 \times 20 = 40 & 225 & \ \ 5 \\
5 & & \overline{25} \\ \hline
\text{Complete divisor, } = 45 & 225 &
\end{array}$$

In practice:

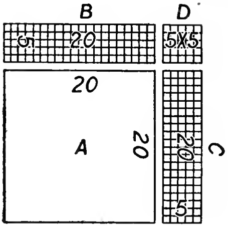
$$\begin{array}{r|l}
6'25 & \underline{25} \\
4 & \\
45 & 225 \\
 & \underline{225}
\end{array}$$

Since 625 has two periods, its square root is composed of two figures, tens and ones. Since the square of *tens* is *hundreds*, 6 hundreds must be the square of at least 2 tens. Two tens or 20 squared is 400, as shown in figure *A*; and $625 - 400$ leaves a remainder of 225. The root 20, therefore, must be so increased as to exhaust this remainder and keep the figure a square.




The necessary additions to enlarge *A*, and keep it a square, are the two equal rectangles *B* and *C*, and the small square *D*.

*B*, *C*, and *D* contain 225 square units; and since the area of *D* is small, if 225 is divided by 40, the combined length of *B* and *C*, the quotient will indicate the approximate width of these additions. The quotient is 5; the entire length of *B*, *C*, and *D* is $20 + 20 + 5 = 45$ units; and the area of the additions is 5 times $45 \times 1$ sq. unit, or 225 sq. units. Since these three additions exhaust the remaining 225 sq. units, and keep the figure a square, the side of the required square is 25 units, and the square root of 625 is 25.



**3.** Square 35 $(30+5)$; then find the square root of 1225.

$$\text{Partial Products:}\left\{\begin{array}{lrl}(1) & 30^2 &= 900\\ (2) & 2(5\times 30) &= 300\\ (3) & 5^2 &= \underline{\ 25}\end{array}\right\}1225$$

$$\begin{array}{lrr|l}
 & & 12'25 & 30\\
 & 30^2 & 9\ 00 & \ \ 5\\
\text{Trial divisor,} & 2\times 30 = 60 & 3\ 25 & \overline{35}\\
 & 5 & & \\
\text{Complete divisor,} & \overline{65} & \underline{3\ 25} &
\end{array}$$

**Study of Problem**

1. How many periods in 1225?
2. How many figures, then, will be in its root?
3. What is the largest partial product in 1225?
4. What is the square root of 900?
5. What two partial products are contained in the 325 remaining?
6. 325 is composed principally of which partial product? $2(5 \times 30)$.

325 is composed principally of 2 *times the first figure of the root by the second*. Hence, if 325 is divided by $(2 \times 30)$, the quotient will give approximately the second figure of the root.

7. What is the quotient? What, then, is probably the second figure of the root?
8. What must be added to the trial divisor $(2 \times 30)$ to form the complete divisor?

Since $5 \times 65 = 325$, 5 is the second root figure, and $30+5$, or 35, is the square root of 1225.

9. How does $(5 \times 60) + (5 \times 5)$ compare with $5 \times 65$?

**4.** Find the square root of 21.16. Separate the number into periods, left and right from the decimal point.

$$\begin{array}{lr|l}
 & '21.16' & \underline{4.0 + .6 = 4.6}\\
 & 40^2 = \underline{16\ 00} & \\
\text{Trial divisor, } 2\times 40 = & 80 \quad 5\ 16 & \\
 & \underline{6} & \\
\text{Complete divisor,} & 86 \quad \underline{5\ 16} &
\end{array}$$

In practice:

$$\begin{array}{r|l}
 & '21.16' \quad \underline{4.6}\\
 & \underline{16}\\
86 & 5\ 16\\
 & \underline{5\ 16}
\end{array}$$

*Beginning at units, separate the number into periods of two figures each.*

*Find the largest square in the first period on the left, and write its root as the first figure in the required root. Subtract its square from the period and annex the second period to the remainder.*

*Double the root found for a trial divisor. Divide the remainder, omitting the last figure, by this trial divisor, and annex the quotient to the trial divisor and also to the root.*

*Multiply the complete divisor by the second figure of the root, and subtract the product from the remainder.*

*Double the root already found for another trial divisor, and proceed as above until all the periods have been used.*

NOTE. — When a naught occurs in the root, annex a naught to the trial divisor, bring down another period, and proceed as before.

In extracting the square root of a *decimal* or a *whole number* and a *decimal*, point off into periods of two figures each, the whole number toward the left and the decimal toward the right of the decimal point.

In extracting the square root of a fraction, extract the square root of the numerator and of the denominator if both terms are perfect squares; or reduce the fraction to a decimal, and extract the root of the decimal.

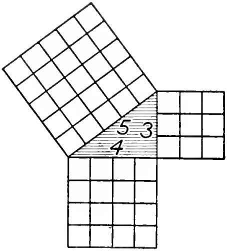
Solve the following :

| | | | |
|---|---|---|---|
| **5.** $\sqrt{484}$ | **9.** $\sqrt{1089}$ | **13.** $\sqrt{\frac{64}{100}}$ | **17.** $\sqrt{.0225}$ |
| **6.** $\sqrt{576}$ | **10.** $\sqrt{1849}$ | **14.** $\sqrt{\frac{196}{225}}$ | **18.** $\sqrt{4.41}$ |
| **7.** $\sqrt{676}$ | **11.** $\sqrt{2601}$ | **15.** $\sqrt{\frac{169}{324}}$ | **19.** $\sqrt{6.25}$ |
| **8.** $\sqrt{961}$ | **12.** $\sqrt{3969}$ | **16.** $\sqrt{.25}$ | **20.** $\sqrt{.0625}$ |

Find the square root to the nearest hundredth of :

| | | | |
|---|---|---|---|
| **21.** 315 | **26.** 178.25 | **31.** 96.8256 | **36.** .8464 |
| **22.** 525 | **27.** 14884 | **32.** $192\frac{33}{64}$ | **37.** .012996 |
| **23.** 1156 | **28.** 26569 | **33.** $540\frac{9}{16}$ | **38.** .000566 |
| **24.** 4356 | **29.** 136161 | **34.** $3192\frac{1}{4}$ | **39.** .43681 |
| **25.** 210.25 | **30.** 20.7936 | **35.** $5643\frac{49}{64}$ | **40.** 112225 |

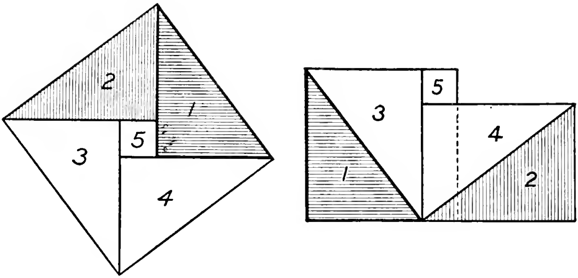
The **hypotenuse** of a right triangle is the side opposite the right angle.




**1.** How many square units are there in the square described upon the hypotenuse? in the square described upon the perpendicular? in the square described upon the base?

**2.** How do the number of square units described upon the hypotenuse compare with the *sum* of the square units described upon the other two sides?

That this is universally true is shown by the following diagram:

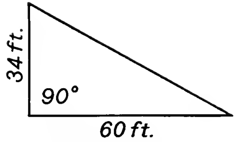



Take any right triangle, as 1; lay it off on a piece of cardboard and draw the square on its hypotenuse. Cut this square into the four equal triangles 1, 2, 3, and 4, and the small square 5, as here shown.

By changing the position of the triangles 1 and 2 as indicated, we change the *first* diagram into the *second*. But the *first is the square on the hypotenuse*, and the second is the *sum of the squares on the other two sides*. Since they are equal, the truth of the proposition is evident.

*The square* **on the hypotenuse** *of a right triangle equals the sum of the squares described on the other two sides.*

### Written Work




**1.** Find the hypotenuse of this right triangle.

$$\text{Hypotenuse}^2 = 34^2 + 60^2, \text{ or } 4756$$
$$\text{Hypotenuse} = \sqrt{4756} = 68.9 + \text{ft.}$$

Draw figures to a convenient scale and find the unknown side:

| | Base | Perpendicular | Hypotenuse |
|---|---|---|---|
| **2.** | 18 in. | 27 in. | $(x)$ |
| **3.** | $(x)$ | 4 ft. | 5 ft. |
| **4.** | 24 ft. | $(x)$ | 40 ft. |
| **5.** | 15 yd. | 20 yd. | $(x)$ |
| **6.** | $(x)$ | 40 in. | 60 in. |

**7.** Find the length of the longest straight line that can be drawn on a table 8 ft. by 4 ft.

**8.** A has a field 40 rd. long and 30 rd. wide. B has a square field whose side equals the diagonal of A's field. What is the difference in the area of the two fields?

**9.** Find the longest straight line in a room 16 ft. in length by 12 ft. in width by 10 ft. in height.

**10.** Two automobiles, A and B, start from the same point. A goes east 10 mi., then north 10 mi.; B goes west 20 mi., then south 20 mi. Draw figure and find distance apart.

**11.** Find the length of the diagonal of a square field containing 225 sq. rd.

**12.** Find the side of the largest square that may be inscribed in a circle 2 ft. in diameter.

**13.** A fireman's ladder just reaches a window 36 ft. above the ground. How long is the ladder if its foot is 27 ft. from the building?

# MENSURATION

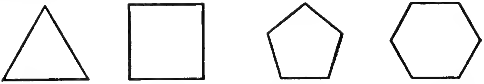
Rectangular surfaces, rectangular solids, and the cylinder have been treated under Practical Measurements.

## REGULAR POLYGONS

A **plane** is a surface such that a straight line joining any two of its points lies wholly in the surface.

A **polygon** is a plane figure bounded by straight lines.

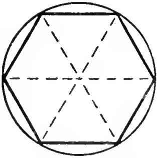
A **regular polygon** is a plane figure having equal sides and equal angles.



TRIANGLE SQUARE PENTAGON HEXAGON

Polygons are named from their sides. A regular polygon of *three* sides is called an **equilateral triangle**; one of *four* sides, a **square**; one of *five* sides, a **pentagon**; one of *six* sides, a **hexagon**, etc.

### Finding the area of a regular polygon.



Draw a circle. Divide the circumference into 6 equal parts by points marked at distances apart equal to the length of the radius. Join these points, thus making a **hexagon**. Connect the opposite points by dotted lines, thus dividing the hexagon into six **equilateral triangles**.

Show by folding together the opposite sides of any equilateral, and the equal sides of any isosceles triangle, that each may be divided into two equal right triangles.

*The* **area of any regular polygon** *equals the sum of the areas of the triangles composing it.*

1. Find the surface of the bottom of a hexagonal silo that is 12 feet on a side, the distance from the middle point of the side to the center of the bottom being 10.3 ft.

2. How far from the corner is the center of a square field that is 40 rods on a side? (Draw the figure.)

3. Find the area of an equilateral triangular design that is 15 inches on a side. (Divide into two right triangles.)

## SOLIDS

A **solid** is anything that has length, breadth, and thickness.

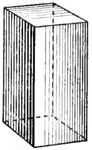
The **faces** of a solid are the surfaces that bound it.

The **lateral** or **convex surface** of a solid is the area of its sides, or faces.

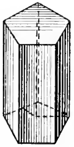
The **volume** of a solid is the number of cubic units it contains.

A **prism** is a solid whose ends are equal and parallel polygons, and whose sides are parallelograms. Prisms are named from their bases, as *triangular*, *square*, *rectangular*, *pentagonal*, *hexagonal*, etc.

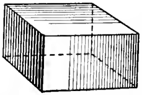
Triangular Prism



Square Prism



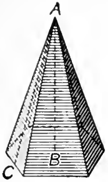
Pentangular Prism



Rectangular Prism

A **cylinder** is a solid with circular ends and uniform diameter. The ends are the bases, and the curved surface is the **convex surface.**

The **altitude** of a prism or of a cylinder is the perpendicular distance between the bases.



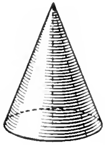
PYRAMID

A **pyramid** is a solid whose base is a regular polygon, and whose faces are triangles that meet at a point called the *vertex*. Pyramids are named from their bases, as, **triangular, square, pentagonal,** etc.


CYLINDER

The **slant height** of a pyramid is the altitude of the triangles that bound it.

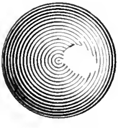
A **cone** is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the *vertex*.


CONE

The **altitude** of a pyramid, or of a cone, is the perpendicular distance from the vertex to the base.

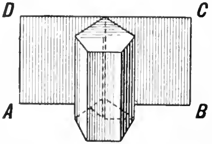
The **slant height** of a cone is the distance between the vertex and any point in the circumference of the base.

A **globe** or **sphere** is a solid bounded by a curved surface, every point of which is equally distant from a point within, called the *center*.


SPHERE

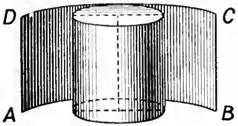
## SURFACES OF SOLIDS

### Surfaces of Prisms and Cylinders




Observe: 1. That if a piece of paper is fitted to cover the convex surface of a prism or a cylinder, and then unrolled, its form will be that of a rectangle, as $ABCD$.

2. That the perimeter of the solid forms one side of the rectangle, and the altitude of the solid the other side.




*The* **convex surface of a prism or of a cylinder** *is found by multiplying the unit of measure by the product of the perimeter and the altitude.*

Find the convex surface of a regular prism of:

**1.** 5 sides; 1 side 10 ft.; height 5 ft.

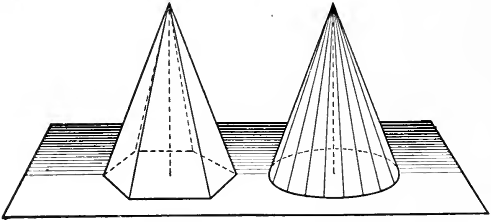
**2.** 3 sides; 1 side 20 in.; height 42 in.

**3.** A steam boiler, diameter 3 ft.; length 10 ft. Entire surface = ?

**4.** A water pail, diameter 11 in.; height 15 in. Entire surface = ?

## Surfaces of Pyramids and Cones

Observe: 1. That the convex surface of a pyramid is composed of triangles.



PYRAMID CONE

2. That the convex surface of a cone may also be considered as made up of small triangles.

3. That the bases of the triangles in both pyramid and cone form the perimeter of the base of the figure, and the altitude of the triangles the slant height. Hence,

*The* **convex surface of a pyramid** *or* **of a cone** *is found by multiplying the unit of measure by one half the product of the perimeter and the slant height.*

Find the convex surface of a pyramid or a cone if :

1. Diameter of base of cone = 9 ft.; slant height = 12 ft.

2. One side of a square pyramid = 16 ft.; slant height = 24 ft.

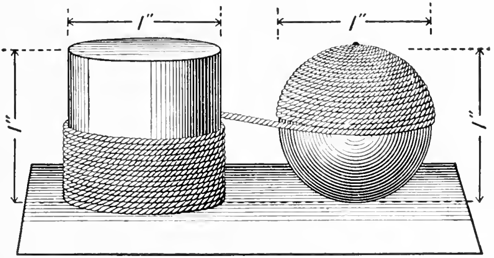
3. One side of a square pyramid = 5 ft.; altitude = 16 ft.

4. Altitude of square pyramid = 24 ft.; one side = 14 ft.

5. A church spire is in the form of a hexagonal pyramid, each side being 10 feet, and the slant height 65 feet. Find the cost of painting it at 25¢ per square yard.

6. A spire on the corner of a church is in the form of a cone. Its base is 12 feet in diameter and its slant height 24 feet. Find the cost of covering it with tin at $13 per square (100 sq. ft. = 1 square).

**Comparative Surfaces of Cylinder and Sphere**




Examine the solids. What is the height of the cylinder? What is the diameter of the cylinder? What is the diameter of the sphere? How does the diameter of each compare with the height of the cylinder? Observe that the *dimensions are equal.*

Geometry shows that the surface of a sphere is equal to the convex surface of a cylinder whose *height* and *diameter* are *each* equal to the diameter of the sphere.

To show this, wind a hard wax cord around a cylinder 1 in. in height and 1 in. in diameter until its convex surface is covered. Unwind the cord from the cylinder on to a sphere 1 in. in diameter as shown in the illustration. When one half the surface of the sphere is covered with the cord, one half of the convex surface of the cylinder is uncovered. Hence,

*The* **surface of any sphere** *equals the convex surface of a cylinder of equal dimensions.*

It may also be shown by geometry that

*The* **surface of a sphere** *equals the square of the diameter multiplied by* 3.1416, or $\pi d^2$ (representing the diameter by $d$ and 3.1416 by $\pi$).

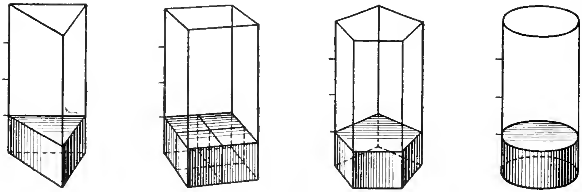
Find the surface of :

1. A globe, *D*. 12 in.
2. A ball, *R*. $1\frac{1}{2}$ in.
3. A sphere, *D*. 13 in.
4. A ball, *D*. 4 in.

5. How much will it cost to paint a dome in the form of a hemisphere, 20 ft. in diameter, at 25 cents per square yard?

## VOLUME OF SOLIDS

### Prisms and Cylinders



SCALE $\frac{1}{4}$ IN. = 1 IN.

Observe: 1. That the solids are all 4 in. high.

2. That the first row in the rectangular prism contains 4 cu. in.

3. That if the first row in each solid contains 4 cu. in., the volume of each solid is 4 times 4 cu. in., or 16 cu. in.

*The* **volume of a prism or of a cylinder** *is found by multiplying the unit of measure by the product of the numbers corresponding to the area of the base and the altitude.*

Find the volume of:

**1.** A prism 4 inches square; altitude 8 inches.

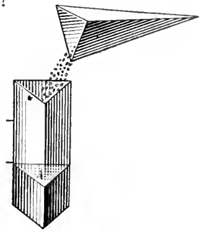
**2.** A square prism, side 12 in.; altitude 24 in.

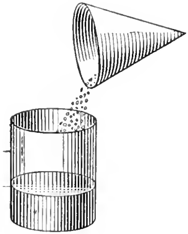
**3.** A hexagonal silo is 25 ft. high, 12 ft. on a side, and 10.3 ft. from the middle point of a side (measuring at the base) to the center of the base. Estimating 50 cu. ft. to a ton of ensilage, how many tons will the silo contain?

**4.** In the rotunda of a building there are 6 cylindrical marble columns, 18 in. in diameter and 18 ft. in height. Estimate the number of cubic feet in all.

## PYRAMIDS AND CONES

**1.** Fill a hollow pyramid with sand. Empty it into a prism having the same base and altitude. How often must the pyramid be filled and emptied to fill the prism? The volume of a pyramid, then, is what part of the volume of the prism?





**2.** Measure in like manner with a cone the volume of a cylinder having the same dimensions. The volume of the cone is what part of the volume of the cylinder?

Observe: 1. That the dimensions of the pyramid and of the prism are the same, and that those of the cone and of the cylinder are the same.

2. That the volume of the pyramid is $\frac{1}{3}$ that of the prism, and the volume of the cone is $\frac{1}{3}$ that of the cylinder.

By geometry, it is shown that the volume of a pyramid is $\frac{1}{3}$ of that of a prism having an equal base and an equal altitude. Hence,

*The* **volume of a pyramid** *equals one third of the volume of a prism of like dimensions.*

*The* **volume of a cone** *equals one third of the volume of a cylinder of like dimensions.*

But we have already learned that the volume of a prism or of a cylinder is found by multiplying its unit of measure by the product of the area of its base by its altitude. Hence,

*The* **volume of a pyramid** *or* **of a cone** *is found by multiplying its unit of measure by one third the product of the altitude and the area of the base.*

**1.** Find the volume of a cone whose altitude is 12 in. and the diameter of the base 8 in.

**2.** How often can a conical cup 8 in. high and 6 in. in diameter be filled from a cylindrical vessel 2 ft. high and 6 in. in diameter?

**3.** Find the volume of a pyramid whose base is 12 in. square and whose altitude is 30 in.

**4.** A square pyramid whose side is 18 in. is 32 in. high. Find its volume.

**5.** Find the volume of a pyramid whose altitude is 12 ft. and whose base is a square 8 ft. on a side.

**6.** Find the contents of a rectangular pyramid 15 ft. high, the sides of whose base are 10 ft. and 12 ft. respectively.

**7.** A pile of grain in the form of a cone is 15 ft. in diameter and 6 ft. high. How many bushels of grain does it contain?

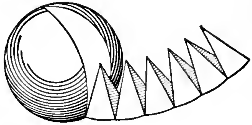
**8.** A concrete mixer, 6 ft. from base to apex, being conical in form, and measuring 3 feet across the base, is filled six times an hour. How many cubic feet of concrete material may be manufactured with it in a week of six working days of 8 hours each?

**9.** A wooden hopper supplying coal to a furnace is in the form of an inverted pyramid. If it is 8 ft. deep and 6 ft. square at the top, how many tons of hard coal will it contain?

**10.** A square pyramid, the perimeter of whose base measures 64 inches, contains 2048 cubic inches. Find its altitude.

**11.** The contents of a cone are 471.24 cu. ft.; the altitude is 18 ft. Find the diameter.

## SPHERES



Examine the figure:

Observe: 1. That the solids formed by the dissected part of the sphere are pyramids.

2. That the radius of the sphere is the altitude of the pyramids.

3. That the combined bases of the pyramids form the convex surface of the sphere.

*The* **volume of a sphere** *is found by multiplying its unit of measure by one third the product of the radius and its convex surface.*

It is also shown by geometry that

*The volume of a sphere equals four thirds of the cube of the radius multiplied by* 3.1416, *or (representing the radius by* $r$ *and* 3.1416 *by* $\pi$*)* $\frac{4}{3}\pi r^3$.

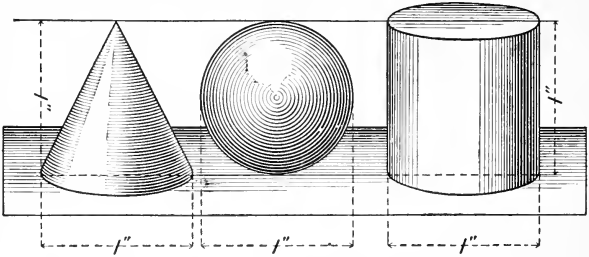
Find the volume of:

**1.** A globe 12 inches in diameter.

**2.** A bowling ball with a radius of 4 inches.

**3.** A cannon ball with a diameter of 8.2 inches.

**Comparative Volumes of Cone, Sphere, and Cylinder**

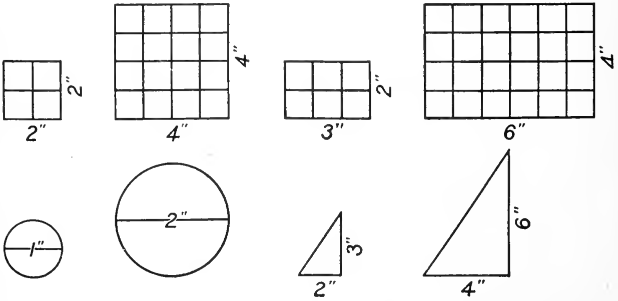



Compare the diameters of the bases and the altitudes of the cone and the cylinder with each other, and with the diameter of the sphere. Observe that the *dimensions* are *all equal.*

By geometry it is shown that the volumes of these three solids are in the ratio of 1, 2, and 3. The volume of the cone is $\frac{1}{3}$, and of the sphere $\frac{2}{3}$ of that of the cylinder.

## SIMILAR SURFACES

**Similar figures** are plane surfaces that have exactly the same shape, but differ in size. Point out similar figures:

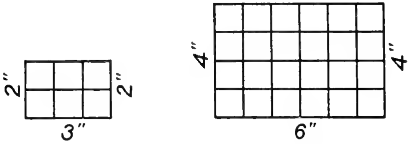


Find the areas of the similar surfaces on p. 376. Square the corresponding lines of the similar figures and express their ratio. Compare the ratio of the areas of the similar figures with the ratio of the squares of their corresponding lines.

In the similar figures observe:

1. That the corresponding sides are proportional; that is, 2:4::3:6.

2. That the ratio of their areas equals the ratio of the squares of their corresponding lines; that is, 6 sq. in. : 24 sq. in. as $2^2:4^2$, or as $3^2:6^2$.




**Corresponding lines of similar plane surfaces** *are proportional.*

*The* **areas of similar plane figures** *are proportional to the squares of their corresponding lines.*

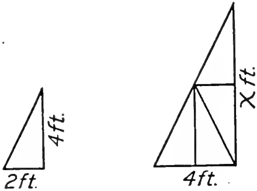
### Written Work

**1.** If a rectangle is 20 ft. by 50 ft., what will be the length of a similar rectangle 30 ft. in width?

**2.** The side of a square field is 40 rods. Find the side of a similar field that contains four times as many acres.

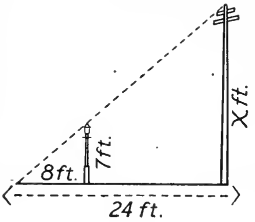
**3.** A lady buys two rugs, one 6 ft. by 9 ft. and a similar rug 18 ft. in length. Find its width.

**4.** In East Park a circular fountain is 40 ft. across and in West Park a circular fountain is 26 ft. across. The area of the first fountain is how many times the area of the second fountain?




**5.** Find the length of the side marked $x$ in the larger of these similar triangles.

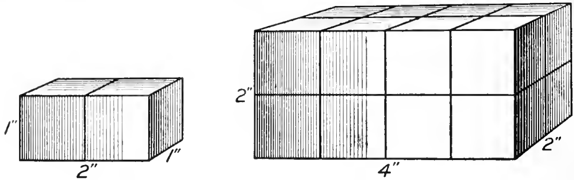
6. It costs $19.50 to gild a sphere 18 inches in diameter. At the same rate estimate the cost of gilding a sphere 12 inches in diameter. Why are the surfaces of all spheres similar?




7. A telephone pole and a lamp post cast shadows as shown in the figure. Find the height of the telephone pole.

## SIMILAR SOLIDS

Similar solids are solids that have the same shape but differ in contents or volume; thus,




Observe: 1. That the length of the first solid is to the length of the second as 1 : 2, and that the heights and widths of the solids are in the same ratio.

2. That the ratio of their contents or volume equals the ratio of the cubes of their corresponding lines; that is, 2 cu. in. : 16 cu. in. as $1^3 : 2^3$; or as $2^3 : 4^3$.

*The* **corresponding lines of similar solids** *are proportional.*

*The* **contents or volumes of similar solids** *are proportional to the cubes of their corresponding lines.*

### Written Work

1. Compare in volume a 5-in. cube with a 10-in. cube.

Compare in volume:

**2.** A $6\frac{1}{4}$-inch globe with a 25-inch globe.

**3.** A $3\frac{1}{8}$-inch cube with a 25-inch cube.

**4.** A $16\frac{2}{3}$-inch sphere with a 50-inch sphere.

**5.** What are corresponding lines in spheres? in triangles?

**6.** The dimensions of a rectangular solid are 30 ft. by 20 ft. by 12 ft. What are the dimensions of a similar solid 20 ft. in length?

**7.** A village has two similar cylindrical water tanks, one 15 ft. in diameter and 24 ft. high; the other 10 ft. in diameter. Find the height of the second tank.

**8.** Make three problems with similar rectangular solids.

**9.** The volume of a sphere is 1600 cu. in. What is the volume of a sphere with one half the diameter?

**10.** A rectangular bin 8 ft. long contains 92 bu. of wheat. How many bushels will a similar bin 10 ft. long contain?

## SPECIFIC GRAVITY

**1.** A piece of timber 12 inches square and 6 feet long floating in water shows 2 inches of the log above water. How many cubic feet of the log are under water? How many cubic feet of water are displaced by the log?

**2.** A cubic foot of water weighs $62\frac{1}{2}$ lb. How many cubic feet of water will a log displace that weighs 250 lb.? If the log in problem 1 is $\frac{2}{3}$ as heavy as the same volume of water, how much of the log is under water?

*Every object floating in water displaces its own weight of water.*

*Every object that sinks in water displaces its own volume of water.*

**3.** A piece of wood when floating is $\frac{1}{2}$ under water. The ratio of the weight of the wood to the weight of an equal volume of water is therefore $1 \div 2$ or $\frac{1}{2}$. What is the ratio of the weight of objects to the weight of the same volume of water if the displacement is $\frac{1}{4}$? $\frac{2}{3}$? $\frac{4}{5}$? $\frac{3}{4}$? .5?

Copper is 8.9 times as heavy as an equal volume of water. The ratio of the weight of copper to the weight of an equal volume of water is, therefore, 8.9.

**Specific gravity** is the ratio of the weight of any substance to the weight of an equal volume of water.

A **cubic foot** of water weighs $62\frac{1}{2}$ pounds or 1000 ounces.

**4.** The specific gravity of ice is .92. Find the weight of a block of ice 2′ by 18″ by 12″.

**5.** The specific gravity of pure cows' milk is 1.03. Find the weight of 50 gallons of milk.

**6.** The specific gravity of cork is .24. Find the weight of 10 cu. ft. of cork.

**7.** The specific gravity of lead is 11.3. Find the weight of 20 bars 16″ long, 4″ wide, and 2″ thick.

Find the weight of 1 cu. ft. of each object whose specific gravity is given:

| | | | | | |
|---|---|---|---|---|---|
| **8.** | Silver | 10.50 | **16.** | Copper | 8.90 |
| **9.** | Milk | 1.03 | **17.** | Lead | 11.30 |
| **10.** | Ice | .92 | **18.** | Sandstone | 2.90 |
| **11.** | Nickel | 8.90 | **19.** | Iron | 7.80 |
| **12.** | Granite | 2.70 | **20.** | Tin | 7.29 |
| **13.** | Mercury | 13.59 | **21.** | Steel | 7.83 |
| **14.** | Gold | 19.30 | **22.** | Marble | 2.70 |
| **15.** | Cork | .24 | **23.** | Ivory | 1.83 |

## REVIEW OF MENSURATION

**1.** From an artesian well 2 inches in diameter the water flows out at the rate of $1\frac{1}{2}$ ft. per second. Find the number of barrels that flow out per hour.

**2.** One cubic inch of gold could be pounded into how many square inches of gold leaf $\frac{1}{1000}$ of an inch in thickness?

**3.** A spherical ball must be how large in order that a cube, each surface of which contains 36 square inches, could be cut from it?

**4.** A copper ingot containing 1 cubic foot is to be drawn into a copper wire $\frac{1}{10}$ of an inch in diameter. Find the length of the wire when drawn.

**5.** A plowman found by measurement that he had plowed a strip 4 rd. in width around a rectangular field 40 rd. long and 20 rd. wide. Find the number of acres he had plowed. Draw figure to illustrate.

**6.** A western farmer has a pile of corn in the ear, 400 ft. long. The pile at the end is in the form of an isosceles triangle, 12 ft. wide at the bottom, and the altitude of the pile is 6 ft. Find the number of bushels, allowing $1\frac{5}{9}$ cu. ft. to a bushel.

**7.** A solid ball 6 inches in diameter is in a cylinder 10 inches in diameter and 10 inches high. How many cubic inches of water will the cyclinder contain?

**8.** What is the convex surface of a piece of stove pipe 2 feet long and 8 inches in diameter?

**9.** The inside diameter of a hollow globe is $1\frac{2}{3}$ ft. How many gallons of water will it contain?

**10.** The flow of water from the same source through two different pipes depends upon the area of the cross section of the openings. Compare the flow through a $\frac{1}{2}$-inch pipe with the flow through a $\frac{3}{4}$-inch pipe.

# METRIC SYSTEM OF WEIGHTS AND MEASURES

By the United States system of money we may write 5 dollars, 9 dimes, and 7 cents thus, \$5.97, because there is a uniform ratio between the dollar and the dime, the dollar and the cent, the dollar and the mill; the dollar being **10** times the dime, **100** times the cent, and **1000** times the mill.

As a mill is $\frac{1}{1000}$ of a dollar, a cent $\frac{1}{100}$ of a dollar, and a dime $\frac{1}{10}$ of a dollar, *United States money* is based on a **decimal system.**

By the **English long measure**, 12 in. = 1 ft., 3 ft. = 1 yd., and $5\frac{1}{2}$ yd. = 1 rd.; thus, we see there is no uniform ratio between the rod and the yard, the rod and the foot, and the rod and the inch, the rod being **$5\frac{1}{2}$** times the yard, **$16\frac{1}{2}$** times the foot, and **198** times the inch.

By the **English measure of weights**, 16 oz. = 1 lb., 100 lb. = 1 cwt., 20 cwt. = 1 ton. The ton equals **20** times the hundredweight, **2000** times the pound, and **32000** times the ounce.

The **metric system** was devised by the *French* government in an effort to establish a system of weights and measures that would be on a uniform decimal scale, so that a unit of one denomination might be changed to a unit of another denomination by simply moving the decimal point.

The **meter** is the fundamental unit of the metric system. Its length (about 39.37 in.) was meant to be .0000001 of the distance from the equator to the pole. Though an error has since been discovered in the measurement of the distance from the equator to the pole, the standard unit has not been changed. The original standard is a bar of platinum 39.37 inches in length, deposited in the archives in Paris.

From the meter every other unit of measure or weight is derived. Thus, the unit of **weight** is the **gram**, which equals the weight of 1 cubic centimeter of pure water.

Draw a cube .01 of a meter on an edge and state the length of the edge in inches.

The unit of **capacity** is the **liter** (lēter), which contains 1 cubic decimeter.

Draw a cube .1 of a meter on an edge and state the length of the edge in inches.

The metric system is now in use in most of the civilized countries except Great Britain and the United States, and in the latter it is in use in some of the departments of the government. It is the official system adopted by Congress for our island possessions. It is universally used by scientists. The United States by a vote of Congress permitted its use in 1866.

Observe:

The **meter** measures length.

The **square meter** measures surface.

The **cubic meter** measures solids or volume.

The **gram** measures weight.

The **liter** measures capacity.

**Latin prefixes.**

To express **.1** of a meter, **.1** of a gram, and **.1** of a liter, we prefix **deci** to each of the words, meter, gram, and liter. Thus, *decimeter* means $\frac{1}{10}$ of a meter; *decigram*, $\frac{1}{10}$ of a gram; and *deciliter*, $\frac{1}{10}$ of a liter.

To express **.01** of a meter, gram, and liter, we prefix **centi** to each of the words, meter, gram, and liter.

To express **.001** of a meter, gram, and liter, we prefix **milli** to each of the words, meter, gram, and liter.

NOTE. — From these Latin prefixes we get our words *dime, cent,* and *mill.*

**Greek prefixes.**

To express **10** times a meter, **10** times a gram, and **10** times a liter, we prefix **deca** to each of the words, meter, gram, and liter. Thus, *decameter* means **10** times a meter; *decagram*, **10** times a gram; and *decaliter*, **10** times a liter.

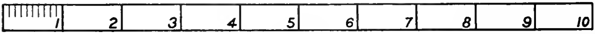
To express **100** times a meter, gram, and liter, we prefix **hecto** to each of the words, meter, gram, and liter.

To express **1000** times a meter, gram, and liter, we prefix **kilo** to each of the words, meter, gram, and liter.

## METRIC MEASURES OF LENGTH

### Comparison of Fundamental Units of Measures of Length

1 Yard.



1 Meter

### Table of Long Measures

| | |
|---|---|
| **1 millimeter** (mm.) | = .001 of a meter |
| **1 centimeter** (cm.) | = .01 of a meter |
| 1 decimeter (dm.) | = .1 of a meter (nearly 4 in.) |
| **1 meter** | = **39.37 in.** |
| 1 decameter (Dm.) | = 10 meters |
| 1 hectometer (Hm.) | = 100 meters |
| **1 kilometer** (Km.) | = 1000 meters **(nearly .6 mi.)** |

*In metric long measure* **10** *times one unit of any denomination equals one unit of the next higher denomination.*

The denominations most frequently used are given in black-faced type.

Approximately:

**1 yard = $\frac{11}{12}$ meter** **1 mile = 1.6 kilometer**

The **kilometer** is used for measuring long distances; the **meter**, for short distances and for measuring cloth, etc.; and the **millimeter** is used in the sciences and to show very small measurements, as the thickness of wire, etc.

### Written Work

**1.** What decimal parts of a meter are expressed by the Latin prefixes? What multiples are expressed by the Greek prefixes?

**2.** Draw a line one meter long. Show the number of decimeters in a meter; the number of centimeters; the number of millimeters.

**3.** Explain why 5 m. = 50 dm. = 500 cm. = 5000 mm.

**4.** In the metric system the fundamental operations are decimal or multiple operations. Thus, 8 m. 5 dm. 6 cm. 25 mm. are added in this manner.

| Added in meters: | | Added in millimeters: | |
|---|---|---|---|
| 8 m. | = 8 m. | 8 m. | = 8000 mm. |
| 5 dm. | = .5 m. | 5 dm. | = 500 mm. |
| 6 cm. | = .06 m. | 6 cm. | = 60 mm. |
| 25 mm. | = .025 m. | 25 mm. | = 25 mm. |
| | 8.585 m. | | 8585 mm. |

**5.** Add 1 m., 3 dm., 6 cm., 3mm. Add 6.5 m., .25 mm., 65 dm.

**6.** The distance between two towns is 5 Km. and 45½ m. After a bicyclist has traveled 3 Km. 57.5 m., how much of the distance remains to be traveled?

**7.** The distance from Paris to Calais is 295.32 Km. Express this distance approximately in miles.

**8.** How many meters of ribbon are necessary to make 150 badges, each 25 cm. in length?

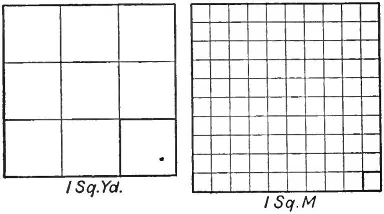
**9.** The distance from Erie, Pa. to Buffalo, N.Y. is 112.651 Km. Express the distance approximately in miles.

**10.** Reduce the decimal in the last problem to meters and lower denominations.

**11.** The distance from New York to San Francisco is 4000 miles. Approximate this distance in kilometers and meters.

## METRIC MEASURES OF SURFACE

### Comparison of Fundamental Units of Square Measure




### Table of Square Measures

| | |
|---|---|
| 1 sq. millimeter (sq. mm.) | = .000001 sq. meter |
| **1 sq. centimeter** (sq. cm.) | = .0001 sq. meter |
| 1 sq. decimeter (sq. dm.) | = .01 sq. meter |
| **1 sq. meter (sq. m.)** | = **1.196 sq. yd.** |
| 1 sq. decameter (sq. Dm.) | = 100 sq. meters |
| 1 sq. hectometer (sq. Hm.) | = 10000 sq meters |
| **1 sq. kilometer** (sq. Km.) | = 1000000 sq. meters **(nearly .4 sq. mi.)** |

*In metric measure of surface* **100** *times one unit of any denomination equals one unit of the next higher denomination.*

**Land Measure.**

The standard unit used for measuring is the **are** (**âr**) containing 100 square meters = 119.6 square yards.

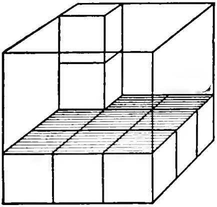
Table:

1 centare = 1 sq. meter.
1 **are** = 100 sq. meters (**nearly 120 square yards**)
1 hectare = 10000 sq. meters (**nearly $2\frac{1}{2}$ acres**).

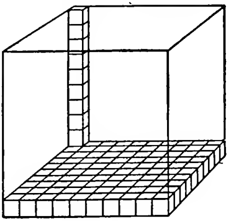
The **square meter** is used in measuring ordinary surfaces, such as are found in houses, lots, farms, etc.; the **square kilometer** for measuring areas of countries and their divisions into states, counties, etc.

## METRIC MEASURES OF VOLUME

### Comparison of Fundamental Units of Cubic Measure



1 Cubic Yard



1 Cubic Meter

### Table of Solid or Cubic Measures

| | |
|---|---|
| 1 cu. millimeter (cu. mm.) | = .000000001 cu. meter |
| **1 cu. centimeter (cu. cm.)** | = .000001 cu. meter |
| 1 cu. decimeter (cu. dm). | = .001 cu. meter |
| **1 cu. meter** | = **1.308 cu. yd.** |

*In metric measure of volume* **1000** *times one unit of any denomination equals one unit of the next higher denomination.*

In measuring wood 1 cu. meter is called a **stere.**

The **cubic meter** is the practical unit of measure of **vol**ume for all purposes.

### Written Work in Square and Cubic Measures

1. Find the number of square meters in the floor of your schoolroom.

2. At 27 cents per cubic meter, find the cost of excavating a cellar 10 m. by 18 m. by $1\frac{1}{2}$ m.

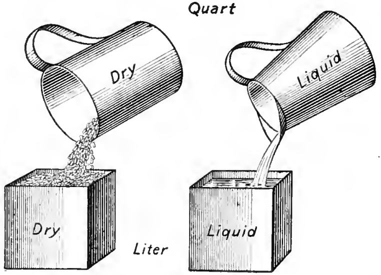
3. How much will it cost to paint one side of a tight board fence 25 meters long and 3 meters wide at $10 per square decameter?

4. How many square meters of linoleum will be required to cover the floor of a hall 8 meters long and 3 meters wide?

5. How many steres are there in a pile of wood 2 meters high, 2 meters wide, and 6 meters long?

## METRIC MEASURES OF CAPACITY

### Comparison of Fundamental Units of Capacity



### Table of Measures of Capacity

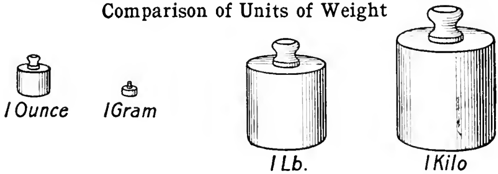
| | |
|---|---|
| 1 millimeter (ml.) | = .001 of a liter |
| **1 centiliter** (cl.) | = .01 of a liter |
| 1 deciliter (dl.) | = .1 of a liter |
| **1 liter** | **= 1.0567 liquid quarts = .908 dry quart.** |
| 1 decaliter (Dl.) | = 10 liters |
| **1 hectoliter** (Hl.) | = 100 liters **(nearly 2.84 bu.)** |

*In metric measure of capacity* **10** *times one unit of any denomination equals one unit of the next higher denomination.*

The **liter** is used for liquid and dry measures.

## METRIC MEASURES OF WEIGHT

### Comparison of Units of Weight




### Table of Measures of Weight

| | |
|---|---|
| 1 milligram (mg.) | = .001 of a gram |
| **1 centigram** (cg.) | = .01 of a gram |
| 1 decigram (dg.) | = .1 of a gram |
| **1 gram** | **= .03527 of an oz. Avoir.** |
| 1 decagram (Dg.) | = 10 grams |
| 1 hectogram (Hg.) | = 100 grams |
| **1 kilogram** (Kg.) | = 1000 grams **(nearly 2.2 lb.)** |
| 1 myriagram (Mg.) | = 10000 grams |
| 1 quintal (Q.) | = 100000 grams |
| **1 tonneau** (T.) | = 1000000 grams **(nearly 2205 lb.)** |

*In the metric measure of weight* **10** *times one unit of any denomination equals one unit of the next higher denomination.*

The *gram* is the weight of 1 cubic centimeter of water, the *kilogram*, of 1 cubic decimeter, and the *metric ton*, of 1 cubic meter; the *gram* is used by druggists and chemists; the *kilogram* (usually called the kilo) for weighing small articles; and the *metric ton* for large, heavy articles.

### Table of Metrical Equivalents

**1 cu. mm. of water weighs 1 mg. and measures .001 ml.**
**1 cu. cm. of water weighs 1 g. and measures 1 ml.**
**1 cu. dm. of water weighs 1 Kg. and measures 1 l.**
**1 cu. m. of water weighs 1 T. and measures 1 Kl.**

### Written Work

Things sold in the United States and England by the *quart* are sold in countries using the metric system by the *liter*.

**1.** Estimate the number of liters in a tank $2\frac{1}{2}$ m. long, $\frac{3}{4}$ m. in width, and $\frac{1}{2}$ m. in depth.

**2.** A cylindrical tank 6 m. in diameter and 10 m. in height is $\frac{2}{3}$ full. Estimate the number of liters it contains. Estimate the weight in metric tons.

**3.** A Paris milkman retailed on an average 110 liters of milk daily at 25 centimes a liter. Find the amount of his sales in our own money for 30 days.

**4.** The rainfall in a certain place in one week was 1 dm. Find the number of liters that fell on 3 hectares of land.

**5.** A horse eats 4 liters of oats 3 times a day. How many hectoliters does it eat in 60 days?

**6.** From an olive orchard, 4.5 Kl. of olive oil was put up in bottles holding .5 l. How many bottles were used?

**7.** Find the amount in United States money from the sale of 2000 Hl. of wheat at 10 francs per hectoliter.

**8.** A German ice dealer retails blocks of ice .8 m. in length, .3 m. in width, and .2 m. in thickness. The weight of ice is .92 that of the same volume of water. Find its cost at 3.5 pfennigs per kilo. Find the amount in our money.

**9.** Change a cubic meter of water to liters.

**10.** Find the weight of a barrel of flour in kilos.

**11.** A stone 8 ft. by 3 ft. by 2 ft. contains how many cubic meters?

**12.** If stone is 2.9 times as heavy as the same volume of water, find the weight of the stone in kilos.

**13.** The Washington Monument is 555 feet high. Express its height in meters.

**14.** Find the cost of laying a cement walk .025 Km. in length and 1.5. m. in width, at $1.70 per sq. m.

**15.** Mr. James bought a tract of land in the Philippine Islands, 3 Km. in length and 2.5 Km. in width, at $15.75 per hectare. Find the cost of inclosing this land with wire fence at 10¢ per meter.

**16.** A hallway is 12 m. in length and 5 m. in width. Estimate the number of tiles 1 cm. square necessary to cover it.

**17.** A railroad in building a retaining wall used 52000 cu. m. of stone. Find its weight in metric tons if stone is 2.7 times as heavy as the same volume of water.

**18.** A city sewer is 1.3 Km. in length, 1.2 m. in width, and averages 3 m. in depth. Estimate the number of cubic meters of earth removed.

**19.** A certain kind of cloth costs 90¢ per meter + 25% ad valorem duty. For how much must it be marked in United States money to gain 25% on the yard?

# AGRICULTURAL PROBLEMS

## FEEDING STOCK

TABLE OF DIGESTIBLE NUTRIENTS

| FEEDS | PROTEIN | CARBOHYDRATES | FAT |
|---|---|---|---|
| Corn fodder, green . | 1.1 | 12.1 | .4 |
| Red clover, green . | 3.1 | 14.8 | .7 |
| Alfalfa, green . . | 3.9 | 11.2 | .4 |
| Corn silage . . . | 1.2 | 14.6 | .9 |
| Corn fodder, dry . | 2.3 | 32.3 | 1.1 |
| Timothy hay . . . | 2.9 | 43.7 | 1.4 |
| Red clover hay . . | 7.4 | 38.1 | 1.8 |
| Alsike clover hay . | 8.1 | 41.7 | 1.4 |
| Alfalfa hay . . . | 10.6 | 37.3 | 1.4 |
| Cowpea hay . . . | 10.8 | 38.4 | 1.5 |
| Wheat straw . . . | .4 | 36.3 | .4 |
| Oat straw . . . . | 1.2 | 38.6 | .8 |
| Corn, grain . . . | 7.1 | 66.1 | 4.8 |
| Oats, grain . . . | 9.2 | 48.3 | 4.2 |
| Wheat bran . . . | 12.0 | 41.2 | 2.9 |
| Oil meal . . . . | 30.6 | 38.7 | 2.9 |
| Cotton-seed meal . | 37.0 | 16.5 | 12.6 |
| Whole milk . . . | 3.4 | 4.8 | 3.7 |
| Skim milk . . . | 3.0 | 5.0 | .3 |

This table shows the number of pounds of digestible nutrients found in 100 pounds of each kind of feed tabulated.

A fair price for these nutrients is as follows: Protein, \$4 per cwt. Carbohydrates, 40¢ per cwt. Fat, \$1 per cwt.

Find the total value of the digestible nutrients in 1 ton of the following:

**1.** Timothy hay. **2.** Red clover hay. **3.** Dry corn fodder. **4.** Corn silage. **5.** Wheat straw. **6.** Alfalfa hay. **7.** Corn, grain. **8.** Oats, grain. **9.** Wheat bran. **10.** Oil meal. **11.** Oat straw. **12.** Cowpea hay. **13.** Corn fodder, green. **14.** Red clover, green.

**Protein** is a *muscle former*, while **carbohydrates** and **fats** are *fat formers*.

The *ratio* of the muscle formers to the fat formers is called the **nutritive ratio**. A nutritive ratio of 1 : 3 is a *narrow ratio*, while one of 1 : 15 is a *wide ratio*.

A desirable nutritive ratio for a dairy cow is about 1 : 5.7; for a work horse, about 1 : 7; for swine, about 1 : 5.9.

Fat, as a food constituent, is regarded as $2\frac{1}{4}$ times more valuable than carbohydrates. Hence, the customary rule for finding the nutritive ratio is to add to the carbohydrates $2\frac{1}{4}$ times the fat, and divide by the protein.

**15.** What is the nutritive ratio of green corn fodder?

Protein, 1.1; carbohydrates, 12.1; fat, .4.

Thus, .4 lb. fat $= 2\frac{1}{4} \times .4$ lb. carbohydrates $= .9$ lb. carbohydrates; 12.1 lb. carbohydrates $+ .9$ lb. carbohydrates $= 13$ lb. carbohydrates; 13 lb. $\div$ 1.1 lb. $= 11.8 +$. Hence, the nutritive ratio is 1 to 11.8 +.

**16.** Find the nutritive ratio of a feed composed of 100 lb. timothy hay and 50 lb. wheat bran.

**17.** A farmer feeds his sheep 100 lb. of alfalfa hay to 10 lb. of bran. Find the nutritive ratio.

**18.** An Ohio farmer feeds his dairy cows 100 lb. red clover hay to 20 lb. of wheat bran. Find the nutritive ratio.

**19.** The best authorities suggest that a dairy cow weighing 1000 lb. and giving daily 22 lb. of milk should be fed 2.5 lb. protein, 13 lb. carbohydrates, and 0.5 lb. fat. Find the nutritive ratio of this feed.

**A balanced ration** is a mixture of different kinds of feed according to a nutritive ratio that will produce the desired end of the feeding, whether that end be growth, fat, milk, or butter.

**20.** A farmer mixed a balanced ration of 3 parts by weight red clover hay and 1 part oats grain. Find the nutritive ratio.

**21.** A feeder prepared a balanced ration for his hogs with the following parts by weight: $1\frac{1}{2}$ parts corn grain, 1 part oats grain. Find the nutritive ratio.

**22.** A balanced ration is as follows: 3 parts by weight timothy hay and 1 part corn grain. Find the nutritive ratio.

**23.** A farmer prepared a balanced ration consisting of 150 lb. of alfalfa hay and 100 lb. corn grain. Find the nutritive ratio.

**24.** Find the nutritive ratio of a balanced ration consisting of 200 lb. corn silage and 100 lb. red clover hay.

**25.** What is the nutritive ratio of a balanced ration that uses 200 lb. of dry corn fodder with 50 lb. of wheat bran?

**26.** A balanced ration for a dairy cow should be about 1 to 5.7. Is the nutritive ratio of the following balanced ration too wide or too narrow for dairy feeding? 200 lb. alsike clover hay; 100 lb. corn grain.

**27.** A balanced ration for work horses should have a nutritive ratio of about 1 to 7. Is the nutritive ratio of the following ration too wide or too narrow for work horses? 200 lb. timothy hay; 100 lb. oats grain.

**28.** A dairy man prepared a ration for his cows consisting of 100 lb. corn silage, 50 lb. timothy hay, and 100 lb. oats grain. Find the nutritive ratio. Is this ration too wide or too narrow?

**29.** A livery man feeds his horses on a ration consisting of 50 lb. timothy hay, 25 lb. corn grain, and 25 lb. oats grain. Find the nutritive ratio. Is this ratio too wide or too narrow?

**30.** A stockman preparing cattle for market feeds 100 lb. red clover hay to 30 lb. corn grain, 10 lb. oats grain, and 1 lb. oil meal. Find the nutritive ratio.

**31.** A Kansas farmer feeds his young cattle 100 lb. alfalfa hay to 10 lb. corn grain and 2 lb. cotton seed meal. Find the nutritive ratio.

## FERTILIZERS

Pounds of Fertilizing Constituents in One Ton

| Materials | Nitrogen | Phosphoric Acid | Potash |
|---|---|---|---|
| Timothy hay . | 19 | 16 | 47 |
| Clover hay . . | 39 | 13 | 44 |
| Alfalfa hay . | 46 | 13 | 35 |
| Corn, grain . | 34 | 13 | 9 |
| Corn, stover . | 12 | 9 | 39 |
| Bran . . . . | 51 | 62 | 35 |
| Oat straw . . | 13 | 7 | 38 |
| Milk . . . . | 12 | 4 | 3 |
| Butter . . . | 1.6 | 1 | 1 |
| Farm animals . | 53 | 37 | 3 |
| Farm manure . | 10 | 5 | 10 |

This table shows the number of pounds of plant food, such as nitrogen, phosphoric acid, and potash in one ton of the article named. All other constituents necessary to plant food can be obtained from the soil, if the water supply is sufficient and the heat conditions proper.

The leading artificial fertilizers are rock phosphate, ground bones, ammonia sulphate, nitrate of soda, animal refuse called "tankage," fish scrap, and potash salts, especially the sulphate and the muriate of potash.

**Lime** is used on land to neutralize the acidity of the soil.

**Nitrogen** in fertilizing material is sometimes, though not often, given as *ammonia*. 14 lb. of nitrogen correspond to 17 lb. of ammonia.

**Phosphoric acid** contains 43.6 % phosphorus; this constituent of fertilizer is sometimes, though rarely, given as phosphorus. A ton of timothy hay contains 16 lb. of phosphoric acid, which is equivalent to 7 lb. of phosphorus.

**Potash** contains 83 % potassium. This constituent is sometimes, though rarely, given as potassium. A ton of timothy hay contains 47 lb. of potash, which is equivalent to 39 lb. of potassium.

The **value** of fertilizing constituents depends largely on the forms in which they occur. An average value per pound is 15 cents for nitrogen, 3 cents for phosphoric acid, and 5 cents for potash.

The **composition** of farm produce varies greatly, so that the same kind of crop may be almost twice as rich in certain constituents in some cases as it is in other cases. The table given here is intended to show the *average* composition of each.

Through the growth of the leguminous plants, such as clover, alfalfa, beans, peas, etc., it is estimated that the farmer may obtain nitrogen from the air at a cost of about $2\frac{1}{2}$ ¢ per pound.

**1.** How much nitrogen is required to produce each of the following: 6 tons clover hay? 3600 lb. corn grain? 5500 lb. timothy hay? 4 tons alfalfa?

**2.** How much potash did a farmer sell in 1500 lb. of butter? in 760 lb. milk? in 5 steers averaging 1450 lb. each?

**3.** A farmer cut on an average 3 tons per acre of clover hay from a 5 acre field. How many tons of manure are required for the field to supply the phosphoric acid in the hay?

**4.** A farmer sold 50 tons of timothy hay in one season. How many tons of farm manure will balance the loss in potash?

**5.** When nitrogen is worth 16¢ per pound, phosphoric acid 5¢, and potash 6¢, find the fertilizing value of 5 tons of farm manure.

**6.** A commercial fertilizer when analyzed was found to contain by weight 3.4 % nitrogen, 6.6 % phosphoric acid, and 9 % potash. Find its value per ton, when nitrogen is worth 15¢ per pound, phosphoric acid 3¢ per pound and potash 5¢ per pound.

**7.** The analysis of a certain brand of bone meal showed the following: 1.6 % nitrogen, and 27.9 % phosphoric acid. What is its value per ton at 15¢ per pound for nitrogen and $3\frac{1}{2}$ ¢ per pound for phosphoric acid.

**8.** A farmer sold 500 bu. of corn grain (shelled corn 56 lb. to the bushel) from a certain field. How many tons of farm manure will supply the field with the phosphoric acid removed by the corn grain?

## SPRAYING PLANTS

**The Bordeaux mixture** used for killing fungus growths — such as black rot and scab of apples, black rot and mildew of grapes, brown rot of plums and cherries, etc. — was first discovered in Bordeaux, France. To make a solution for spraying: first, dissolve 4 pounds of copper sulphate, (blue vitrol) in 25 gallons of water; second, dissolve 4 pounds of freshly slaked stone lime in 25 gallons of water until it forms a milk of lime; third, pour the two solutions together and they are ready for use. The solution should be applied to the fruit while small. In this way a coating is formed over the fruit through which the fungus cannot grow.

NOTE. — In all directions for spraying, 50 gallons are considered a barrel.

**1.** For a vineyard of 25 acres of grapes suffering from black rot of the fruit, how many pounds of sulphate of copper and lime are necessary, if 100 gallons of Bordeaux mixture will spray $\frac{2}{3}$ of an acre?

**2.** If sulphate of copper can be bought at 6¢ per pound, and stone lime at $\frac{1}{2}$¢ per pound, what is the cost of materials for 500 gallons of Bordeaux mixture?

**3.** If a man with a team of horses at $3.50 per day, using a geared sprayer, can spray 3 acres of vineyard in a day, and Bordeaux mixture costs 27¢ per barrel, using 10 barrels per day, what is the cost of spraying 30 acres of grapes?

**4.** 600 bushels of apples from 40 trees without spraying for fungus growth were sold as follows: 100 bu. of perfect fruit at $1 per bushel, and 500 bu. of scabby and wormy fruit at 30¢ per bushel. If spraying at 15¢ per tree reversed the quantities of perfect and imperfect fruit, what would be gained by spraying?

**5.** The estimated value of a crop of grapes is $400 per acre. Thorough spraying costs $6 per acre. If without spraying, fungus diseases destroy 30 % of the crop, what is the net value of the spraying of 10 acres?

**6.** The average yield from 4 acres of sprayed grapes was 4770 lb. per acre, and the yield from unsprayed grapes was 3108 lb. The grapes were sold for $2\frac{1}{2}$¢ per pound. The cost of spraying was $8.60 per acre. What was the net gain on the 4 acres as a result of spraying?

A **Paris green** solution is used for killing chewing insects that destroy the plants by eating the leaves; and for destroying the codling moth to prevent wormy fruit. A solution for this purpose is made by mixing 4 ounces of Paris green with 50 gallons of water, a sufficient amount to spray 20 apple trees. When fungus diseases are to be controlled also, the Paris green is added to Bordeaux mixture.

**7.** What is the cost of materials to spray an apple orchard of 250 trees, for the apple worm, with Paris green at 32¢ per pound, using a barrel of the solution to 20 trees?

**8.** Mr. Wagner sprayed $8\frac{1}{4}$ acres of potatoes 4 times with Bordeaux mixture containing Paris green to control blight, rot, and insects. His expense account was as follows: 183 lb. copper sulphate at 8¢; 204 lb. lime at $1.15 per hundred; 10 lb. Paris green at 35¢; 48 hr. labor for a man at 20¢; 40 hr. labor for team at 15¢; wear of sprayer, $1.50. What was the cost of spraying per application per acre?

**9.** The $8\frac{1}{4}$ acres mentioned above yielded 1567.5 bu. potatoes. A portion of the same field that was left unsprayed yielded at the rate of 156 bu. per acre. Mr. Wagner sold his crop for 55¢ per bushel. What was his net profit per acre resulting from spraying?

**10.** Mr. Gould sprayed an orchard of 240 trees five times during the season, using altogether 3013 gal. Bordeaux mixture containing Paris green. The cost of materials was: Copper sulphate, 256 lb. at 8¢; lime, 347 lb. at $\frac{1}{2}$¢; Paris green, $15\frac{1}{2}$ lb. at 35¢; and the cost of applying was labor, 3 men, $4\frac{1}{2}$ days at $1.50; 1 team, $4\frac{1}{5}$ days at $1.50; wear on

spray machinery, \$5. Determine the cost of materials and the cost of applying per gallon and also per tree, including wear on machinery.

**11.** Thirty-four apple trees sprayed to control scab and codling moth yielded 90 bu. merchantable fruit and 32 bu. culls and windfalls. In the same orchard 21 unsprayed trees yielded 11 bu. merchantable fruit and 40 bu. culls and windfalls. The merchantable apples were sold for 67¢ per bushel and the culls and windfalls for 22¢ per bushel. The cost of spraying was 25¢ per tree. What was the net gain from spraying per tree?

**Kerosene Emulsion** is used to kill insects that suck the juices of plants. A solution for this purpose is made as follows: first, dissolve 2 ounces of soap in one quart of water by heating until the soap is dissolved; second, add two quarts of kerosene oil and stir for five minutes; third, add to this mixture 17 quarts of water to make 5 gallons of a 10% mixture. After thorough stirring, it is ready for use.

**12.** What would be the cost, at $12\frac{1}{2}$¢ per gallon, for enough kerosene to make 500 gallons of a 10% kerosene emulsion that might be used on plants in leaf? To make 128 gallons of an 8% emulsion?

**13.** A farmer has $5\frac{1}{4}$ A. of cabbage. He estimates that the yield will be 6500 heads per acre, and that the selling price will be $2\frac{1}{2}$¢ per head. Cabbage lice threaten a loss of 10%. If by applying kerosene emulsion at a cost for materials and labor of \$7.25 per acre, the loss can be reduced to 2%, what will be the gain from applying the treatment to the $5\frac{1}{4}$ A.?

**14.** A nurseryman used 10% kerosene emulsion to kill plant lice on his roses and young fruit stock. His expenditure for kerosene was \$4.50. If kerosene was 15¢ per gallon, how many gallons of the 10% mixture did he use? How many pounds of soap were used in making it?

The **San José scale** is a very small insect not larger than the head of a pin. It injures the tree by sucking the juice of the bark. A **lime-sulphur solution** for killing the insect is made as follows: first, dissolve 20 pounds of freshly slaked lime and 15 pounds of sulphur in 12 gallons of water, boiling the mixture 1 hour; second, add to this enough water to make 50 gallons of spraying mixture. This forms a hard covering over the tree, thereby preventing the scale insect from doing further harm.

**15.** If sulphur is worth 3¢ per pound and lime is worth $\frac{1}{2}$¢ per pound, and the labor of making lime-sulphur solution is worth 15¢ per barrel, what will be the cost of making enough solution to spray an orchard of 252 peach trees, 25 gallons being sufficient for 9 trees?

**16.** In the above mentioned orchard, the cost of applying is 85% of the cost of solution. How much does it cost per tree to spray the orchard? If one application per year for ten years prolongs the life of each tree four years and the average annual yield of fruit is worth $1.25, what is the net gain per tree due to prolonged life?

**17.** A lime-sulphur solution costs 75¢ per barrel to make, and a soluble oil preparation for killing scale insects costs $1.25 per barrel. A barrel of the former will cover 18 trees and a barrel of the latter 15 trees. What would be the saving from using a lime-sulphur solution on a ten acre orchard of pear trees set 140 to the acre, the cost of applying being equal and the solutions being equally effective insecticides?

**18.** The market value of a crop of peaches is $225 per acre. The total cost of production, including interest on investment, cost of marketing, etc., is $175 per acre. What would be the per cent of loss on the market value from fungus diseases and insect injuries, if the market value is so reduced as to equal the cost of production?

## TEST PROBLEMS

**1.** J. P. Black & Co.'s bank account is overdrawn $182.50. They place to their credit a 90-day note for $1500 with interest at 6 %, discounted 30 days after date. They then deposit $84.30 and check on their account to the extent of $341.65. Find their balance in bank, if the custom of this bank in discounting is to count both the day of maturity and the day of discount.

**2.** A cow is tied in the corner of a square field by a rope 2 rods long. Find the extent of the surface over which it can graze.

**3.** A cold air register is 18 in. by 30 in. What are the dimensions of a similar register to let in double the volume of cold air?

**4.** A clothier sold a suit marked $24, at 10 % off for cash, making a profit of 20 %. How much did the suit cost?

**5.** There are 50 persons in a schoolroom 36 ft. long, 30 ft. wide, and 15 ft. high. How many cubic feet of air space are there for each person? The law fixes the minimum of fresh air at 30 cubic feet per minute per person. How frequently must the room be filled with air to meet this requirement?

**6.** A municipality paved a street 48 ft. from curb to curb at $1.60 per square yard, under an ordinance that assessed property holders abutting on the street $\frac{1}{3}$ of the cost. Mr. James owns two lots each 25 ft. wide fronting on the street. He puts in a curb at 75¢ per linear foot and a sidewalk 12 ft. wide at $1.80 per square yard. How much is his bill for the improvements?

**7.** A conical glass is 4 inches in diameter and 6 inches high. How often can it be filled from a cylindrical vessel 4 inches in diameter and 12 inches high?

**8.** An article was sold at 25 % advance on the cost. The proceeds were invested in a second purchase which was afterwards sold for \$240. The latter sale was at a loss of 20 %. Find the selling price of the first article.

**9.** It cost \$280 to fence a field 80 rods long and 60 rods wide. How much less will it cost to fence a square field of equal area with the same kind of fence?

**10.** How much will it cost to bronze a globe 12 inches in diameter with gold leaf at 5¢ per square inch?

**11.** The diameter and altitude of a cone, the diameter of the base and the altitude of a cylinder, and the diameter of a globe are each 3 ft. Find the volume of each.

**12.** A town assessed at \$2,450,000 must raise a tax of \$7766.25. The poll tax is \$825. Find the tax rate, if the collector is allowed 5 % for collecting.

**13.** When the *Florida* collided with the *Republic* off the coast of Massachusetts, the wireless message summoned the *Baltic* to the wreck from a point 62 miles north and 84 miles east. What was the distance in a direct line?

**14.** A merchant sold 20 lb. more than $\frac{1}{3}$ of his butter. He then reduced the price 5¢ per pound, thereby reducing his profit \$2. How many pounds had he at first?

**15.** Mr. Adams sold 50 % of his stock at 20 % gain, and 80 % of the remainder at 25 % gain. What was his total gain, if the stock unsold cost \$1000?

**16.** A cow and a horse cost \$190, and $\frac{3}{5}$ the cost of the cow plus \$12 equal $\frac{3}{10}$ the cost of the horse. Find the cost of each.

**17.** A piece of steel in the form of a cylinder is 4 ft. long and 2 inches in diameter. How long is it when rolled into a bar 1 inch square?

**18.** A rectangular field that contains 40 acres is four times as long as it is wide. Find its dimensions.

**19.** A man bought a tenement containing 8 apartments for \$35,000. His taxes, repairs, and insurance cost him \$1157 annually. He rented 4 suites at \$40 per month, 2 suites at \$35 per month, and 2 suites at \$28 per month. What per cent did he realize on his investment?

**20.** If a piano is marked 80 % above cost, what per cent discount can be allowed from the marked price to realize 20 % profit?

**21.** The edge of a cube is 10 inches. Find its diagonal.

**22.** Find the shortest distance on the surface of the cube mentioned in Ex. 21 between two diagonally opposite corners.

**23.** A rectangular box is 8 ft. long, 6 ft. wide, and 4 ft. high. Find the dimensions of a similar box whose length is 12 ft.

**24.** What is the ratio of the volume of the two boxes mentioned in Ex. 23?

**25.** Find the duty in U.S. money on an invoice of leather from Paris, if the leather costs 12,350 francs and the duty is 35 % ad valorem.

**26.** Rose Adhorn and Co. imported 10 cases of woolen goods from England of 385 pounds each, invoiced at £408 per case. Find the total duty at 40¢ per pound and 60 % ad valorem.

**27.** A merchant sold goods for \$1242; one half he sold at an advance of 20 % on the cost, three tenths at an advance of $16\frac{2}{3}$ %, and the remainder at cost. How much did he pay for the goods?

**28.** Two successive trade discounts of 20 % and 10 % reduced a bill \$560. What was the original bill?

**29.** I sold my horse for \$180 and took in payment a 90-day note bearing interest at 5 %, dated Feb. 1, 1909. On April 5, I had the note discounted at the bank at 6 %. What were the proceeds of the note?

**30.** I asked for a grain binder 40 % more than it cost. I accepted $\frac{4}{5}$ of what I asked, and gained \$15. How much did I ask?

**31.** I have a rectangular field, the perimeter of which is 240 rods. It is twice as long as it is wide. How many acres does it contain?

**32.** A, B, & C have 1400 acres of land. $\frac{1}{2}$ of A's share is equal to $\frac{1}{3}$ of B's, and $\frac{5}{6}$ of B's share is equal to $\frac{2}{3}$ of C's. How many acres has each?

**33.** Mr. Fair bought five \$1000, 5 % San Francisco water bonds, due in 6 years, at 104, brokerage $\frac{1}{4}$ %. He sold the bonds one year before they were due at 102, brokerage $\frac{1}{4}$ %. Find his average annual rate of income.

**34.** A 90-day note for \$640, without interest, is discounted at the bank at 6 % on the day of issue. Find the proceeds of the note. What would be the proceeds, if the note read "with interest"?

**35.** I buy furniture listed at \$4400, getting trade discounts of 20 %, 10 %, and 5 %; I sell at 40 % above cost, taking a 120-day note in payment without interest. I have the note discounted at 6 % on the day of sale. How much do I gain?

**36.** A solid bar of silver weighs $6\frac{3}{4}$ lb. avoirdupois. Estimate its weight by Troy, and its value at 53¢ per ounce.

**37.** A steel ingot is 16 in. square and 8 ft. long. What length of steel bar will it make 4 in. thick and 6 in. wide?

**38.** Find the cost at 27¢ per load, to excavate the ground for a cellar 24 ft. wide, and 42 ft. long, the ground being excavated to a depth of $7\frac{1}{3}$ ft. at one end and 2 ft. 4 in. at the other end.

**39.** Mr. Samuels bought eight $1000, 4%, ten-year bonds when issued, interest payable semiannually, at $97\frac{1}{2}$, brokerage $\frac{1}{4}$%. After keeping the bonds $3\frac{1}{2}$ years, he sold them at $110\frac{1}{4}$, brokerage $\frac{1}{4}$%, and loaned the money at 6% interest for $2\frac{1}{2}$ years. Find his average annual income on the original investment for the 6 years.

**40.** A boy climbs a flag pole to the height of 40 feet. Another boy is standing on the ground 120 feet from the foot of the flag pole. If the second boy is 165 feet from a ball on the top of the pole, how far is the first boy from the ball?

**41.** The product of two equal factors multiplied by a third factor, 7, and that product by a fourth factor, 8, is 35,000. What are the equal factors?

**42.** Mr. Adams buys two $1000, 5-year, 5% bonds, interest payable semiannually, that have $3\frac{1}{2}$ years to run, for $1020 each, brokerage $\frac{1}{4}$%. Find his average annual income, if he keeps the bonds until due?

**43.** Mr. Brown sells five $1000, 10-year, 4% bonds, interest payable semiannually, that have $4\frac{1}{2}$ years to run, at 96¢ on the dollar, brokerage $\frac{1}{4}$%. Find the buyer's average rate of income, if the bonds are redeemed when due without brokerage.

# GENERAL REVIEW

## Oral Problems

1. A boy buys oranges at 25 cents per dozen and retails them at 3 for 10 cents. Find his gain per cent.

2. Mr. Henderson was earning $3.50 per day of 10 hours. His wages were reduced 20 %. Find the rate of wages per hour he then earned and his daily wages.

3. A man purchased 4 % bonds at par. How many dollars' worth did he purchase if his income from the bonds was $1200 per year?

4. A merchant buys potatoes at 60 cents per bushel and retails them at 30 cents per peck. Find his gain per cent.

5. A can do $\frac{2}{3}$ of a piece of work in 10 hours and B can do $\frac{3}{4}$ of it in 9 hours. In how many hours working together can they do the work?

6. If I sell $\frac{4}{5}$ of an article for $\frac{3}{4}$ of the cost, what per cent do I lose?

7. A book dealer's selling price is 25 % above cost. A special discount of 10 % is allowed to teachers. A teacher pays him $14.85 for books. Find the dealer's profit.

8. A merchant buys goods at a discount of 20 %, 5 % off, and sells them at the list price. Find the per cent of profit.

9. Read as per cents: $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{16}$, $\frac{1}{12}$, $.16\frac{2}{3}$, .05, $.06\frac{1}{4}$, $.37\frac{1}{2}$, .40, .25.

**10.** M and N have a profit of \$1700 in business. $\frac{2}{3}$ of M's capital equals $\frac{3}{4}$ of N's capital. How should the profit be divided?

**11.** A man sold $\frac{1}{4}$ of his farm to B, $\frac{2}{3}$ of the remainder to C, and the remaining 60 acres to D. How many acres were in the farm at first?

**12.** A coal merchant pays \$2 per ton for coal. The freight and delivery cost him 50¢ per ton. If he retails the coal at \$3 per ton, find his per cent of profit on the entire cost of the coal.

**13.** Wishing to find whether a corner was square, a man measured 8 feet along one side from the corner and 6 feet along the other side from the corner. What should be the distance between the ends of the lines if the corner is square?

**14.** Explain why, when it is 12 o'clock M. at Washington, it is approximately 9 o'clock A.M. at San Francisco and 5 o'clock P.M. at London (standard time).

**15.** A commission house received a consignment of peaches at 10% and retailed them at \$1.50 per bushel crate. If the commission was \$45, how many bushel crates were sold?

**16.** After depositing $\frac{3}{8}$ of his month's salary, a man pays with the remainder bills of \$8, \$12, and \$25, and has \$30 left. Find his monthly salary.

**17.** 30%, 10%, 5% off is equivalent to what single commercial discount?

**18.** 27 cents is $\frac{3}{4}$% of how many dollars?

**19.** A retailer bought pencil tablets at \$2.40 per hundred and sold them at 5 cents each. Find the gain per cent.

**20.** An automobile was sold for \$1200 at a loss of 25%. What would have been the loss per cent if it had been sold for \$1500?

**21.** The surface of a cube is 96 square inches. Find its volume.

**22.** A house and lot cost \$6000. It rents for \$360 a year. The taxes average 1%, insurance $\frac{1}{2}$%, and repairs \$30. What is the rate per cent of net income?

**23.** A, B, and C gain \$2100 in business. A's share of the profits is $\frac{3}{4}$ of B's, and C's share of the profits is equal to A's and B's together. Find the profits of each.

**24.** \$2900 is divided between two partners in the ratio of $1\frac{3}{4}$ to $1\frac{7}{8}$. Find each one's share.

**25.** A withdraws $\frac{2}{3}$ of his deposit in a bank. He then deposits $\frac{1}{3}$ as much as he has drawn out, and still has \$2500 in the bank. Find the amount in the bank at first.

**26.** A block of business houses was insured for $\frac{4}{5}$ of its value at 3%. If the premium was \$600, what was the value of the block?

### Written Work

**27.** Simplify: *a.* $\dfrac{4\frac{1}{2} \times 4\frac{1}{2} - 5\frac{1}{4}}{\frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{4}{5}}$ *b.* $\dfrac{\frac{2}{3} \text{ of } \frac{4}{5} + \frac{2}{3}}{\frac{3}{4} \text{ of } \frac{5}{6} + \frac{7}{8}}$

**28.** A commission of \$43.47 was charged for selling \$1242 worth of goods. What was the rate of commission?

**29.** How many yards are there in 3.1 mi.? in 7.9 Km.?

**30.** What are the proceeds of a note for \$400, when discounted for 95 days at 6%?

**31.** A lawyer's commission for collecting a bill, at 5%, was \$125.50. Find the amount of the bill.

**32.** The square of a number is 1024. What is its cube?

**33.** Simplify: *a.* $\dfrac{4 \times \frac{2}{5} \times 25}{\frac{3}{4} \times 1\frac{1}{5} \times 10}$ *b.* $\left(\dfrac{37\frac{3}{5}}{2\frac{2}{5}} \div \dfrac{2\frac{1}{3}}{2\frac{1}{3}}\right) \times \dfrac{1\frac{1}{2}}{2\frac{1}{3}}$

34 A bankrupt's liabilities are \$15000 and his assets \$9000. How much does a creditor receive whose claim is \$6000, no allowance being made for court costs?

35. How many cakes of soap 4 in. long, 2 in. wide, and $1\frac{1}{2}$ in. thick can be packed in a box 2 ft. long, 1 ft. wide, and 1 ft. high?

36. Write in figures the number eleven million, eleven thousand eleven, and eleven millionths.

37. What is the square root of $14\frac{1}{16}$?

38. Find the sum of the quotients:

| | | |
|---|---|---|
| $3 \div 3$ | $3 \div .03$ | $300 \div 30$ |
| $3 \div .3$ | $.003 \div 3$ | $30 \div 300$ |
| $.03 \div 3$ | $30 \div .03$ | $.03 \div 30$ |

39. In a town \$3500 was raised from a tax of 14 mills. What was the assessed valuation of the property?

40. The perimeter of a square field is 320 rods. How many acres does it contain?

41. Reduce to decimals: $\frac{2}{5}$, $\frac{3}{8}$, $\frac{7}{25}$ $\frac{9}{16}$, $\frac{3}{4}$, $\frac{5}{32}$.

42. A man borrowed \$360 September 1, 1903, and paid the note Jan. 16, 1906, with interest at 7 %. How much was paid at settlement?

43. Find the omitted term in each:

$$5 : 8 = 15 : (\ ) \qquad \tfrac{1}{2} : \tfrac{5}{8} = (\ ) : \tfrac{5}{9}$$
$$(\ ) : 12 = 5 : 4 \qquad 1\tfrac{1}{2} : (\ ) = 3\tfrac{3}{8} : 4\tfrac{1}{2}$$

44. I pay \$36 for insuring my house at $\frac{3}{4}$ %. For how much is the house insured?

45. Find the exact interest of \$180 from April 1, 1904 to August 25, 1904, at 6 %.

46. The assessed valuation of a town is \$875000. What rate must be levied to raise \$10937.50?

**47.** A 60-day note for \$1000 is discounted at a bank at 8 % on the date of issue. Find the proceeds.

**48.** A speculator buys a tract of land $4\frac{1}{2}$ miles square. How many sections does it contain? How much will it cost to put a fence around it at \$0.50 per rod?

**49.** Find the cost of the following bill of lumber:

| | | |
|---|---|---|
| 30 pieces | $30' \times 6'' \times 12''$ | at \$30 per M. |
| 60 pieces | $24' \times 6'' \times 8''$ | at \$24 per M. |
| 120 pieces | $18' \times 4'' \times 6''$ | at \$20 per M. |
| 150 pieces | $16' \times 3'' \times 4''$ | at \$18 per M. |

**50.** A commission merchant sold 2240 pounds of butter at 24¢ a pound. His commission for selling was 5 % and he paid freight charges amounting to \$5.72. How much did he remit to the shipper?

**51.** A barn valued at \$1800 is insured for $\frac{3}{4}$ of its value, at $1\frac{1}{2}$ % for a term of 3 years. Find the average annual cost of insurance.

**52.** \$200 was borrowed at 5 % on Oct. 1, 1904. When it was paid, it amounted to \$217.50. On what date was it paid?

**53.** Two successive discounts of 20 % and 15 % reduce a bill to \$306. How much was the original bill?

**54.** From a piece of land 45 rods square, I sold 145 square rods. What is the value of the remainder, at \$60 an acre?

**55.** Simplify: *a.* $\dfrac{1\frac{1}{2} - \frac{2}{3} + 1\frac{1}{4} - 1\frac{5}{6}}{2\frac{1}{4} + \frac{7}{8} - 1\frac{1}{3} + \frac{1}{12}}$ *b.* $\dfrac{3\frac{3}{5} \times 4\frac{1}{6} \times \frac{8}{15}}{7\frac{1}{2} \times 8\frac{1}{4} \times \frac{5}{11}}$

**56.** A note for \$320, dated June 1, 1902, falls due September 16, 1904. What amount will pay the note if it draws $4\frac{1}{2}$ % simple interest?

**57.** What is the tax on property assessed for \$12480 at \$13.50 a thousand?

**58.** A broker's purchase at 105, with his brokerage at $\frac{1}{8}$%, was \$6307.50. How many shares were bought?

**59.** In what time will any principal double itself at 5%? at 6%? at 8%?

**60.** A street 40 rods long and 40 feet wide is to be graded down on an average $1\frac{1}{2}$ feet. How much will the excavating cost at 27¢ a cubic yard?

**61.** A coal dealer bought 260 tons of coal at \$3.50 per long ton and sold it at \$4.20 per short ton. Find the gain.

**62.** A wholesale merchant imports 30000 yards of Brussels carpet, 27 inches wide, purchased in Belgium at 40¢ per yard. The duty is 18¢ per yard and 40% ad valorem. Find the wholesale price in the United States, if a profit of 25% on the yard is made.

**63.** I invested \$5000 in bank stock at 156 and sold it at 167, brokerage $\frac{1}{4}$% in each case. Find the gain.

**64.** The specific gravity of oak is .934. Find the weight of an oak sill 24 ft. × 1 ft. × 1 ft.

**65.** The residence of Mr. Daniels valued at \$8000 was insured for 3 years at 90 cents per \$100 on 80% of its valuation. How much was the average annual premium?

**66.** A man 45 years old takes out a \$5000 life insurance policy payable in 20 years. If he pays an annual premium of \$36.87 on the \$1000, and lives till the policy falls due, how much will his insurance cost him, estimating that he gets back in dividends an average of 20% of the premiums paid?

**67.** A train running 30 miles an hour is 54 minutes in going from one city to another. If it makes 3 stops of 4 minutes each, how far apart are the cities?

**68.** A square field contains 10 acres. How much longer is its diagonal than its side?

**69.** A man desires to realize 6 % on his investment. How much should he pay for borough bonds bearing $4\frac{1}{2}$ % interest?

**70.** In an examination 150 questions were asked each of 5 members of a class. The first answered 140, the second 135, the third and fourth 120 each, and the fifth 110. Find the average per cent of the class.

**71.** A farm roller 8 ft. long and $2\frac{1}{2}$ ft. in diameter will pass over how much surface in 100 revolutions?

**72.** The floor of a room is 16 ft. × $13\frac{1}{2}$ ft. Find the cost of carpeting it with carpet $\frac{3}{4}$ of a yard wide, laid lengthwise and costing $1.20 a yard, allowing 9 inches for matching on each strip except the first.

**73.** A grocer pays $16 for 5 bushels of cranberries, and sells them so as to gain 30 %. What is the selling price per quart?

**74.** I loaned $450 at 5 %, and received $502.50 when it was paid. For how long was it loaned?

**75.** The face of one side of a cube is a surface of 100 square feet. Find the volume.

**76.** A tax of $6181.40 is to be assessed upon a town. There are 620 persons subject to a poll tax of $1.25 each. The property assessment is $675800. Find the tax on Mr. Anderson's property, which is assessed at $4750 if he pays for one poll.

**77.** A man buys a house and lot for $3500; he pays $1000 cash and gives a mortgage for the balance at 6 %. At the end of 9 months he sells the house and lot for $4500, paying the interest due on the mortgage at the time of sale. How much does he realize on his investment?

**78.** I desire to invest \$9000. I can buy 7 % stock at 20 % premium or loan the money at 6 %. Which is the better, both being safe investments?

**79.** Solve the following equations:

$$6\,\%\text{ of }\$100 = 5\,\%\text{ of }(\$\quad).$$
$$37\tfrac{1}{2}\,\%\text{ of }\quad 64 = (\quad\%)\text{ of }120.$$

**80.** \$1650 yields \$530.75 interest in 5 years, 4 months, 10 days. Find the rate.

**81.** What sum of money, at 6 % simple interest, will produce in 2 years, 6 months, the same interest that \$900 will produce in 3 years, 4 months, at 5 %?

**82.** The slant height of a church spire is 48 feet, and its base is a hexagon 6 feet on each side. Find the cost of painting it at 40¢ a square yard.

**83.** On a bill of goods amounting to \$900, I am offered a discount of 25 %, or two successive discounts of 15 % and 10 %. Which would be more advantageous for me to accept, and how much more?

**84.** My gas meter, Jan. 1, registered 11800 cu. ft.; Feb. 1, 35800 cu. ft. I paid the bill before Feb. 10, receiving a discount of 2¢ on the even thousand. At 27¢ a thousand cubic feet, how much did I pay for the gas used in January?

**85.** Simplify: *a.* $\dfrac{1\frac{4}{5} \div 3\frac{1}{2}}{4\frac{2}{3} \div 5\frac{5}{6}} \times \dfrac{2\frac{2}{9} \div 3\frac{1}{3}}{3\frac{1}{8} \div 7\frac{1}{2}}$  *b.* $\dfrac{7\frac{1}{2} + \frac{2}{3}}{5\frac{3}{8} - 1\frac{1}{4}} \times \dfrac{22}{49}$

**86.** I imported from Canada 7500 yards of flannel valued at 80¢ a yard, and weighing 1480 pounds. Specific duty 22¢ per pound, and ad valorem duty 30 %. Find the amount of duty paid.

**87.** The taxes on a property last year were \$42, which were $\frac{1}{8}$ less than this year. Find the per cent of increase in taxes.

**88.** A street $\frac{1}{2}$ mile long and 30 feet wide is paved and curbed. The paving costs \$3 a square yard, and the curbing 30¢ a linear foot. Find the entire cost.

**89.** Find the sum of the quotients:

| | | |
|---|---|---|
| $.01 \div .001$ | $.1 \div 10$ | $.001 \div .1$ |
| $10 \div .01$ | $.01 \div 10$ | $.01 \div .001$ |

**90.** The specific gravity of milk is 1.032. Find the weight of 48 gallons of milk.

**91.** A hotel and farm sold for \$6000 each. The hotel was sold at a gain of 20%, and the farm at a loss of 20%. Find the gain or loss on both sales.

**92.** The cost of insuring a dwelling at $\frac{3}{4}$% is \$33.75 a year, and the cost of insuring the furniture at 1% is \$12.75. Find the amount of each policy.

**93.** Which produces the greater per cent of income and how much, 4% bonds at 75 or 5% bonds at 90?

**94.** Find the sum of the square roots of:

| | | |
|---|---|---|
| .5625 | 226.5025 | $110\frac{1}{4}$ |
| .0016 | 100.2001 | $148\frac{21}{25}$ |

**95.** I bought stock at $89\frac{7}{8}$, brokerage $\frac{1}{8}$%, and after receiving a dividend of 4%, sold at $104\frac{7}{8}$, brokerage $\frac{1}{8}$%, clearing \$150. Find the amount of stock purchased.

**96.** A town expends for improvements \$6894. The assessed valuation is \$480000. Find the rate levied to cover the expense, including the collector's commission estimated at \$306.

**97.** A swimming pool is 30 meters long, 14 meters wide, and averages 1.5 meters in depth. Find the number of kiloliters of water it contains and its weight in kilograms.

**98.** A Mexican ranchman purchased a tract of land in the form of a rectangle 10 kilometers in length and 4 kilometers in width. Find the number of acres in it.

**99.** The oxygen in the air is to the nitrogen as 21 to 79. Find the number of cubic feet of each gas in a schoolroom whose inside dimensions are 30 ft. $\times$ 24 ft. $\times$ $12\frac{1}{2}$ ft.

**100.** What sum will cancel a note for \$122.50 bearing interest at 6%, dated April 10, 1903, and maturing September 4, 1905?

**101.** A garrison of 800 men have provisions for 90 days. A reënforcement arrives at the end of 40 days, and the provisions last only 40 days longer. Find the number of the reënforcement.

**102.** Simplify: $\dfrac{\frac{2}{3}}{(1\frac{1}{5} \times \frac{5}{9}) \div 10\frac{1}{2}} \times \dfrac{3}{4} \text{ of } \dfrac{1\frac{1}{2} \times 4\frac{1}{5}}{2\frac{1}{4} \times 5\frac{1}{3}}.$

**103.** What decimal bears the same ratio to .05 that $\frac{3}{8}$ does to $1\frac{1}{2}$?

**104.** A savings bank pays 4% interest compounded semi-annually, the interest periods being April 1, and October 1. I deposited \$100, April 1, 1903, and an equal amount semi-annually to and including October 1, 1905. What amount had I on deposit April 1, 1906?

**105.** An insolvent debtor pays 40 cents on the dollar. How much will a creditor receive whose claim is \$960, after paying his attorney 10% for collecting it?

**106.** A rectangular field whose width is $\frac{3}{4}$ of its length contains $7\frac{1}{2}$ acres. Find the distance between the opposite corners.

**107.** I loaned $\frac{3}{8}$ of a certain sum at 6%, and the remainder at 5%. The entire income was \$322.50. Find the sum loaned.

**108.** A gentleman wishes to invest in $4\frac{1}{2}$ % bonds, selling at 102, so as to provide for a permanent income of \$1620. How much should he invest?

**109.** From one tenth take one thousandth; multiply the remainder by 10000; divide the product by one million, and write the answer in words.

**110.** A drugget 9 ft. by 12 ft. covers 50 % of the floor of a room $13\frac{1}{2}$ ft. wide. Find the length of the room.

**111.** A mechanic had his wages twice increased 10 %. Find his wages before the first increase, if he now receives \$4.84 per day.

**112.** A house which had been insured for \$3000 for 9 years at $1\frac{3}{4}$ % for a term of three years was destroyed by fire. How much did the money received exceed the premiums paid?

**113.** A natural gas company declares a semiannual dividend of 4 % on a capital stock of \$150,000. Find the yearly dividend of a stockholder who owns 36 shares, par value \$50 a share.

**114.** A merchant in Denver, Col., buys a New York draft for \$600, at $\frac{1}{4}$ % exchange, and mails it in payment of a bill in Memphis, Tenn. Find the amount paid for the draft.

**115.** How long must \$120 be at interest at 6 % to earn \$34.64?

**116.** If a clerk's wages are \$48 a month, when he works 8 hours a day, how much should he receive for 9 months' work, of 10 hours a day?

**117.** Each side of a roof is 30 ft. long and 18 ft. wide. How many shingles 16 in. long and 4 in. wide, laid $\frac{1}{4}$ to the weather, will be required to cover the roof?

**118.** City bonds bearing 4 % interest are sold at 12 % premium. Find the rate per cent the buyer gets on his investment.

**119.** The capital stock of a company is \$ 50,000. There is a deficit of \$ 4000 in the earnings. I own 80 shares. Find the amount I must pay if an assessment is levied.

**120.** A lawyer in collecting a note of \$ 3000, compromised by taking 80 % and charged 5 % for his fee. Find his commission.

**121.** A man's income is $\frac{1}{6}$ of his capital. His taxes are $2\frac{2}{5}$ % of his income. Find the amount of his capital if he pays \$ 24 taxes.

**122.** The specific gravity of iron is 7.80. Find the weight of an iron bar 12 ft. long and 2 in. square.

**123.** Find the result of $\frac{(.05 \times 1.25 + .1875)}{(.4 \div 5 + .17)} \times 96 - 16.5.$

**124.** The inside measure of a cubical box is 4 in. on each side. A sphere 4 in. in diameter is placed in the box. Find the per cent of space unfilled.

**125.** At \$ $1\frac{1}{2}$ a rod, it cost \$ 240 to fence a square field. Find the cost of fencing a rectangular field of equal area whose sides are to each other as 1 is to 4.

**126.** A piano listed at \$ 650 was sold at a discount of 40 % and 20 %. If the freight was \$ 3.25 and drayage \$ 5, what was the net cost of the piano?

**127.** A merchant sold a bill of goods amounting to \$ 3600 and took a 90-day note for it. Fifteen days later he sold the note at a bank at 6 % discount. How much did he receive for the note?

**128.** A boat in crossing a river 400 feet wide, drifted with the current 300 feet. How far did the boat go?

**129.** Mr. Brown owed Mr. Smith \$2000 which he was unable to pay; but he gave him two 90-day notes covering the amount. One was for \$1000 without interest; the other with interest at 6 %. Mr. Smith had both notes discounted at a bank at 7 % on the day they were given. How much cash did he receive?

**130.** A merchant owing a bill of \$1250 in New York is asked to send a draft in settlement of the account. The merchant has only \$868 in bank and holds a note of \$900 due in 30 days without interest. This he has discounted at 6 % for 20 days and the proceeds is placed to his credit. He buys a draft at $\frac{1}{8}$ % exchange, giving his check for the amount. How much does he then have in bank?

**131.** Mr. Franks has a promissory note of \$800 dated July 1, 1906, due in one year with interest at 6 %, against Boyd Emerson, on which are the following endorsements:

| | |
|---|---|
| Jan. 1, 1907, \$50.00 | Jan. 1, 1908, \$150.00 |
| Dec. 1, 1907, 25.00 | Apr. 1, 1908, 200.00 |

Write the note with the indorsements and find the balance due Aug. 1, 1908.

**132.** A San Francisco banker discounts a draft for \$3000 payable at Portland, Oregon, 90 days after sight. Exchange $\frac{1}{10}$ %, discount 8 %. Find the proceeds.

**133.** The United States government pays exact interest at 5 % from April 1 to Oct. 10 on a claim of \$63500. Find the interest the government pays.

**134.** Find the cost of carpeting a hall 30 ft. by 50 ft. with carpet 27 inches wide, laid lengthwise, at \$1.10 per yd., surrounded by a carpet border 18 inches wide, at \$1.10 per yd., allowing 6 inches for matching on each strip except the first.

**135.** A cable message was sent at 6:15 A.M. from New York to London. It was delivered 23 minutes 20 seconds after being sent. At what time was it delivered?

**136.** Mr. Coll leased some property for three years at \$2400 per year. His commission for leasing was 1 % of the first year's rent and his commission for collecting 5 % of each year's rent. Find the agent's entire commission for the three years, if all the rent was collected.

**137.** A commission merchant was offered \$1800 per year salary and 2 % on all sales above \$40000, or 5 % on all sales. He chose the latter and sold \$85000 worth of goods. Did he gain or lose, and how much, by so doing?

**138.** 2000 yards of silk when imported cost 215 ¢ per meter. If sold at \$2.25 per yard, find the gain.

**139.** A railroad tank along the line of the Paris and Lyons railway is 3.5 meters in diameter and 6 meters in height. Find the number of kiloliters it contains.

**140.** The specific gravity of iron is 7.80. Find the weight in kilograms of a bar of iron 1 meter long, 1 decimeter wide, and 5 centimeters in thickness.

**141.** A bookkeeper's income is \$2700 per year. His expenses average \$50 per month. If he deposits the balance every six months in a savings bank, how much will he have in the bank, at 4 % interest, compounded semiannually, after 3 deposits?

**142.** A ship sets sail at Seattle, Nov. 15, at 1 P.M., 120th meridian time, and arrives at Canton in 21 da. 5 hr. 18 min. Find the solar time of arrival in Canton.

**143.** Sound travels 1120 ft. per second. The thunder from a flash of lightning was heard 8 seconds after the flash was seen. How far distant was the cloud?

**144.** At what price must a bank stock paying 6 % annual dividends be purchased so as to net the purchaser 5 % income on his investment?

**145.** An agent's commission at $2\frac{1}{2}$ % is $57.85. What must be the amount of a check mailed to cover the purchase?

**146.** A railway company declared a $1\frac{3}{4}$ % quarterly dividend. How much did the purchaser pay for the stock, if it yielded him 10 % on the amount invested?

**147.** Ames Bros., brokers, bought for me 10 shares of Delaware & Hudson, selling at $129\frac{7}{8}$ % premium, par $100. Write the check in payment to Ames Bros. for the stock.

**148.** A clerk made the following deposits in a savings bank, at 4 % interest, payable January 1 and July 1: January 1, 1906, $500; July 1, 1906, $350; July 1, 1907, $200. On January 2, 1907 he drew out $100. What was his balance July 1, 1908?

**149.** In one section of the Bessemer Railroad there was laid in one year 6 miles of double track. The rails weighed 100 pounds to the yard and the market price was $32.50 per long ton. The ties cost delivered 69 cents each and were laid on an average of one tie to every two feet. Find the cost of the rails and ties.

**150.** A commission broker was to receive 5 % on the first $50000 from a sale of coal land and 2 % for the remaining amount of the sale. His entire commission amounted to $4000. Find the total amount of the sale.

**151.** Mr. Adams bought a property for $20000. He expended $4000 in improvements. The repairs each year averaged $250, the insurance and taxes $2\frac{1}{2}$ % on $\frac{4}{5}$ of the original cost of the property. For how much a year must he rent the property to realize 6 % net on his investment?

# OPTIONAL SUBJECTS

## PRESENT WORTH AND TRUE DISCOUNT

A owes B \$106, due in one year without interest. If A pays the debt to-day, \$100 will cancel it, since \$100 at 6 % will amount to \$106 when the debt is due. \$106 is the *debt;* \$100 is the *present worth;* and \$106 minus \$100, or \$6, is the *true discount.*

The **present worth** of a debt, due at a future time without interest, is a sum of money which, at a given interest, will amount to the debt when it becomes due.

The **true discount** is the difference between the *debt* and its *present worth.* True discount is seldom used in business.

### Written Work

**1.** Find the present worth and true discount of a debt of \$287.50, due in 2 yr. 6 mo. without interest, money being worth 6 %.

$\$.15$ = the interest of \$1 for $2\frac{1}{2}$ yr. at 6 %.

$\$1.00 + \$.15 = \$1.15$ = the amount of \$1 for $2\frac{1}{2}$ yr. at 6 %.

$\$287.50 \div \$1.15 = 250$, and

$250 \times \$1.00 = \$250$, the present worth of \$287.50.

Since the present worth of \$1.15 is \$1, the present worth of \$287.50, which is 250 times \$1.15, is 250 times $\$1 = \$250$.

$\$287.50 - \$250.00 = \$37.50$, the true discount.

### Comparative Study

**Bank Discount** is the interest paid in advance upon the *value of a note, or a debt, at maturity;* **true discount** is the interest on the *present worth* of the note, or debt, for the given time. In *bank discount* notes *may, or may not, bear interest;* in *true discount* debts are *without interest.*

The *present worth* corresponds to the *principal;* the *true discount,* to the *interest;* the sum due at a future time, to the *amount.*

**2.** What principal will, in 3 yr. and 6 mo., at 6 %, amount to \$344.85? (The principal is the present worth; the interest is the true discount.)

**3.** Find the present worth of \$517.50, due in 2 yr. and 6 mo., without interest, money being worth 6 %.

**4.** A merchant buys goods amounting to \$355.25 and agrees to pay for them in 3 mo. What cash sum will pay the bill, money being worth 6 %?

**5.** A farmer is offered \$5000 for his farm, or \$5600, payable one half in cash and the balance in 1 yr. without interest. How much is the second offer better than the first, money being worth 6 %?

**6.** Find the true discount of \$1350, due in 1 yr., 4 mo., without interest, money being worth 6 %.

**7.** What is the difference between the true discount of \$575 due in $2\frac{1}{2}$ yr. without interest, money being worth 6 %, and the simple interest of \$575 for $2\frac{1}{2}$ yr. at 6 %?

**8.** A purchaser is offered a horse for \$195 cash, or \$206 due in 6 mo. without interest. Which is the better offer and how much?

## FOREIGN EXCHANGE

**Foreign exchange** is a method of paying or collecting bills in foreign countries without the actual transfer of money.

A bill may be paid in a foreign country by a **postal money order**, by an **international express money order**, by a **telegraphic money order**, or by a foreign draft, called a **bill of exchange**.

Bills of exchange are usually issued in duplicate and numbered *first* and *second* of exchange. By sending them by different mails, the payee is almost certain to receive one. Each contains a condition that it shall be void after the other is paid.

Bills in foreign countries are collected by **commercial drafts** in the same manner as in domestic exchange.

A **foreign draft**, or **bill** of exchange, is similar to a domestic bank draft and is payable in the money of the country on which it is drawn. Thus, a bill of exchange on Paris is payable in *francs.*

**Premium and discount.**

In *domestic exchange*, there is practically no premium or discount on money, except during financial panics.

In *foreign exchange*, the premium or discount varies according to the demand for, and the supply of, money.

**English** exchange is quoted as so many dollars to the pound. Thus, a quotation of 4.91 means that a foreign draft for £ 1 will cost $ 4.91.

**French** exchange is quoted either at so many francs to the dollar, or at so many cents to the franc. Thus, a quotation of 5.8 means that a draft for $1 will purchase 5.8 francs; or a quotation of $20\frac{1}{10}$ means that a draft for 1 franc will cost $20\frac{1}{10}$¢.

**German** exchange is quoted at so many cents to the 4 marks or to the mark. Thus, a quotation of 98 means that 98¢ will purchase a draft for 4 marks; or a quotation of 24.2 means that a draft for 1 mark will cost 24.2 cents.

The **par of exchange** between two countries is the standard value of the monetary unit of one expressed in that of the other. The **English par of exchange** is $4.8665. A quotation of 4.90 is *above par* and one of 4.84 is *below par.* The **French par of exchange** is about $5.18\frac{1}{8}$, or 19.3. The **German par of exchange** is about 95.2, or 23.8.

The following is a newspaper quotation of commercial foreign exchange:

| | 60 Days | Demand |
|---|---|---|
| Sterling | 4.81 . . . . | 4.87 |
| Germany, reichsmarks | $.93\frac{3}{4}$ . . . . | $.94\frac{7}{8}$ |
| France, francs | 5.21 . . . . | $5.16\frac{7}{8}$ |

This means that a £ 1 draft payable on demand will cost $ 4.87; or $4.81 payable in 60 days.

## Written Work

**1.** Find the cost of a demand draft, at the quotation given, for £ 25.

Cost of £ 1 = $ 4.87; cost of a £ 25 draft = 25 × $ 4.87 = $ 121.75.

**2.** Find the cost of a 60-day draft for 4480 marks.

Cost of 4 marks = $93\frac{3}{4}$¢; cost of 4480 marks = $\frac{4480}{4} \times 93\frac{3}{4}$¢ = \$1050.

**3.** Find the cost of a 60-day draft for 3200 francs.

Cost of 5.21 francs = \$1; cost of 3200 francs = $\frac{3200}{5.21} \times \$1 = \$614.21$.

Find the cost of a 60-day draft for:

**4.** £ 25

**5.** 275 M.

**6.** 6500 fr.

**7.** £ 255.5

**8.** 635.5 M.

**9.** 398.2 fr.

**10.** Find the cost of a sight draft on London for £ 400, at the foreign quotation given.

**11.** Find the face of a demand bill of exchange on Paris for \$2500, at the foreign quotation given.

**12.** What is the cost of a demand draft on Hamburg for 1125 marks, exchange $94\frac{3}{4}$?

**13.** What is the cost of a draft on Lyons for 14,000 francs at 5.19?

**14.** A merchant buys a London draft 60 days after sight for £ 95. If exchange is 4.82, find the cost of the draft.

**15.** How large a bill of exchange on Berlin can be purchased for \$1590, exchange being 98?

**16.** A mechanic has \$1500 with which to purchase a London draft. If exchange is 4.845, how much is the face of the draft in English money?

**17.** An American tourist bought a *letter of credit* on London for £ 300 at \$4.88 and 1 % commission. How much, in United States money, did this letter of credit cost him? If he drew £ 50 in Paris, how many francs did he get in exchange, the pound being valued at 25.2 francs?

## COMPOUND PROPORTION

A **simple ratio** is a ratio of two numbers; thus, $9:3$ is a simple ratio.

A **compound ratio** is the product of two or more simple ratios; thus, $(9:3) \times (6:2)$, or $\left(\frac{9}{3} \times \frac{6}{2}\right)$, or $\left\{\begin{matrix}9:3\\6:2\end{matrix}\right\}$ is a compound ratio.

A **compound proportion** is a proportion in which one or both ratios are compound; thus, $\left\{\begin{matrix}9:3\\6:2\end{matrix}\right\} = 18:2$ is a compound proportion.

### Written Work

**1.** If 3 men earn \$24 in 4 days, how much can 6 men earn in 3 days?

Since the answer is to be *dollars*, the second ratio in this compound proportion is \$24 : \$$x$.

The first ratio in this proportion is compound. If 3 men earn \$24 in a certain time, 6 men can earn *more* in the same time; $x$, then, represents a *larger* sum than \$24, and the *first simple ratio* in the compound ratio is $3:6$. If a given number of men earn \$24 in 4 days, in 3 days they will earn *less*; $x$, then, represents a smaller sum than \$24, and the *second simple ratio* in the compound ratio is $4:3$.

Combining these two ratios, the *compound ratio* is $\left\{\begin{matrix}3:6\\4:3\end{matrix}\right\}$ and the compound proportion is $\left\{\begin{matrix}3:6\\4:3\end{matrix}\right\} = \$24 : x$. Then, $x = \frac{6 \times 3 \times \$24}{4 \times 3} = \$36$.

**2.** If 6 men earn \$75 in 5 days, how much can 12 men earn in 3 days?

**3.** If 8 men, in 10 days of 9 hours each, earn \$280, how much can 9 men earn in 5 days of 8 hours each?

**4.** If 24 men dig a trench 72 rods long, 3 feet wide, and 5 feet deep in 12 days, how long a trench, $2\frac{1}{2}$ feet wide and 3 feet deep, can 18 men dig in 6 days?

## CUBE ROOT

### Comparing Roots and Periods

The **cubes** of the smallest and the largest integers composed of one, two, and three figures are as follows:

| | | |
|---|---|---|
| $1^3 = 1$ | $10^3 = 1000$ | $100^3 = 1{,}000{,}000$ |
| $9^3 = 729$ | $99^3 = 970{,}299$ | $999^3 = 997{,}002{,}999$ |

**1.** Separate each of these perfect cubes into periods of three figures each, beginning at the right; thus, 997′002′999.

**2.** How does the *number of periods* in each cube compare with the number of figures in the corresponding roots?

*The number of periods of three figures each, beginning at units, into which a number can be separated, equals the number of figures in the cube root of the number.*

NOTE. — The left-hand period may contain one, two, or three figures.

**3.** How many figures are there in the cube root of 46,656? 1,030,301? 12,326,391?

**4.** Cube 25. $25 = 20 + 5$; hence, it may be cubed in two ways, thus:

$$
\begin{array}{rcr}
25 & = & 20 + 5 \\
\underline{25} & = & \underline{20 + 5} \\
125 & = & (20 \times 5) + 5^2 \\
\underline{50 \text{ tens}} & = & \underline{20^2 + (20 \times 5)} \\
625 & = & 20^2 + 2(20 \times 5) + 5^2 \\
\underline{25} & = & \underline{20 + 5} \\
3125 & = & (20^2 \times 5) + 2(20 \times 5^2) + 5^3 \\
\underline{1250 \text{ tens}} & = & \underline{20^3 + 2(20^2 \times 5) + (20 \times 5^2)} \\
15625 & = & 20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3
\end{array}
$$

Or, representing the tens by $t$ and the units by $u$, we have the formula:

$$(t + u)^3 = t^3 + 3\,t^2u + 3\,tu^2 + u^3.$$

Observe that 15,625, the cube of 25, is composed of four **partial products :**

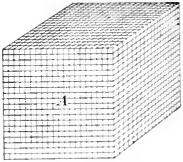
$$25^3 = \left\{\begin{array}{lr} (1) & t^3 = 20^3 = 8000 \\ (2) & 3\,t^2u = 3(20^2 \times 5\,) = 6000 \\ (3) & 3\,tu^2 = 3(20\ \times 5^2) = 1500 \\ (4) & u^3 = 5^3 = \ \ 125 \end{array}\right\} = 15{,}625$$

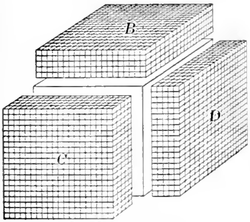
**5.** Find the cube root of 15,625, or find the edge of a cube whose volume is 15,625 cubic units.

| | | | |
|---|---|---|---|
| | | 15′625 | 20 |
| | $20^3 =$ | 8 000 | 5 |
| Trial divisor, | $3 \times 20^2 =$ 1200 | 7 625 | 25 |
| | $3 \times 20 \times 5 =$ 300 | | |
| | $5^2 =$ 25 | | |
| Complete divisor, | 1525 | 7 625 | |

Separate the number into periods of three figures each.

Since 15,625 contains two periods, its cube root is composed of two figures, *tens* and *ones*. As the cube of *tens* is *thousands*, the largest cube found in 15 *thousands* is 2 *tens*, or 20 *ones*. $20^3 = 8000$ (1st partial product, or $t^3$) as shown in figure *A*; $15{,}625 - 8000$ leaves a remainder of 7625 ($3\,t^2u + 3\,tu^2 + u^3$). The root 20, therefore, must be so increased as to exhaust this remainder, and keep the figure a perfect cube.



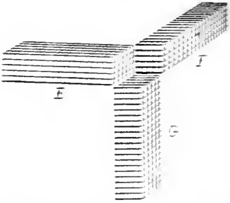




The necessary additions to enlarge *A* and keep it a cube are: First, the three equal square solids *B*, *C*, and *D* (2d partial product, or $3\,t^2u$); second, the three equal rectangular solids (p. 428) *E*, *F*, and *G* (3d partial product, or $3\,tu^2$); and third, the small cube (p. 428) *H* (4th partial product, or $u^3$).

The sum of these three additions is 7625 cubic units; and, since the square solids *B*, *C*, and *D* (2d partial product, or $3\,t^2u$) contain the greatest part of the additions, their volume is *nearly* 7625 cubic units. If we

divide 7625 by the surface of the square solids, we shall find their *approximate* thickness.

Each of the solids A, B, and C is 20 units square, and the surface of the three solids is 3 × 20², or 1200 square units. 7625 ÷ 1200 gives 6 as the *approximate* thickness of the additions. But if the solids *B, C, D, E, F, G,* and *H* were each 6 units thick, their volume (9576 cubic units) would be greater than 7625 cubic units. Hence, the thickness of these additions cannot be more than 5 units.

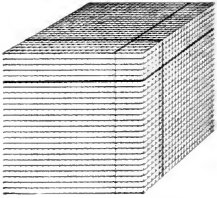



Besides the square solids, *A, B, C,* there are three equal rectangular solids, *E, F,* and *G,* to be added. Each of them is 20 units long and 5 units wide; hence, the surface of a face is 20 × 5, or 100 square units, and the surface of the three is 3 × 20 × 5 square units, or 300 square units.

The last addition is the small cube *H,* whose edge is 5 units and whose surface is 5 × 5, or 25 square units.


H

The sum of the areas of these three additions is 1200 + 300 + 25, ($3t^2 + 3tu + u^2$), or 1525 square units, which multiplied by 5 ($u$), the thickness of the additions, gives 7625 cubic units ($3t^2u + 3tu^2 + u^3$). And since these three additions exactly exhaust the remainder, the second figure of the root is 5, and the cube root of 15,625 is 20 + 5, or 25.



NOTE. — Since 2 × 2 × 2 = 8, the cube root of 8 is 2, that is, *one of the three equal factors* of 8 is 2.

Concrete numbers cannot be separated into *two or three equal* factors; hence, they have no roots. The factors of 8 cu. yd. are 2 × 2 × 2 cu. yd.; of 27 cu. ft. are 3 × 3 × 3 cu. ft. In each case, *one* of the *factors* is concrete; hence, they are not *equal*. In extracting the root of such numbers as 64 sq. in., 125 cu. yd., etc., the numbers are regarded as *abstract*.

The cube root of a number equals the number of units in *the edge of a cube* whose volume is a number of cubic units equal to the given number.

6. Cube 35 $(30+5)$; then find the cube root of 42875.

$$\text{Partial products:}\begin{cases}(1) & t^3 = 30^3 = 27000\\ (2) & 3\,t^2u = 3\,(30^2\times 5) = 13500\\ (3) & 3\,tu^2 = 3\,(30\times 5^2) = \phantom{0}2250\\ (4) & u^3 = 5^3 = \phantom{00}125\end{cases}$$

$$\overline{42875}$$

$$\begin{array}{lr|r|r} & 42'875 & & 30\\ \text{Trial divisor,} & 30^3 = 27000 & & 5\\ \hline & 3\times 30^2 = 2700 & 15875 & 35\\ & 3\times 30\times 5 = \phantom{0}450 & & \\ & 5^2 = \phantom{00}25 & & \\ \hline \text{Complete divisor,} & 3175 & 15875 & \end{array}$$

**Study of Problems**

1. How many periods are there in 42,875?

2. How many figures are in its cube root?

3. The left period is 42 (42,000). The largest cube found in it is 27 (27,000), the cube of the **tens**.

4. What, then, is the tens' figure of the root? $(\sqrt[3]{27}=3.)$

5. Subtract; annex the next period, and the new dividend is 15,875. The cube, 30, or 3 *tens*, is to be so enlarged as to exhaust this remainder and yet preserve it a cube.

6. This dividend is composed mainly of what partial product? (The second, $3\,t^2u) = 3\,(30^2\times 5)$.

7. What factors in the second partial product are already known? $(3\times 30^2.)$

8. If 15,875 is divided by $(3\times 30^2)$ as a trial divisor, the quotient (5) will be (approximately) the other factor in the second partial product.

9. What, then, is probably the *units'* figure of the root? (5.)

10. Observe that the trial divisor, 2700, is equal to three times the square of the root found, considered as *tens*.

11. The first addition to the trial divisor is $3\times 30\times 5$, or 450 (three times the root found, considered as *tens*). The second addition is $5^2$, or 25.

12. What, then, is the complete divisor?

$$3\,t^2+3\,tu+u^2 = 3\,(30)^2+(3\times 30\times 5)+5^2 = 3175.$$

13. Multiplying 3175 by 5 gives 15,875; or, $u\,(3\,t^2+3\,tu+u^2) = 3\,t^2u + 3\,tu^2+u^3$. This exactly exhausts the remainder of 15,875. Hence, the *units'* figure of the root is 5, and the cube root of 42,875 is $30+5$, or 35.

**7.** Find the cube root of 34012.224. (Separate the number into periods, to the left and right from the decimal point.)

| | | | | |
|---|---|---|---|---|
| | | | 34′012.′224′ | 32.4 |
| | $3^3 =$ | | 27 | |
| Trial divisor, | $3 \times 30^2 =$ | 2700 | 7012 | |
| | $3 \times 30 \times 2 =$ | 180 | | |
| | $2^2 =$ | 4 | | |
| Complete divisor, | | 2884 | 5768 | |
| Trial divisor, | $3 \times 320^2 =$ | 307200 | 1244224 | |
| | $3 \times 320 \times 4 =$ | 3840 | | |
| | $4^2 =$ | 16 | | |
| Complete divisor, | | 311056 | 1244224 | |

When the cube root consists of more than two figures, *three times the square of the root already found* (considered as *tens*), is used as a *trial divisor* in finding the next figure of the root.

**8.** Find the cube root of 5 to thousandths.

| | | | | |
|---|---|---|---|---|
| | | | 5. | 1.709+ |
| | $1^3 =$ | | 1 | |
| Trial divisor, | $3 \times 10^2 =$ | 300 | 4000 | |
| | $3 \times 10 \times 7 =$ | 210 | | |
| | $7^2 =$ | 49 | | |
| Complete divisor, | | 559 | 3913 | |
| Trial divisor, | $3 \times 170^2 =$ | 86700 | 87000 | |
| Trial divisor, | $3 \times 1700^2 =$ | 8670000 | 87000000 | |
| | $3 \times 1700 \times 9 =$ | 45900 | | |
| | $9^2 =$ | 81 | | |
| Complete divisor, | | 8715981 | 78443829 | |

When a cipher occurs in the root, *annex two more ciphers to the trial divisor*, bring down the next period, and proceed as before.

*Separate the number into periods of three figures each, to the left and right from the decimal point.*

*Find the largest cube in the left-hand period, and write its root as the first root figure sought. Subtract this cube from the left-hand period and annex the next period to the remainder.*

*For a trial divisor, take* **three times the square of the root already found**, *considered as tens, or three times the square of the root with two naughts annexed. Divide the dividend by it, and the quotient, or the quotient diminished, will be the second part of the root.*

*To the trial divisor, add* **three times the product of the first part of the root**, *considered as tens,* **by the second part**, *and also* **the square of the second part.** *Their sum will be the complete divisor. Multiply the complete divisor by the second part of the root, and subtract the product from the dividend.*

*When other periods remain, take* **three times the square of the root already found**, *considered as tens, for a trial divisor, and proceed as before.*

NOTE. — 1. When a number is not a perfect cube, annex periods of naughts, and continue the work as far as desired.

2. Decimals are separated into periods of three figures each, beginning at the decimal point and passing to the right.

3. To find the cube root of a common fraction, take the cube root of the numerator and the denominator separately, or reduce the fraction to a decimal and then extract its cube root.

Find the cube root of:

| | | |
|---|---|---|
| **1.** 2744 | **7.** 373,248 | **13.** .729 |
| **2.** 4096 | **8.** 941,192 | **14.** 15.625 |
| **3.** 13,824 | **9.** 1,860,867 | **15.** 39.304 |
| **4.** 19.683 | **10.** $\frac{27}{343}$ | **16.** 226.981 |
| **5.** 32,768 | **11.** $\frac{1728}{15625}$ | **17.** .003375 |
| **6.** 91,125 | **12.** $\frac{125}{1331}$ | **18.** $3\frac{3}{8}$ |

## Written Work

**1.** The volume of a cubical box is 5832 cu. in. What is its edge?

NOTE. — Since the volume of a cube is a number of cubic units equal to the product of its *three equal* dimensions, the cube root of the volume of a cube gives the length of its edge.

**2.** If 9261 cubic inches are built into one cube, what is the area of its base?

**3.** Find the depth of a cubical cistern that will contain 2197 cu. ft.

**4.** What is the edge of a cubical box that will contain 50 bushels of 2150.42 cu. in. each?

**5.** A box is 16 ft. long, 8 ft. wide, 4 ft. high. What is the edge of a cubical box of the same volume?

**6.** Find the edge of a cube whose volume is equal to the volume of three cubes whose edges are respectively 3, 4, and 5 inches.

Geometry shows that:

The **corresponding lines of similar solids** are proportional to the cube roots of their volumes.

**7.** If a ball 10 in. in diameter weighs 125 lb., what is the diameter of a similar ball that weighs 216 lb.?

**8.** Of two similar solids, one contains 8 times the volume of the other. The diameter of the smaller is $8\frac{1}{2}$ feet; what is the diameter of the other?

Suggestion: $\sqrt[3]{1} : \sqrt[3]{8} = 8\frac{1}{2}$ ft. $: x$.

**9.** The weights of two balls of metal are as 125 to 343. What is the ratio of their diameters?

Suggestion: D. of 1st : D. of 2d $= \sqrt[3]{125} : \sqrt[3]{343}$.

**10.** If a stack of hay 12 ft. high contains 8 tons, how high is a similar stack that contains 27 tons?

**11.** A square grain bin which contains 1200 bushels of wheat has a depth of only $\frac{1}{2}$ of its width. Find its dimensions.

# REFERENCE TABLES OF MEASURES

## Liquid Measures

| | |
|---|---|
| 4 gills | = 1 pint |
| 2 pints | = 1 quart |
| 4 quarts | = 1 gallon |
| 1 gal. | = 4 qt. = 8 pt. |

The **gill** is now seldom used.

The standard unit of liquid measure is the **gallon.**

| | | |
|---|---|---|
| 1 gallon | = 231 cubic inches | |
| 1 cubic foot | = nearly $7\frac{1}{2}$ gallons | |
| $31\frac{1}{2}$ gallons | = 1 barrel | in measuring the capacity of cisterns and vats |
| 63 gallons | = 1 hogshead | |

1 gallon of water weighs nearly $8\frac{1}{3}$ pounds

1 cubic foot of water weighs nearly $62\frac{1}{2}$ pounds

## Dry Measures

| | |
|---|---|
| 2 pints | = 1 quart |
| 8 quarts | = 1 peck |
| 4 pecks | = 1 bushel |
| 1 bu. | = 4 pk. = 32 qt. = 64 pt. |

Our standard unit, the **Winchester bushel,** used for measuring shelled grains, = 2150.42 cu. in., or nearly $1\frac{1}{4}$ cubic feet. In form it is a cylinder $18\frac{1}{2}$ inches in diameter and 8 inches deep.

The dry gallon = 268.8 cu. in.

The heaped bushel, used for measuring corn in the ear, apples, potatoes, etc., = 2747.71 cu. in., or nearly $1\frac{5}{9}$ cu. ft.

The standard **English bushel** = 2218.192 cu. in.

## Measures of Length

| | |
|---|---|
| 12 inches | = 1 foot |
| 3 feet | = 1 yard |
| $5\frac{1}{2}$ yards, $16\frac{1}{2}$ feet | = 1 rod |
| 320 rods, 5280 feet | = 1 mile |

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63360 in.

The standard unit of length is the **yard.**

A **nautical mile (knot)** = 6080.27 ft. or nearly 1.15 common miles. A **league** = 3 nautical miles; a **fathom,** used in measuring the depth of water, = 6 ft.; a **hand,** used in measuring the height of horses, = 4 in. A **furlong** = $\frac{1}{8}$ mi.

### Measures of Surface

| | | |
|---|---|---|
| 144 square inches | = | 1 square foot |
| 9 square feet | = | 1 square yard |
| $30\frac{1}{4}$ square yards | = | 1 square rod |
| 160 square rods, 43560 square feet | = | 1 acre |
| 640 acres | = | 1 square mile |
| 1 mile square | = | 1 section |
| 36 square miles | = | 1 township |

The **acre** is not a square unit like the square foot, the square yard, etc. When in the form of a square, it is nearly 209 feet on a side.

### Surveyors' Measures

**Surveyors** and **engineers** formerly used the *Gunter's Chain.* It is 66 feet long and divided into 100 links of 7.92 inches each. The tables are as follows:

**Length**

| | | |
|---|---|---|
| 7.92 inches | = | 1 link |
| 100 links | = | 1 chain |
| 80 chains | = | 1 mile |

**Surface**

| | | |
|---|---|---|
| 16 square rods | = | 1 square chain |
| 10 square chains | = | 1 acre |

They now generally use a steel tape 50 ft. to 100 ft. long divided into feet and tenths of a foot; or a chain 50 ft. to 100 ft. long having links each a foot in length, divided into tenths of a foot.

*Land Measure* is computed by dividing the number of square feet of surface by 43560, the number of square feet in an acre, and changing the decimal of an acre to square rods, etc.

### Measures of Volume

| | | |
|---|---|---|
| 1728 cubic inches | = | 1 cubic foot |
| 27 cubic feet | = | 1 cubic yard |

A cubic yard of earth is considered a **load.**

A **cord of 4 foot wood** is a pile of wood 8 feet long and 4 feet high, the sticks averaging 4 feet in length, making 128 cubic feet in the pile.

A **cord of short wood** is a pile of wood 8 feet long and 4 feet high, the number of cords in a pile being computed by multiplying the length of the pile in feet by the height in feet, and dividing the product by 32.

## Avoirdupois Weight

16 ounces = 1 pound
100 pounds = 1 hundredweight
2000 pounds = 1 ton
2240 pounds = 1 *long* ton
1 T. = 20 cwt. = 2000 lb. = 32000 oz.

The standard unit of weight is the **pound** = 7000 grains.
1 Av. oz. = $437\frac{1}{2}$ grains.

* 60 pounds = 1 bu. of wheat or potatoes
* 56 pounds = 1 bu. of shelled corn or rye
* 32 pounds = 1 bu. of oats
196 pounds = 1 bbl. of flour
200 pounds = 1 bbl. of beef or pork

* In most states.

The long ton is used in the United States custom houses and in the wholesale transactions in coal and iron. The long cwt. = 112 lb.

## Troy Weight

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound
1 pound = 12 oz. = 240 pwt. = 5760 gr.

The unit generally used for weighing diamonds, gems, etc., is the **carat**, which is about 3.2 Troy grains. It is used also to express the fineness of gold. 18 carats fine means $\frac{18}{24}$ pure gold and $\frac{6}{24}$ baser metal.

The Troy pound = 5760 grains
The Troy ounce = 480 grains

## Apothecaries' Weight

This is used only in filling medical prescriptions.

20 grains = 1 scruple — sc. or ℈
3 scruples = 1 dram — dr. or ʒ
8 drams = 1 ounce — oz. or ℥
12 ounces = 1 lb. or ℔

## Apothecaries' Liquid Measures

This is used only in filling medical prescriptions.

10 minims (m) = 1 fluid dram (fʒ)
8 fluid drams = 1 fluid ounce (f℥)
16 fluid ounces = 1 pint (0)

## Counting

12 things = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross
20 things = 1 score.

## Stationers' Measures

24 sheets = 1 quire
20 quires = 1 ream

Paper is frequently sold by the pad or bulk of 100, 500, or 1000 sheets, or by the pound.

### Measures of Time

| | |
|---|---|
| 60 seconds | = 1 minute |
| 60 minutes | = 1 hour |
| 24 hours | = 1 day |
| 7 days | = 1 week |
| 12 months / 365 days | = 1 common year |
| 366 days | = 1 leap year |
| 10 years | = 1 decade |
| 100 years | = 1 century |

Thirty days have September,
April, June, and November.
All the rest have thirty-one
Save February, which alone
Has 28, and one day more
We add to it one year in four.

The true solar year is 365 days, 5 hr., 48 min., 46 sec. The standard unit of time is the **day** which is divided into 24 hours counting from midnight to midnight. In business transactions 30 days are considered a month, and 12 months are regarded as a year.

The centennial years divisible by 400, and all other years divisible by 4, are **leap years.**

### Measures of Angles and Arcs

| | |
|---|---|
| 60 seconds | = 1 minute |
| 60 minutes | = 1 degree |
| 360 degrees | = 1 circumference |
| 1 right angle | = 90 degrees |

### United States Money

| | |
|---|---|
| 10 mills | = 1 cent |
| 10 cents | = 1 dime |
| 10 dimes | = 1 dollar |
| 10 dollars | = 1 eagle |

### English Money

| | | |
|---|---|---|
| 4 farthings | = 1 penny | = \$.02025 |
| 12 pence | = 1 shilling | = \$.243 |
| 20 shillings | = 1 pound | = \$4.8665 |

The unit of English money is the **pound.**

The value in United States money of other foreign coins is as follows:

| | | |
|---|---|---|
| Ruble | Russia | = \$.515 |
| Yen | Japan | = .498 |
| Franc | France (Belgium) | = .193 |
| Mark | Germany | = .238 |
| Crown | Austria-Hungary | = .203 |
| Lira | Italy | = .193 |
| Peseta | Spain | = .193 |
| Peso | Chile | = .365 |
| Crown | Sweden | = .268 |

# INDEX

# ANSWERS

**Page 11.** — **1.** 43; 449; 1,000,502; 1492. **2.** 99; 1908; 1576; 1861. **3.** XLI; LXIII; LXXXIV; XCIX; CVII; CCXVIII; DLXXII; DCCXXXV; CMXCVI; MCMVII; MDLXIV; MDCXVI; M; CCLX.

**Page 15.** — **2.** $3603.18. **3.** 2,595,237. **4.** 3,303,261. **5.** 3,996,337.

**Page 16.** — **7.** 5182. **8.** 5261. **9.** 6684. **10.** 5269. **11.** 2799. **12.** 83,302. **13.** 82,276. **14.** 76,385. **15.** 84,503. **16.** 62,872. **17.** 71,809.

**Page 18.** — **1.** 913 mi. **2.** $11,276. **3.** 3,805,074 sq. mi. **4.** $2,376,000,000. **5.** $1,254,060,661. **6.** $748,152,215.

**Page 20.** — **7.** 5997. **8.** 22,962. **9.** 27,434. **10.** 2600. **11.** 7999.

**Page 21.** — **3.** 14,571. **4.** 36,906. **5.** 18,393. **6.** 30,996. **7.** 38,131. **8.** 28,984. **9.** $329.04. **10.** $379.21. **11.** $561.47. **12.** $194.39. **13.** $190.39. **14.** $190.33. **15.** 300,884. **16.** 565,065. **17.** 295,404. **18.** 186,418. **19.** 433,983. **20.** 225,545. **21.** 3,472,273. **22.** 2,698,987. **23.** 3,125,092. **24.** 4,197,541. **25.** 2,304,902. **26.** 2,919,062.

**Page 22.** — **1.** 2029. **2.** 7290. **3.** 966,779. **4.** 144,752,500 bu. **5.** A, $1375; B, $3075. **6.** $9 gained. **7.** $4800. **8.** 52,407,000 sq. mi. **9.** James, $6750; Henry, $11,925; Frank, $5000.

**Page 27.** — **2.** 6000. **3.** 3745. **4.** 12,880. **5.** 35,000. **6.** 21,584. **7.** 51,134. **8** *a.* 1,574,125; **8** *b.* 2,210,200; **8** *c.* 3,623,700; **8** *d.* 4,793,050; **8** *e.* 5,165,700; **8** *f.* 6,174,425; **8** *g.* 5,583,325; **8** *h.* 5,114,300; **8** *i.* 6,219,400; **8** *j.* 5,769,650. **9** *a.* 250,880; **9** *b.* 352,256; **9** *c.* 577,536; **9** *d.* 763,904; **9** *e.* 823,296; **9** *f.* 984,064; **9** *g.* 889,856; **9** *h.* 815,104; **9** *i.* 991,232; **9** *j.* 919,552. **10** *a.* 2,136,400; **10** *b.* 2,999,680; **10** *c.* 4,918,080; **10** *d.* 6,505,120; **10** *e.* 7,010,880; **10** *f.* 8,379,920; **10** *g.* 7,577,680; **10** *h.* 6,941,120; **10** *i.* 8,440,960; **10** *j.* 7,830,560. **11** *a.* 2,364,740; **11** *b.* 3,320,288; **11** *c.* 5,433,728; **11** *d.* 7,200,392; **11** *e.* 7,760,208; **11** *f.* 9,275,572; **11** *g.* 8,387,588; **11** *h.* 7,682,992; **11** *i.* 9,343,136; **11** *j.* 8,667,496. **12** *a.* 2,122,925; **12** *b.* 2,980,760; **12** *c.* 4,887,060; **12** *d.* 6,464,090; **12** *e.* 6,966,660; **12** *f.* 8,327,065; **12** *g.* 7,529,885; **12** *h.* 6,897,340; **12** *i.* 8,387,720; **12** *j.* 7,781,170. **13.** *a.* 1,934,030; **13** *b.* 2,715,536; **13** *c.* 4,452,216; **13** *d.* 5,888,924; **13** *e.* 6,346,776; **13** *f.* 7,586,134; **13** *g.* 6,859,886; **13** *h.* 6,283,624; **13** *i.* 7,641,392; **13** *j.* 708,812. **14** *a.* 2,073,925; **14** *b.* 2,911,960; **14** *c.* 4,774,060; **14** *d.* 6,314,890; **14** *e.* 6,805,860; **14** *f.* 8,134,865; **14** *g.* 7,356,085; **14** *h.* 6,738,140; **14** *i.* 8,194,120; **14** *j.* 7,601,570. **15** *a.* 1,885,275; **15** *b.* 2,647,080; **15** *c.* 4,339,980; **15** *d.* 5,740,470; **15** *e.* 6,186,780; **15** *f.* 7,394,895; **15** *g.* 6,686,955; **15** *h.* 6,125,220; **15** *i.* 7,448,760; **15** *j.* 6,910,110. **16** *a.* 2,064,125; **16** *b.* 2,898,200; **16** *c.* 4,751,700; **16** *d.* 6,285,050; **16** *e.* 6,773,700; **16** *f.* 8,096,425; **16** *g.* 7,321,325; **16** *h.* 6,706,300; **16** *i.* 8,155,490; **16** *j.* 7,565,650. **17** *a.* 2,321,620; **17** *b.* 3,259,744; **17** *c.* 5,344,464; **17** *d.* 7,069,096; **17** *e.* 7,618,704; **17** *f.* 9,106,436; **17** *g.* 8,234,644; **17** *h.* 7,542,896; **17** *i.* 9,172,768; **17** *j.* 8,509,448.

**Page 28**. — **18**. $155. **19**. $7750. **20**. $866.95. **21**. $84. **22**. 1,254,400 lb. **23**. $3729.38. **24**. $26,020. **25**. 48,720 rd. **26**. 747 mi. **27**. $13.65.

**Page 30**. — **2**. 114, rem. 2. **3**. 127, rem. 1 ft. **4**. 51, rem. 7¢. **5** *a*. $101.87, rem. 1¢; **5** *b*. $67.91, rem. 2¢; **5** *c*. $50.93, rem. 3¢; **5** *d*. $40.75; **5** *e*. $33.95, rem. 5¢; **5** *f*. $29.10, rem. 5¢; **5** *g*. $25.46. rem. 7¢; **5** *h*. $22.63, rem. 8¢; **5** *i*. $20.37, rem. 5¢; **5** *j*. $18.52. rem. 3¢. **6** *a*. $339.17; **6** *b*. $226.11, rem. 1¢; **6** *c*. $169.58, rem. 2¢; **6** *d*. $135.66, rem. 4¢; **6** *e*. $113.05, rem. 4¢; **6** *f*. $96.90, rem. 4¢; **6** *g*. $84.79, rem. 2¢; **6** *h*. $75.37, rem. 1¢; **6** *i*. $67.83, rem. 4¢; **6** *j*. $61.66, rem. 8¢. **7** *a*. $104.53, rem. 1¢; **7** *b*. $69.69; **7** *c*. $52.26, rem. 3¢; **7** *d*. $41.81, rem. 2¢; **7** *e*. $34.84, rem. 3¢; **7** *f*. $29.86, rem. 5¢; **7** *g*. $26.13, rem. 3¢; **7** *h*. $23.23; **7** *i*. $20.90, rem. 7¢; **7** *j*. $19.00, rem. 7¢. **8** *a*. $195.04; **8** *b*. $130.02, rem. 2¢; **8** *c*. $97.52; **8** *d*. $78.01, rem. 3¢; **8** *e*. $65.01, rem. 2¢; **8** *f*. $55.72, rem. 4¢; **8** *g*. $48.76; **8** *h*. $43.34, rem. 2¢; **8** *i*. $39.00, rem. 8¢; **8** *j*. $35.46, rem. 2¢. **9** *a*. $360.46, rem. 1¢; **9** *b*. $240.31; **9** *c*. $180.23, rem. 1¢; **9** *d*. $144.18, rem. 3¢; **9** *e*. $120.15, rem. 3¢; **9** *f*. $102.99; **9** *g*. $90.11, rem. 5¢; **9** *h*. $80.10, rem. 3¢; **9** *i*. $72.09, rem. 3¢; **9** *j*. $65.53, rem. 10¢. **10** *a*. $189.69; **10** *b*. $126.46; **10** *c*. $94.84, rem. 2¢; **10** *d*. $75.87, rem. 3¢; **10** *e*. $63.23; **10** *f*. $54.19, rem. 5¢; **10** *g*. $47.42, rem. 2¢; **10** *h*. $42.15, rem. 3¢; **10** *i*. $37.93, rem. 8¢; **10** *j*. $34.48, rem. 10¢. **11** *a*. $148.67; **11** *b*. $99.11, rem. 1¢; **11** *c*. $74.33, rem. 2¢; **11** *d*. $59.46. rem. 4¢; **11** *e*. $49.55, rem. 4¢; **11** *f*. $42.47, rem. 5¢; **11** *g*. $37.16, rem. 6¢; **11** *h*. $33.03, rem. 7¢; **11** *i*. $29.73, rem. 4¢; **11** *j*. $27.03, rem. 1¢. **12** *a*. $213.92; **12** *b*. $142.61, rem. 1¢; **12** *c*. $106.96; **12** *d*. $85.56, rem. 4¢; **12** *e*. $71.30, rem. 4¢; **12** *f*. $61.12; **12** *g*. $53.48; **12** *h*. $47.53, rem. 7¢; **12** *i*. $42.78, rem. 4¢; **12** *j*. $38.89, rem. 5¢. **13** *a*. $459.03, rem. 1¢; **13** *b*. $306.02, rem. 1¢; **13** *c*. $229.51, rem. 3¢; **13** *d*. $183.61, rem. 2¢; **13** *e*. $153.01, rem. 1¢; **13** *f*. $131.15, rem. 2¢; **13** *g*. $114.75. rem. 7¢; **13** *h*. $102.00, rem. 7¢; **13** *i*. $91.80, rem. 7¢; **13** *j*. $83.46, rem. 1¢. **14** *a*. $423.56; **14** *b*. $282.37, rem. 1¢; **14** *c*. $211.78; **14** *d*. $169.42, rem. 2¢; **14** *e*. $141.18, rem. 4¢; **14** *f*. $121.01, rem. 5¢; **14** *g*. $105.89; **14** *h*. $94.12, rem. 4¢; **14** *i*. $84.71, rem. 2¢; **14** *j*. $77.01, rem. 1¢.

**Page 31**. — **5**. 118, rem. 15; 11, rem. 175. **6**. 1892, rem. 5; 756, rem. 45; 473, rem. 5; 75, rem. 345. **7**. 2512; 1004, rem. 40; 628; 100, rem. 240. **8**. 4510; 1804; 1127, rem. 40; 180, rem. 200. **9**. 3703, rem. 19; 1481, rem. 29; 925, rem. 79; 148, rem. 79. **10**. 20,490, rem. 5; 8196, rem. 5; 5122, rem. 45; 819, rem. 305. **11**. 39,504, rem. 6; 15,801, rem. 36; 9876, rem. 6; 1580, rem. 86. **12**. 19,503, rem. 15; 7801, rem. 25; 4875, rem. 75; 780, rem. 75. **13**. 49,250; 19,700; 12,312. rem. 40; 1970.

**2**. 12. **3**. 25. **4**. 32, rem. 20. **5**. 20, rem. 22. **6**. 11, rem. 4. **7**. 105, rem. 9. **8**. 209, rem. 3. **9**. 41. **10**. 32. **11**. 32. **12**. 26. **13**. 31. **14**. 43. **15**. 77, rem. 3. **16**. 84, rem. 70.

**Page 32**. — **17**. 62. **18**. 45. **19**. 41, rem. 6. **20**. 43. **21**. 41. **22**. 41. **23**. 66, rem. 55. **24**. 85, rem. 57. **25**. 81, rem. 51. **26**. 96, rem. 44. **27**. 77, rem. 3. **28**. 79, rem. 16. **29**. 84, rem. 70. **30**. 84, rem. 14. **31**. 81, rem. 78. **32**. 91, rem. 34. **33**. 90, rem. 78. **34**. 87, rem. 77. **35**. 108, rem. 12. **36**. 145, rem. 14. **37**. 83, rem. 29. **38**. 106, rem. 38. **39**. 82, rem. 37. **40**. 109, rem. 47. **44** *a*. 24,120, rem. 181; **44** *b*. 18,260, rem. 301;

**44** *c*. 9562, rem. 429; **44** *d*. 8528, rem. 117; **44** *e*. 7473, rem. 196; **44** *f*. 6569, rem. 445; **44** *g*. 8193, rem. 64; **44** *h*. 7206, rem. 559; **44** *i*. 12,700, rem. 41; **44** *j*. 7263, rem. 271. **45** *a*. 29,278, rem. 256; **45** *b*. 22,165, rem. 350; **45** *c*. 11,607, rem. 428; **45** *d*. 10,351, rem. 702; **45** *e*. 9071, rem. 345; **45** *f*. 7974, rem. 344; **45** *g*. 9945, rem. 155; **45** *h*. 8747, rem. 701; **45** *i*. 15,416, rem. 16; **45** *j*. 8816, rem. 520. **46** *a*. 21,882, rem. 172; **46** *b*. 16,566, rem. 184; **46** *c*. 8675, rem. 248; **46** *d*. 7736, rem. 660; **46** *e*. 6779, rem. 713; **46** *f*. 5959, rem. 892; **46** *g*. 7432, rem. 700; **46** *h*. 6537, rem. 859; **46** *i*. 11,521, rem. 359; **46** *j*. 6589, rem. 338. **47** *a*. 32,258, rem. 197; **47** *b*. 24,421, rem. 307; **47** *c*. 12,788, rem. 653; **47** *d*. 11,405, rem. 351; **47** *e*. 9994, rem. 531; **47** *f*. 8785, rem. 901; **47** *g*. 10,957, rem. 268; **47** *h*. 9638, rem. 55; **47** *i*. 16,984, rem. 485; **47** *j*. 9713, rem. 771. **48** *a*. 35,913, rem. 188; **48** *b*. 27,188, rem. 320; **48** *c*. 14,237, rem. 660; **48** *d*. 12,697, rem. 546; **48** *e*. 11,127, rem. 17; **48** *f*. 9781, rem. 368; **48** *g*. 12,198, rem. 650; **48** *h*. 10,730, rem. 62; **48** *i*. 18,909, rem. 191; **48** *j*. 10,814, rem. 412. **49** *a*. 29,045, rem. 40; **49** *b*. 21,988, rem. 348; **49** *c*. 11,514, rem. 636; **49** *d*. 10,269, rem. 198; **49** *e*. 8998, rem. 830; **49** *f*. 7910, rem. 660; **49** *g*. 9865, rem. 615; **49** *h*. 8677, rem. 831; **49** *i*. 15,292, rem. 472; **49** *j*. 8746, rem. 160. **50** *a*. 25,411, rem. 256; **50** *b*. 19,238, rem. 152; **50** *c*. 10,074, rem. 380; **50** *d*. 8984, rem. 532; **50** *e*. 7873, rem. 259; **50** *f*. 6921, rem. 140; **50** *g*. 8631, rem. 545; **50** *h*. 7592, rem. 380; **50** *i*. 13,379, rem. 493; **50** *j*. 7652, rem. 124. **51** *a*. 29,492, rem. 169; **51** *b*. 22,327, rem. 267; **51** *c*. 11,692, rem. 233; **51** *d*. 10,427, rem. 359; **51** *e*. 9137, rem. 520; **51** *f*. 8032, rem. 537; **51** *g*. 10,017, rem. 612; **51** *h*. 8811, rem. 558; **51** *i*. 15,528, rem. 273; **51** *j*. 8880, rem. 825. **52** *a*. 18,160, rem. 165; **52** *b*. 13,748, rem. 253; **52** *c*. 7199, rem. 521; **52** *d*. 6420, rem. 685; **52** *e*. 5625, rem. 555; **52** *f*. 4946, rem. 181; **52** *g*. 6168, rem. 493; **52** *h*. 5425, rem. 820; **52** *i*. 9561, rem. 496; **52** *j*. 5468, rem. 525. **53** *a*. 12,068, rem. 343; **53** *b*. 9137, rem. 69; **53** *c*. 4784, rem. 583; **53** *d*. 4267, rem. 181; **53** *e*. 3739, rem. 332; **53** *f*. 3287, rem. 159; **53** *g*. 4099, rem. 456; **53** *h*. 3605, rem. 882; **53** *i*. 6354, rem. 381; **53** *j*. 3634, rem. 307.

**Page 33.**—**2.** $199.92. **3.** 40 hr. **4.** 16 da. **5.** $289.50. **6.** $63.58+. **7.** B, $4800; A, $5075.

**Page 34.**—**9.** $270. **10.** $217.05. **11.** 1710 T. **12.** $5.30, gain. **13.** $1660, total sale; $.87+, average cost. **14.** $5247.45.

**Page 35.**—**2.** $22.50. **3.** $5. **4.** $37.50. **5.** $2.40. **6.** $25. **7.** $10.

**Page 36.**—**1.** 44. **2.** 37. **3.** 144. **4.** 2832. **5.** 88,894. **6.** 546. **7.** 8. **8.** 130. **9.** 62. **10.** 75. **11.** 24. **12.** 47. **13.** 109. **14.** 80. **15.** 72. **16.** 137.

**Page 40.**—**2.** 5, 5, 5. **3.** 2, 3, 5, 7. **4.** 3, 3, 5, 5. **5.** $2^4$, $5^2$. **6.** $3^3$, 5, 7. **7.** 2, $3^2$, 163. **8.** $2^2$, 3, 5, 7, 11. **9.** 2, 3, $7^2$, 13. **10.** 11, 221. **11.** $2^5$, $3^2$, $5^2$. **12.** $2^2$, $5^2$, 7, 11. **13.** $3^2$, $5^2$, 29. **14.** $2^{10}$, $5^2$. **15.** $2^7$, 5, 101. **16.** 3, $5^3$, 7, 37. **17.** $2^4$, 3, $5^3$, 13.

**Page 41.**—**2.** 21. **3.** 18. **4.** 12. **5.** 4. **6.** 3. **7.** 14. **8.** 22. **9.** 7. **10.** 17. **11.** 144.

**Page 42.**—**3.** 144. **4.** 1260. **5.** 480. **6.** 1400. **7.** 420. **8.** 96. **9.** 630. **10.** 576. **11.** 720. **12.** 255. **13.** 5400. **14.** 882. **15.** 1080. **16.** 2200.

**Page 43.** — **2.** 108. **3.** 7. **4.** 10. **5.** 8. **6.** 12. **7.** 37. **8.** 40 lb. **9.** 80 bu.

**Page 48.** — **2.** $\frac{15}{20}$. **3.** $\frac{49}{56}$. **4.** $\frac{88}{96}$. **5.** $\frac{54}{78}$. **6.** $\frac{90}{132}$. **7.** $\frac{68}{72}$. **8.** $\frac{63}{135}$. **9.** $\frac{132}{276}$. **10.** $\frac{187}{275}$. **11.** $\frac{216}{372}$. **12.** $\frac{273}{494}$. **13.** $\frac{697}{765}$. **14.** $\frac{255}{415}$. **15.** $\frac{234}{315}$.

**Page 49.** — **3.** $\frac{3}{4}$. **4.** $\frac{5}{11}$. **5.** $\frac{6}{7}$. **6.** $\frac{8}{9}$. **7.** $\frac{7}{12}$. **8.** $\frac{6}{7}$. **9.** $\frac{7}{15}$. **10.** $\frac{3}{8}$. **11.** $\frac{7}{8}$. **12.** $\frac{11}{12}$. **13.** $\frac{3}{4}$. **14.** $\frac{9}{14}$. **15.** $\frac{7}{8}$. **16.** $\frac{6}{7}$. **17.** $\frac{29}{37}$. **18.** $\frac{8}{11}$. **19.** $\frac{47}{72}$. **20.** $\frac{27}{32}$. **21.** $\frac{5}{13}$. **22.** $\frac{73}{121}$. **23.** $\frac{8}{11}$. **24.** $\frac{13}{14}$. **25.** $\frac{88}{111}$. **26.** $\frac{21}{23}$. **27.** $\frac{4}{9}$. **28.** $\frac{11}{13}$. **29.** $\frac{24}{37}$. **30.** $\frac{91}{106}$. **31.** $\frac{10}{11}$. **32.** $\frac{41}{63}$. **33.** $\frac{11}{12}$. **34.** $\frac{28}{39}$. **35.** $\frac{9}{11}$. **36.** $\frac{21}{23}$. **37.** $\frac{21}{34}$. **38.** $\frac{23}{36}$. **39.** $\frac{46}{111}$. **40.** $\frac{2}{3}$. **41.** $\frac{29}{42}$. **42.** $\frac{3}{4}$. **43.** $\frac{4}{5}$. **44.** $\frac{13}{14}$. **45.** $\frac{11}{12}$. **46.** $\frac{21}{29}$. **47.** $\frac{31}{35}$.

**Page 50.** — **2.** $\frac{51}{4}$. **3.** $\frac{95}{6}$. **4.** $\frac{183}{8}$. **5.** $\frac{309}{10}$. **6.** $\frac{587}{12}$. **7.** $\frac{797}{14}$. **8.** $\frac{2115}{28}$. **9.** $\frac{2396}{25}$. **10.** $\frac{3325}{32}$. **11.** $\frac{5341}{45}$. **12.** $\frac{9838}{56}$. **13.** $\frac{22651}{105}$. **14.** $\frac{32257}{120}$. **15.** $\frac{25319}{78}$. **16.** $\frac{52733}{105}$. **17.** $\frac{15743}{144}$. **18.** $\frac{18017}{30}$. **19.** $\frac{2138}{7}$. **20.** $\frac{21941}{56}$. **21.** $\frac{1451}{90}$. **22.** $\frac{28837}{32}$. **23.** $\frac{1507}{15}$. **24.** $\frac{19543}{50}$. **25.** $\frac{17339}{75}$. **27.** $2\frac{1}{15}$. **28.** $47\frac{10}{11}$. **29.** $136\frac{4}{7}$. **30.** $9\frac{13}{16}$. **31.** $32\frac{5}{8}$. **32.** $15\frac{7}{25}$. **33.** $22\frac{11}{32}$. **34.** 22. **35.** $18\frac{1}{2}$. **36.** 25. **37.** 24. **38.** 15. **39.** $21\frac{31}{34}$. **40.** $23\frac{1}{19}$. **41.** 24. **42.** $10\frac{11}{42}$. **43.** $5\frac{9}{16}$. **44.** $51\frac{7}{8}$. **45.** $157\frac{13}{15}$. **46.** $45\frac{13}{15}$. **47.** $110\frac{1}{8}$. **48.** $92\frac{9}{10}$. **49.** $183\frac{1}{3}$. **50.** 112.

**Page 51.** — **2.** $\frac{4}{12}, \frac{3}{12}, \frac{5}{12}$. **3.** $\frac{10}{15}, \frac{3}{15}, \frac{2}{15}$. **4.** $\frac{15}{18}, \frac{12}{18}, \frac{7}{18}$. **5.** $\frac{4}{24}, \frac{9}{24}, \frac{17}{24}$. **6.** $\frac{4}{36}, \frac{8}{36}, \frac{5}{36}$. **7.** $\frac{8}{40}, \frac{12}{40}, \frac{7}{40}$. **8.** $\frac{8}{24}, \frac{10}{24}, \frac{7}{24}$. **9.** $\frac{16}{40}, \frac{15}{40}, \frac{9}{40}$. **10.** $\frac{2}{18}, \frac{3}{18}, \frac{5}{18}$. **11.** $\frac{12}{20}, \frac{14}{20}, \frac{1}{20}$. **12.** $\frac{24}{36}, \frac{6}{36}, \frac{19}{36}$. **13.** $\frac{2}{6}, \frac{3}{6}, \frac{1}{6}$. **14.** $\frac{12}{28}, \frac{22}{28}, \frac{19}{28}$. **15.** $\frac{27}{45}, \frac{25}{45}, \frac{21}{45}$. **16.** $\frac{16}{28}, \frac{6}{28}, \frac{1}{28}$. **17.** $\frac{35}{42}, \frac{18}{42}, \frac{27}{42}$. **18.** $\frac{16}{48}, \frac{15}{48}, \frac{34}{48}$. **19.** $\frac{24}{60}, \frac{40}{60}, \frac{9}{60}$. **20.** $\frac{32}{56}, \frac{35}{56}, \frac{1}{56}$. **21.** $\frac{40}{60}, \frac{48}{60}, \frac{45}{60}$.

**Page 52.** — **2.** $\frac{10}{40}, \frac{16}{40}, \frac{15}{40}, \frac{20}{40}$. **3.** $\frac{32}{48}, \frac{15}{48}, \frac{28}{48}, \frac{12}{48}$. **4.** $\frac{42}{60}, \frac{36}{60}, \frac{32}{60}, \frac{27}{60}$. **5.** $\frac{80}{120}, \frac{96}{120}, \frac{105}{120}, \frac{100}{120}$. **6.** $\frac{250}{450}, \frac{216}{450}, \frac{425}{450}$. **7.** $\frac{198}{693}, \frac{385}{693}, \frac{504}{693}$. **8.** $\frac{24}{60}, \frac{57}{60}, \frac{25}{60}, \frac{34}{60}$. **9.** $\frac{36}{252}, \frac{126}{252}, \frac{81}{252}, \frac{154}{252}$. **10.** $\frac{140}{560}, \frac{504}{560}, \frac{128}{560}, \frac{105}{560}$. **11.** $\frac{35}{90}, \frac{54}{90}, \frac{160}{90}, \frac{39}{90}$. **12.** $\frac{42}{756}, \frac{180}{756}, \frac{243}{756}, \frac{434}{756}$. **13.** $\frac{252}{72}, \frac{44}{72}, \frac{132}{72}, \frac{66}{72}$. **14.** $\frac{42}{60}, \frac{25}{60}, \frac{27}{60}, \frac{58}{60}$. **15.** $\frac{72}{336}, \frac{189}{336}, \frac{1134}{336}, \frac{944}{336}$. **16.** $\frac{240}{840}, \frac{408}{840}, \frac{36}{840}, \frac{357}{840}$. **17.** $\frac{105}{126}, \frac{112}{126}, \frac{30}{126}, \frac{74}{126}$. **18.** $\frac{200}{225}, \frac{126}{225}, \frac{780}{225}$. **19.** $\frac{54}{132}, \frac{64}{132}, \frac{38}{132}, \frac{67}{132}$.

**Page 54.** — **2.** $2\frac{61}{120}$. **3.** $1\frac{4}{7}$. **4.** $1\frac{633}{700}$. **5.** $1\frac{5}{12}$. **6.** $2\frac{1}{24}$. **7.** $2\frac{1}{20}$. **8.** $1\frac{19}{28}$. **9.** $1\frac{191}{315}$. **10.** $2\frac{7}{20}$. **11.** $2\frac{7}{20}$. **12.** $2\frac{31}{96}$. **13.** $1\frac{15}{16}$. **14.** $1\frac{229}{490}$. **15.** $2\frac{23}{100}$. **16.** $1\frac{49}{120}$. **18.** $20\frac{25}{48}$. **19.** $31\frac{11}{28}$. **20.** $35\frac{7}{48}$. **21.** $44\frac{7}{10}$. **22.** $92\frac{13}{18}$. **23.** $115\frac{13}{42}$. **24.** $39\frac{1}{84}$. **25.** $70\frac{361}{420}$. **26.** $36\frac{167}{840}$. **27.** $224\frac{13}{180}$. **28.** $22\frac{184}{315}$. **29.** $194\frac{17}{252}$. **30.** $30\frac{11}{16}$ mi. **31.** $283\frac{1}{4}$ rd. **32.** $7\frac{29}{150}$. **33.** $56\frac{9}{28}$.

**Page 55.** — **2.** $\frac{1}{36}$. **3.** $\frac{1}{6}$. **4.** $\frac{7}{36}$. **5.** $\frac{1}{108}$. **6.** $\frac{11}{96}$. **7.** $\frac{2}{35}$. **8.** $\frac{1}{40}$. **9.** $\frac{1}{2}$. **10.** $\frac{23}{156}$. **11.** $\frac{107}{900}$. **12.** $\frac{32}{315}$. **13.** $\frac{11}{168}$. **14.** \$ $\frac{3}{20}$. **15.** $\frac{1}{20}$. **16.** $\frac{2}{5}$. **17.** $\frac{1}{8}$ A. **18.** $\frac{5}{16}$.

**Page 56.** — **19.** $\frac{34}{63}$. **20.** \$6.

**2.** $4\frac{1}{3}$. **3.** $7\frac{20}{63}$. **4.** $10\frac{23}{63}$. **5.** $9\frac{37}{60}$. **6.** $20\frac{11}{18}$. **7.** $38\frac{43}{78}$. **8.** $39\frac{71}{153}$. **9.** $51\frac{549}{550}$. **10.** $54\frac{65}{72}$. **11.** $63\frac{101}{330}$. **12.** $53\frac{21}{64}$. **13.** $48\frac{151}{864}$. **14.** $24\frac{41}{48}$. **15.** $16\frac{25}{26}$. **16.** $20\frac{86}{135}$. **17.** $37\frac{11}{96}$. **18.** $17\frac{89}{144}$. **19.** $5\frac{4}{35}$. **20.** \$ $8\frac{1}{2}$. **21.** $55\frac{1}{8}$.

**Page 57.** — **22.** $\frac{7}{144}$. **23.** Increased $\frac{5}{36}$. **24.** \$$41\frac{1}{10}$; \$$43\frac{13}{20}$. **25.** $32\frac{19}{40}$ lb.; $6\frac{29}{40}$ lb. **26.** $5\frac{7}{8}$ mi.; $1\frac{1}{8}$ mi. **27.** 23 ¢. **28.** $1\frac{7}{20}$ hr.; $\frac{7}{15}$ hr.; $\frac{19}{20}$ hr. **29.** $84\frac{163}{168}$ ¢. **30.** $17\frac{1}{60}$ T.

**Page 60.** — **2.** $2\frac{2}{3}$. **3.** $6\frac{1}{4}$. **4.** $6\frac{3}{4}$. **5.** $6\frac{6}{7}$. **6.** $5\frac{2}{3}$. **7.** $25\frac{2}{3}$. **8.** $8\frac{1}{2}$. **9.** 105. **10.** 36. **11.** 54. **12.** 44. **13.** 9. **14.** 126. **15.** 81. **16.** $4\frac{1}{12}$. **17.** $2\frac{2}{3}$. **18.** $3\frac{1}{3}$. **19.** $2\frac{2}{21}$. **20.** $13\frac{5}{7}$. **21.** $3\frac{3}{4}$. **22.** $1\frac{29}{34}$. **23.** 9. **24.** $7\frac{1}{2}$. **25.** $15\frac{5}{7}$. **26.** $27\frac{9}{13}$. **27.** $11\frac{1}{2}$. **28.** 51.

**Page 61.** — **1.** 552,000 T. **2.** 23,005,284. **3.** 655,200 T.

**Page 62.** — **2.** \$328.50. **3.** \$7.62. **4.** \$192.19. **5.** \$90.57. **6.** \$68.43. **7.** \$181.86. **8.** \$125.38. **9.** \$10.11. **10.** \$97.95. **11.** \$41.40. **12.** \$13.28. **13.** \$61.43. **14.** $966\frac{2}{3}$ mi. **15.** \$85.73. **16.** \$468.31. **17.** \$2.66; \$2.34; \$3.44. **18.** \$74,131.20.

**Page 63.** — **19.** \$10.12.
**2.** 344. **3.** 530. **4.** 865. **5.** 1926. **6.** 5832. **7.** 3620. **8.** $627\frac{1}{3}$. **9.** 4950. **10.** 9373. **11.** 32,202. **12.** 6391. **13.** 15,420. **14.** $4553\frac{1}{7}$. **15.** 12,798. **16.** 5173. **17.** $13,063\frac{1}{2}$. **18.** $16,672\frac{1}{2}$. **19.** 15,449.

**Page 64.** — **20.** $129,808\frac{2}{7}$. **21.** $29,743\frac{1}{5}$. **22.** $80,726\frac{3}{4}$. **23.** $24,970\frac{2}{5}$. **24.** $19,929\frac{3}{5}$. **25.** 33,823. **26.** 31,644. **27.** $31,221\frac{1}{3}$. **28.** $84,458\frac{2}{3}$. **29.** 35,502. **30.** $18,836\frac{3}{10}$. **31.** 22,810. **32.** $39,201\frac{3}{4}$. **33.** $361,150\frac{2}{3}$. **34.** $416,050\frac{4}{5}$. **35.** 198 ft. **36.** \$$1356\frac{1}{4}$. **37.** \$56. **38.** \$1.20. **39.** \$18. **40.** 190 T. **41.** \$15.86. **42.** \$109.50. **43.** \$176,276. **44.** 14,898 lb.

**Page 65.** — **45.** $25\frac{7}{8}$ da. **46.** \$881.25. **47.** \$25,521.88. **48.** 2,187,000 gal. **49.** \$105.

**Page 67.** — **2.** $\frac{2}{9}$. **3.** $\frac{2}{9}$. **4.** $\frac{3}{10}$. **5.** $\frac{3}{8}$. **6.** $\frac{10}{63}$. **7.** $\frac{9}{32}$. **8.** $\frac{5}{8}$. **9.** $\frac{2}{33}$. **10.** $\frac{15}{28}$. **11.** $\frac{5}{12}$. **12.** $\frac{1}{6}$. **13.** $\frac{71}{76}$. **14.** $\frac{27}{37}$. **15.** $\frac{57}{65}$. **16.** $\frac{21}{40}$. **17.** $\frac{1}{8}$. **18.** $9\frac{15}{32}$. **19.** 5. **20.** $10\frac{5}{8}$. **21.** $12\frac{4}{21}$. **22.** $6\frac{3}{10}$. **23.** 3. **24.** $6\frac{5}{9}$. **25.** $5\frac{5}{64}$. **26.** $5\frac{3}{8}$. **27.** $20\frac{31}{32}$. **28.** $45\frac{3}{8}$. **29.** $25\frac{23}{36}$. **30.** $16\frac{29}{36}$. **32.** $118\frac{1}{3}$. **33.** $64\frac{4}{9}$. **34.** $108\frac{1}{2}$. **35.** $2125\frac{1}{5}$. **36.** 7450. **37.** $262\frac{1}{2}$. **38.** $9269\frac{83}{128}$. **39.** 12,222. **40.** $75\frac{3}{80}$. **41.** 86,060. **42.** $3430\frac{175}{176}$. **43.** $1485\frac{4}{5}$.

**Page 68.** — **44.** $450\frac{37}{40}$ mi. **45.** $228\frac{23}{24}$ T. **46.** $90\frac{27}{40}$ mi. **47.** \$841.50.

**Page 70.** — **2.** $\frac{3}{20}$. **3.** $\frac{7}{72}$. **4.** $\frac{15}{112}$. **5.** $\frac{7}{180}$. **6.** $\frac{11}{65}$. **7.** $\frac{2}{75}$. **8.** $\frac{16}{161}$. **9.** $\frac{3}{29}$. **10.** $\frac{9}{85}$. **11.** $\frac{21}{520}$. **12.** $\frac{1}{93}$. **13.** $\frac{1}{63}$. **14.** $\frac{3}{52}$. **15.** $\frac{2}{57}$. **16.** $\frac{2}{85}$. **17.** $\frac{3}{37}$. **18.** $\frac{11}{216}$. **19.** $\frac{5}{258}$. **20.** $\frac{3}{56}$. **21.** $\frac{12}{415}$. **22.** $\frac{3}{226}$. **23.** $\frac{3}{122}$. **24.** $\frac{11}{652}$. **25.** $\frac{9}{508}$. **26.** $\frac{3}{112}$. **27.** $\frac{16}{1375}$. **28.** $\frac{18}{985}$. **30.** $\frac{3}{7}$. **31.** $\frac{3}{5}$. **32.** $\frac{4}{11}$. **33.** $\frac{3}{50}$. **34.** $\frac{6}{25}$. **35.** $2\frac{2}{9}$. **36.** $1\frac{1}{7}$. **37.** $1\frac{9}{16}$. **38.** $1\frac{43}{77}$. **39.** $\frac{7}{8}$. **40.** $\frac{13}{18}$. **41.** $\frac{14}{33}$. **42.** $\frac{11}{26}$. **43.** $\frac{7}{10}$. **44.** $\frac{245}{638}$. **45.** $1\frac{1}{3}$. **46.** $\frac{3}{4}$. **47.** $\frac{7}{18}$.

**Page 71.** — **49.** $3668\frac{27}{77}$. **50.** $1613\frac{29}{32}$. **51.** $816\frac{59}{96}$. **52.** $1375\frac{49}{50}$. **53.** $3306\frac{37}{88}$. **54.** $666\frac{137}{156}$. **55.** $369\frac{13}{180}$. **56.** $2733\frac{9}{10}$. **57.** $883\frac{23}{33}$. **58.** $787\frac{79}{90}$.

**Page 73.** — **1.** 21. **2.** 15. **3.** $28\frac{1}{8}$. **4.** $45\frac{1}{3}$. **5.** $57\frac{3}{5}$. **6.** $51\frac{1}{5}$. **7.** $193\frac{1}{2}$. **8.** $293\frac{3}{4}$. **9.** 1050. **10.** 2058. **11.** 1242. **12.** 2624. **13.** $1\frac{1}{8}$. **14.** $\frac{8}{9}$. **15.** $1\frac{1}{6}$. **16.** $1\frac{1}{2}$. **17.** $1\frac{1}{12}$. **18.** $\frac{7}{8}$. **19.** $2\frac{1}{22}$. **20.** $\frac{3}{4}$. **21.** $\frac{20}{33}$. **22.** $1\frac{1}{2}$. **23.** $\frac{20}{27}$.

**24.** $1\frac{1}{9}$. **25.** $1\frac{4}{9}$. **26.** $1\frac{13}{15}$. **27.** $\frac{18}{19}$. **28.** $1\frac{11}{70}$. **29.** $\frac{77}{117}$. **30.** $1\frac{17}{28}$. **31.** $5\frac{5}{6}$. **32.** $6\frac{2}{3}$. **33.** $1\frac{81}{94}$. **34.** $16\frac{3}{7}$. **35.** $1\frac{19}{25}$. **36.** $21\frac{5}{9}$. **37.** $11\frac{1}{15}$. **38.** $6\frac{1}{4}$. **39.** $14\frac{1}{6}$. **40.** $2\frac{7}{9}$. **41.** $1\frac{1}{8}$. **42.** $8\frac{2}{3}$. **43.** $8\frac{1}{3}$. **44.** $\frac{33}{46}$. **45.** $7\frac{225}{553}$. **46.** $341\frac{1}{3}$. **47.** 65. **48.** 16. **49.** $\frac{9}{10}$. **50.** $1\frac{1}{2}$. **51.** $3\frac{9}{32}$. **52.** 3. **53.** $\frac{3}{8}$. **54.** $3\frac{87}{121}$. **55.** $\frac{275}{378}$. **56.** $\frac{8}{8}$.

**Page 74.** — **1.** 45 da. **2.** $1\frac{3}{4}$. **3.** $\frac{21}{55}$. **4.** \$5000; $\frac{15}{32}$. **5.** \$5.45. **6.** $9\frac{9}{10}$. **7.** $9\frac{63}{95}$. **8.** $25\frac{29}{55}$ hr. **9.** 6125 gal. **10.** $204\frac{13}{14}$ min. **11.** 3090 pupils. **12.** \$2.20.

**Page 75.** — **2.** $\frac{7}{66}$. **3.** $\frac{2}{45}$. **4.** $\frac{8}{9}$. **5.** $\frac{1}{2}$. **6.** $2\frac{1}{9}$. **7.** $\frac{77}{100}$. **8.** $1\frac{5}{16}$. **9.** $\frac{1}{2}$. **10.** $24\frac{24}{25}$. **11.** $7\frac{14}{33}$. **12.** $\frac{9}{16}$. **13.** $\frac{9}{16}$. **14.** $1\frac{1}{35}$. **15.** $33\frac{17}{18}$. **16.** $18\frac{3}{4}$.

**Page 76.** — **3.** $\frac{8}{45}$. **4.** $\frac{5}{12}$. **5.** $\frac{9}{100}$. **6.** $\frac{3}{4}$. **7.** $\frac{5}{28}$. **8.** $\frac{13}{44}$. **9.** $\frac{4}{5}$. **10.** $\frac{8}{69}$. **11.** $\frac{1}{9}$. **12.** $\frac{64}{125}$. **13.** $\frac{22}{25}$. **14.** $\frac{1}{40}$.

**Page 77.** — **3.** 96. **4.** 539. **5.** $4\frac{1}{2}$. **6.** $13\frac{1}{3}$. **7.** $1\frac{7}{9}$. **8.** $1\frac{1}{13}$. **9.** $\frac{23}{24}$. **10.** 7. **11.** \$1300. **12.** \$14,400. **13.** 10,824 books. **14.** 64,725 men. **15.** \$4500.

**Page 78.** — **1.** 35. **2.** Increased $\frac{3}{20}$. **3.** $\frac{20}{23}$. **4.** $25\frac{1}{12}$. **5.** \$64.76. **6.** \$505. **7.** 24. **8.** $149\frac{7}{16}$. **9.** $\frac{19}{144}$. **10.** 14 da. **11.** 160 sq. rd. **12.** $7\frac{13}{16}$ mi. **13.** \$365,250. **14.** $7\frac{19}{40}$. **15.** \$12,800. **16.** \$382.50.

**Page 79.** — **17.** 8 bu. **18.** \$2480. **19.** \$43.20. **20.** \$57. **21.** \$6408. **22.** 354 mi. **23.** $\frac{75}{88}$. **24.** 16 da. **25.** $104\frac{67}{120}$ mi.; $20\frac{547}{600}$ mi. **26.** $30\frac{1}{2}$ rd.; $142\frac{1}{3}$ rd. **27.** \$66.50. **28.** \$6.30. **29.** 108 bu. **30.** $\frac{300}{320}$.

**Page 80.** — **31.** \$12,500. **32.** 168 trees. **33.** 300. **34.** $\frac{11}{80}$. **35.** \$200 gain. **36.** \$70. **37.** \$12, room; \$20, board. **38.** \$16,000, son's; \$14,000, daughter's. **39.** \$900. **40.** $13\frac{2}{5}$. **41.** $8\frac{4}{9}$ hr. **42.** 6 doz.

**Page 81.** — **43.** \$3289.60. **44.** $9\frac{7}{8}$. **45.** \$214.50. **46.** 120 lemons. **47.** 33 ft. **48.** \$40. **49.** $1\frac{1}{134}$. **50.** \$16,000, B's; \$42,000, A's. **51.** $\frac{16}{25}$. **52.** \$67,500. **53.** $4\frac{1}{2}$. **54.** \$1000 gain; 60 A. **55.** 120 A.

**Page 82.** — **56.** \$6000. **57.** $7\frac{1}{2}$ da. **58.** \$7800. **59.** \$1120. **60.** \$2400. **61.** \$5 per barrel. **62.** \$$145\frac{1}{2}$. **63.** \$60. **64.** $262\frac{1}{2}$ lb.

**Page 83.** — **65.** \$151.05. **66.** \$.$81\frac{3}{8}$ gain. **67.** \$1075 gain. **68.** \$5.43. **69.** $76\frac{2}{3}$ ft.; $60\frac{1}{2}$ ft.; $59\frac{1}{3}$ ft.; $67\frac{5}{6}$ ft.; 109 ft. **70.** $55\frac{17}{36}$. **71.** $3906\frac{1}{10}$ mi.; $123\frac{193}{316}$ hr. **72.** 18 hr.; 8 hr.

**Page 84.** — **1.** 31.5 T. **2.** \$.75. **3.** \$120. **4.** 10. **5.** $\frac{1}{3}$. **6.** 9¢. **7.** 510 lots. **8.** 84 sheep. **9.** \$100.

**Page 85.** — **10.** \$18. **11.** \$1.56. **12.** \$7.80. **13.** \$21. **14.** \$1.86. **15.** \$250. **16.** \$350. **17.** 5 lb. 3 oz. **18.** \$2.17. **20.** \$2.35.

**Page 86.** — **21.** \$70.50. **22.** \$11. **23.** $\frac{1}{8}$, $\frac{1}{32}$, $\frac{1}{4}$. **25.** 85 pupils. **26.** \$450. **27.** 128 cu. ft. **28.** \$1690. **29.** 462 cu. in. **30.** \$50. **31.** 56¢. **32.** 224 bu. **33.** \$770.

**Page 87.** — **34.** 6 da. **35.** $\$330\frac{2}{3}$. **36.** \$600. **37.** 2000 bu. **38.** \$1200. **39.** 100 da. **40.** 34 head. **41.** \$1.60. **42.** \$81. **44.** 35 and 5.

**Page 88.** — **45.** \$6000. **46.** 600 pupils. **47.** 300 A.; 200 A. **48.** \$160. **49.** \$720. **50.** 30. **51.** 117. **52.** 32. **53.** 150 A. **54.** 16. **55.** A, \$35; B, \$45. **56.** \$12. **57.** 55. **58.** \$4000.

**Page 89.** — **59.** \$4800. **60.** \$180. **61.** $\frac{3}{4}$. **62.** \$60. **63.** 12 da. **64.** \$96. **65.** \$1050. **66.** \$18. **67.** $\frac{1}{2}$. **68.** $\frac{1}{7}$, $\frac{3}{7}$, $\frac{2}{21}$, $\frac{1}{4}$. **69.** \$300. **70.** \$4800. **71.** 96 gal. **72.** \$2.40.

**Page 93.** — **1.** .34. **2.** .0675. **3.** 16.000075. **4.** 400.045. **5.** 6006.0066. **6.** 89.005. **7.** .0746. **8.** 900.000084. **9.** 5,000,009.0000409. **10.** .005095. **11.** 8.0017. **12.** .000125. **13.** 896.00301. **14.** 1000.001. **15.** 18,051.957. **16.** 97.0003. **17.** .009864. **18.** 2135.000032. **19.** 1.000001. **20.** 1,000,000.1. **21.** 90,000.071. **22.** 1830.11684. **23.** 429,000.0046. **24.** 7035.97. **25.** 67,375.00035. **26.** .05815. **27.** 375.069.

**Page 94.** — **28.** 419,863.023456. **29.** 81.00921. **30.** 2986.0298643. **31.** 3020.00302. **32.** 70.07. **33.** 8000.008. **34.** 645,000,000.000009.

**Page 95.** — **1.** $\frac{1}{4}$. **2.** $\frac{7}{10}$. **3.** $\frac{9}{200}$. **4.** $\frac{1}{40}$. **5.** $\frac{1}{2}$. **6.** $\frac{3}{4}$. **7.** $\frac{1}{400}$. **8.** $\frac{31}{400}$. **11.** $\frac{3}{8}$. **12.** $\frac{5}{12}$. **13.** $\frac{2}{3}$. **14.** $\frac{29}{120}$. **15.** $\frac{5}{6}$. **16.** $\frac{1}{16}$. **17.** $\frac{1}{24}$. **18.** $\frac{1}{12}$. **19.** $\frac{5}{8}$. **20.** $\frac{1}{3}$. **21.** $\frac{5}{16}$. **22.** $\frac{1}{30}$. **23.** $\frac{1}{8}$. **24.** $\frac{1}{7}$. **25.** $\frac{7}{12}$. **26.** $\frac{53}{60}$.

**Page 96.** — **5.** 111.55. **6.** 187.64. **7.** 66.51. **8.** 105.9. **9.** 168.507. **10.** 19.679. **11.** 179.992. **12.** 103.422. **13.** 471.6. **14.** 1.266. **15.** 2.2249. **16.** 210.80. **17.** 16.2. **18.** 230.4909. **19.** 25.823. **20.** 45.3407. **21.** 35.755.

**Page 97.** — **22.** 11.1751. **23.** 354.32685. **24.** 189.61752. **25.** 343.9706. **26.** 2.282566. **27.** 984.37936. **28.** 298.09398. **29.** 266.1172. **30.** 454.4365. **31.** 858.3827. **32.** 47.0625 lb. **33.** 23.0875 A. **34.** 230.7177. **35.** \$7154.95. **36.** 866.9375 sq. ft.

**Page 98.** — **37.** .5675. **38.** .0466. **39.** 2.4194. **40.** 9.993. **41.** 19.724. **42.** 33.58. **43.** 38.068. **44.** 71.125. **45.** 113.8909. **46.** .101. **47.** 100.192. **48.** 26.483. **49.** .101. **50.** 4.3935. **51.** 8.2415. **52.** .01. **53.** 13.5. **54.** 18.021. **55.** 99.91. **56.** 58.625 yr. **57.** 161.6.

**Page 100.** — **3.** .055. **4.** .0476. **5.** .00081. **6.** 3.24. **7.** 14.7. **8.** 80.214. **9.** 4.25. **10.** .486. **11.** .011055. **12.** 911.2. **13.** 339. **14.** 2759.1. **15.** 719.44. **16.** .0003424. **17.** .030335. **18.** 125.472. **19.** 42.504. **20.** .1183952. **21.** 2.0736. **22.** 42.6725. **23.** 1203.03208. **24.** 1.925. **25.** 1.296. **26.** 6. **27.** 7.936. **28.** 31.296. **29.** 489.06. **30.** 73.8. **31.** 2.352. **32.** 37.468. **33.** 10.7325. **34.** 59.2144. **35.** 3135.65. **36.** 12.054. **37.** .0428796. **38.** .12. **39.** 28.65. **40.** .000225. **41.** 7.762536. **42.** 2.75804. **43.** 328.252. **44.** .12048. **45.** .760125. **46.** 404.02944. **47.** .61548. **48.** 14.8356. **49.** .04936. **50.** .000608. **51.** 1.006008. **52.** .00025. **53.** 30.04812. **54.** \$3888.75. **55.** \$1200; \$3600; \$4800. **56.** \$20,530.25. **57.** 472.44 in.; 13.123+ yd.

**Page 102.** — **3.** 12.02. **4.** 141.6. **5.** 6.07. **6.** 1.5. **7.** 5.06. **8.** .1025. **9.** .03002. **10.** 20.03. **11.** .084. **12.** .061. **13.** .87. **14.** .009. **15.** .075. **16.** .456. **17.** .15. **18.** 50.6. **19.** 10.25. **20.** 3.002. **21.** 200.3. **22.** .904. **23.** .0906. **24.** .704. **25.** 9.2945+. **26.** .0027. **27.** .084. **28.** .0704.

**Page 105.** — **3**. 15. **4**. 1.8. **5**. 2.21. **6**. 2.05. **7**. 1200. **8**. .012 **9**. 560. **10**. 1.24. **11**. 12.5. **12**. .0207. **13**. .8. **14**. 745. **15**. 9.7. **16**. 248. **17**. .018. **18**. .0055. **19**. 1,000,000. **20**. 2.5. **21**. 2.44. **22**. 115.8. **23**. 16.4. **24**. .0027. **25**. 17,500. **26**. .033. **27**. .000011. **28**. 111,000. **29**. 100. **30**. .0002. **31**. 210. **32**. .00137. **33**. .00023. **34**. 125. **35**. 26.175. **36**. 1700.82. **37**. 14,096.4043.

**Page 106.** — **38**. .001. **39**. 83 da. **40**. \$.405. **41**. 88.5. **42**. \$18.25. **43**. .009. **44**. 570.8108+. **45**. 1.25. **46**. 18.5 sq. rd. **47**. \$20,000.
**1**. .625. **2**. .6. **3**. .416$\frac{2}{3}$. **4**. .3125. **5**. .875. **6**. .44. **7**. .5625. **8**. .59375. **9**. .4375. **10**. .68.

**Page 107.** — **11**. .6363+. **12**. .2903+. **13**. .6164+. **14**. .5471+. **15**. .6296+. **16**. .4736+. **17**. 1.5714+. **18**. 1.4705+. **19**. 1.2962+. **20**. .7551+. **21**. .5384+. **22**. .4545+. **23**. .0166+. **24**. .4615+. **25**. 1.5882+. **26**. .75.
**1**. 19.97. **2**. 1.4714. **3**. 214.065. **4**. 4.9222. **5**. 5.996+. **6**. .02+. **7**. 96.309125. **8**. 16.2921. **9**. 130.5808. **10**. 2.31225. **11**. 15.5225. **12**. 104.1635. **13**. 10.5805$\frac{7}{12}$. **14**. 105.4865. **15**. 9.7416$\frac{2}{3}$. **16**. 245.494275.

**Page 109.** — **2**. \$20. **3**. \$6.25. **4**. \$60. **5**. \$6.25. **6**. \$10. **7**. \$45. **8**. \$200. **9**. \$300. **10**. \$200. **11**. \$100. **12**. \$5.37$\frac{1}{2}$. **13**. \$1000. **14**. \$16.

**Page 110.** — **15**. \$585. **16**. \$528. **17**. \$760.

**Page 111.** — **2**. 2560 lb. **3**. 3000 lb. **4**. 60 lb. **5**. 200 yd. **6**. 260 yd. **7**. 150 yd. **8**. 12,800 qt. **9**. 800 lb. **10**. 32 collars. **11**. 60 bananas. **12**. 1600 lb.; 750 bu.; 2400 cakes. **13**. 48 yd.; 48 lb. **14**. 260 lb.

**Page 113.** — **1**. \$5.02. **2**. \$4.22. **3**. \$1.75. **4**. \$.90. **5**. \$2.80. **6**. \$6.20.

**Page 114.** — **7**. \$6.50. **8**. \$10.65. **9**. \$10.60. **10**. \$28.65. **11**. \$21.65.

**Page 117.** — **3**. 248 oz. **4**. 1157 in. **5**. 176 qt. **6**. 56 pt. **7**. 1825 lb. **8**. 1396 min. **9**. 14,608 yd. **10**. 27 in. **11**. 9600 lb. **12**. 465 sec. **13**. 14,560 lb. **14**. 508 pt. **15**. 28 qt. **16**. 43 pk. **18**. 1 rd. 4 yd. **19**. 3 mi. 244 rd. 3 yd. **20**. 1 mi. 1 ft. **21**. 34 min. 3 sec. **22**. 64 lb. 9 oz. **23**. 17 T. 7 cwt. **24**. 3 T. 95 lb. **25**. 52 mi. 217 rd. **26**. 346 bu. 3 pk. 1 qt. **27**. 119 gal. **28**. 221 rd. 4 yd. 1 ft. 6 in. **29**. 1 hr. 54 min. 35 sec. **30**. 14 bu. .5 pk. **31**. 2 mi. 44.5 rd. **32**. 16 hr. 4$\frac{3}{5}$ min. **33**. 46$\frac{7}{8}$ gal. **34**. \$3.84.

**Page 118.** — **35**. \$39.83. **36**. \$26.88. **37**. \$41.60. **38**. 1920 hr. **39**. 32 mi. **40**. 44 mi. 80 rd. **41**. 3 mi. 40 rd.
**3**. $\frac{1}{9}$. **4**. $\frac{5}{11}$. **5**. $\frac{7}{8}$. **6**. $\frac{1}{4}$. **7**. $\frac{1}{48}$. **8**. $\frac{3}{16}$.

**Page 119.** — **9**. $\frac{3}{4}$. **10**. $\frac{7}{64}$. **11**. $\frac{7}{8}$. **12**. $\frac{3}{8}$. **13**. $\frac{1}{4}$. **14**. .7. **15**. .1. **16**. .09. **17**. .28. **18**. \$2.95.
**2**. 29 bu. 3 qt. 1 pt. **3**. 40 T. 11 cwt. 4 lb. **4**. 21 yr. 239 da. 1 hr. **5**. 16 wk. 1 da. 14 hr.

**Page 120.** — **7**. 22 mi. 194 rd. 4 yd. 2 ft. 6 in. **8**. 13 gal. 1 qt. 1 pt. **9**. 8 hr. 43 min. 15 sec. **11**. 6 yr. 9 mo. 27 da. **12**. 15 yr. 16 da. **14**. Lee, 58 yr. 2 mo. 20 da.; Grant, 42 yr. 11 mo. 12 da.; dif. 15 yr. 3 mo. 8 da.

**Page 121.** — **2.** 10 gal. 3 qt. 1 pt. **3.** 77 bu. 2 qt. **4.** 152 T. 10 cwt. 7 lb. 8 oz. **5.** 247 wk. 4 da. 6 hr. **6.** 280 mi. 216 rd. 1 yd. 1 ft. **8.** 3 wk. 5 da. 9 hr. **9.** 26 gal. 3 qt. 1 pt. **10.** 9 bu. 3 pk. 4 qt. **11.** 5 yr. 7 mo. 18 da. **12.** 14 T. 7 cwt. 24 lb. **13.** $22.80. **14.** 26 rd. 1 yd. **15.** 16 packages.

**Page 122.** — **1.** 6 min. 18 sec. **2.** 32 bbl. **3.** $55\frac{15}{32}$ bu. **4.** $18.30. **5.** 137 T. 12 cwt. 88 lb. **6.** 50 ft. 8 in. **7.** 110,340 gal. **8.** 35.8605 T. **9.** 3.45 lb.; $1.29+. **10.** 54 mi. 200 rd. **11.** 20,007 doz.

**Page 126.** — **20.** 272.25 sq. ft. **21.** 2800 sq. rd.; $17\frac{1}{2}$ A. **23.** 24,300 sq. ft. **24.** 7200 sq. in. **25.** 10 A. **26.** 260 sq. rd. **27.** $26\frac{54}{121}$ sq. rd. **28.** 250,470 sq. ft. **29.** $33\frac{3}{4}$ A.; $2025. **30.** 8488 sq. ft. **31.** $76. **32.** $170.10. **33.** $40,425.

**Page 127.** — **34.** $6450. **35.** 640 A.; $54,400. **36.** $17\frac{3}{5}$ bu. **37.** $500; $ $.16\frac{2}{3}$. **38.** $88.32. **39.** 40 rd. **40.** (3) $2\frac{103}{160}$ A. **41.** (3) $1\frac{97}{160}$ A.

**Page 128.** — **1.** $3. **2.** $8. **3.** 146 sq. yd. **4.** $29.55. **5.** $181.

**Page 129.** — **5.** 40 sq. in. **6.** 36 sq. in. **7.** 225 sq. in. **8.** 432 sq. in. **9.** 10 A.

**Page 133.** — **2.** 628 sq. ft. **3.** 120 ft. **4.** $37.21. **5.** $62.80. **6.** 72. **7.** 288 cu. ft. **8.** $166\frac{2}{3}$ loads; $ $41\frac{2}{3}$. **9.** 99 sq. ft. **10.** 810,000 cu. in.; $3506\frac{38}{77}$ gal. **11.** $462.

**Page 134.** — **13.** $\frac{27}{64}$; 48 oz. **14.** $153\frac{3}{5}$ bu. **15.** $4308\frac{60}{77}$ gal. **16.** 2800 sq. rd. larger. **17.** 4 ft. **18.** 64 sq. ft.; 384 sq. ft.

**Page 135.** — **4.** 24 bd. ft. **5.** 9 bd. ft. **6.** 45 bd. ft. **7.** 48 bd. ft. **8.** 100 bd. ft. **9.** 360 bd. ft.

**1.** $200. **2.** $12. **3.** $260.40. **4.** $400.

**Page 136.** — **6.** $68.75. **7.** $7.17. **8.** $13.44. **9.** $2.52. **10.** $8.96. **11.** $67.20. **12.** $40.32. **13.** $58.80. **14.** $26.88. **15.** 16 ft. **16.** 2112 bd. ft. **17.** 224 bd. ft. **18.** $25.96.

**Page 137.** — **1.** 20 cords. **2.** $440. **3.** $15. **4.** $44.80. **5.** $12\frac{1}{2}$ cords.

**Page 138.** — **1.** 540 tiles. **2.** $289\frac{2}{7}$ ft. **3.** $277\frac{1}{3}$ cu. yd. **4.** $105. **5.** 1,760,000 times. **6.** $24. **7.** $327.68. **8.** $7181\frac{23}{77}$ gal. **9.** $168. **10.** $16.33. **11.** 50 rd. **12.** $4.84. **13.** 10 rd.

**Page 139.** — **14.** 1728 cakes. **15.** $116.67. **16.** 792 bd. ft.; $23.76. **17.** 15 A. **18.** 2048 bu. **19.** $4847\frac{29}{77}$ gal. **20.** 150 ft. **21.** $96.80. **22.** 40 ft. **23.** $800.

**Page 140.** — **25.** 400 ft.; 292 ft.; 292 ft.; 26 ft.; 216 ft. **26.** $1111\frac{1}{9}$ sq. yd. **27.** $511\frac{1}{9}$ sq. yd. **28.** $4\frac{4}{9}$ sq. yd. **29.** $88\frac{8}{9}$ sq. yd. **30.** $ $90\frac{10}{11}$. **31.** $137.78. **32.** $.22. **33.** $126.67. **34.** $\frac{9}{16}$ ft. **35.** $23.11. **36.** $2154\frac{30}{77}$ gal. **37.** $16.80. **38.** 60 farms. **39.** 3000 sods. **40.** $62.40.

**Page 144.** — **2.** 375. **3.** 21.375. **4.** 3.5. **5.** 235.2. **6.** 320. **7.** 735. **8.** $16.20.

**Page 145.** — **9.** 312. **10.** $30.

**Page 146.** — **12.** \$12. **13.** 2 mo. **14.** 18 da. **15.** 1 A. **16.** 5 A. **17.** 32. **18.** 28 horses. **19.** $3\frac{3}{10}$. **20.** 9 lb. **21.** \$$1\frac{1}{8}$. **22.** 20. **23.** \$200. **24.** \$50. **25.** 8 words. **26.** \$.20. **27.** 230. **28.** \$3125. **29.** \$60. **30.** 90 A. **31.** \$400. **32.** \$150. **33.** 1209 people.

**Page 147.** — **2.** \$90. **3.** \$79.70. **4.** \$97.70. **5.** \$70.04. **6.** \$76.40. **7.** \$118.53. **8.** \$532.37. **9.** \$135, commission; \$6615, net proceeds. **10.** \$1800. **11.** \$456.

**Page 148.** — **1.** \$12. **2.** \$12. **3.** \$3. **4.** \$20. **5.** \$6. **6.** \$7.50. **7.** \$16. **8.** \$36. **9.** \$5.10. **10.** \$3. **11.** \$8. **12.** \$3.
**1.** \$126.56. **2.** \$1145. **3.** \$1248. **4.** \$174. **5.** \$1818. **6.** \$180.52. **7.** \$612.85. **8.** \$251.01. **9.** \$2283.36. **10.** \$6436.44. **11.** \$176.64. **12.** \$52.90.

**Page 149.** — **14.** \$288. **15.** \$267.54. **16.** \$157.03. **17.** \$6.13. **18.** \$4.59. **19.** \$7.65. **20.** \$129.60. **21.** \$4080. **22.** \$1890. **23.** \$1353.75. **24.** \$53.55. **25.** \$64.60. **26.** \$291.60; \$21.60 dif. **27.** \$641.25. **28.** Both, \$360.

**Page 150.** — **30.** \$6643. **31.** \$31.82. **32.** \$583.20; \$648. **33.** \$203.74.

**Page 152.** — **2.** \$15. **3.** \$128. **4.** \$29.25. **5.** \$126. **6.** \$15. **7.** \$12. **8.** \$25.67. **9.** \$18. **10.** \$6.50. **11.** \$52.50. **12.** \$43.20. **13.** \$173.25. **14.** \$29.43. **15.** \$15.40.

**Page 153.** — **16.** \$3. **17.** \$10. **18.** \$19.50. **19.** \$12.48. **20.** \$6.80. **21.** \$7.69.

**Page 154.** — **2.** \$63; \$363. **3.** \$40.83; \$290.83. **4.** \$34; \$194. **5.** \$4.17; \$54.17. **6.** \$152; \$952. **7.** \$3.81; \$54.61. **8.** \$.64; \$16.64. **9.** \$3; \$78. **10.** \$35; \$455. **11.** \$2.63; \$43.13. **12.** \$8.76; \$309.16. **13.** \$4.08; \$104.08. **14.** \$27.50; \$527.50. **15.** \$5; \$1005. **16.** \$3.23; \$63.83. **17.** \$69.44; \$319.44. **18.** \$15.64; \$91.44. **19.** \$251.25; \$1751.25. **20.** \$24.85; \$150.35. **21.** \$296.78; \$1436.78. **22.** \$130.68; \$1043.28. **23.** \$701.17; \$3910.17. **24.** \$48.22; \$682.72. **25.** \$297.

**Page 155.** — **1.** \$33. **2.** \$800. **3.** \$3. **4.** 30 girls. **5.** .0125; .125; 1.375; .0625. **6.** \$12.50. **7.** \$64. **8.** \$22.50. **9.** \$.83. **10.** \$76.50. **11.** \$.24. **12.** \$1.50.

**Page 156.** — **13.** \$1.32. **14.** \$45. **15.** \$2. **16.** \$420. **17.** \$20. **18.** 20,000. **19.** \$325. **20.** \$3. **21.** \$9.

**Page 158.** — **7.** \$8.75. **4.** \$274.32.

**Page 159.** — **1.** 110,092. **2.** 37. **3.** 50.601. **4.** 25.061; 125.5; 300.0002. **5.** 4742. **6.** .0119. **8.** $8\frac{5}{6}$. **10.** 55.12. **11.** \$.30. **12.** $277\frac{1}{2}$ ft. **13.** 39 mi. 319 rd. 5 yd. 1 ft. 1 in.

**Page 160.** — **14.** \$132. **15.** \$2160. **16.** \$103.74. **17.** $2\frac{2}{9}$. **18.** .1. **19.** $\frac{75}{128}$. **20.** \$42. **21.** $38\frac{106}{145}$ mi. **22.** $2\frac{3}{4}$ T. **23.** \$375.75. **24.** \$1080. **25.** \$100. **26.** \$272.50. **27.** 56 A.

**Page 161.** — **28.** \$3892.50. **29.** \$16.80. **30.** 1 da. 2 hr. 53 min. 20 sec. **31.** \$750. **32.** \$20.17. **33.** $\frac{25}{4}$, $\frac{27}{8}$, $\frac{14}{3}$, $\frac{41}{20}$. **34.** \$288. **35.** $2\frac{3803}{5280}$ mi. **36.** 648,945 cu. in.; $375\frac{35}{64}$ cu. ft. **37.** $26\frac{2}{3}$% gain.

**Page 162.** — **38.** 8; \$1.60; \$20.83$\frac{1}{3}$. **39.** \$5625. **40.** \$101.25. **41.** \$4.50. **42.** 20 rd. **43.** \$707$\frac{3}{8}$. **44.** \$55.08. **45.** \$2.

**Page 163.** — **47.** 156$\frac{4}{9}$ cu. yd. **48.** 4 rd. 1 ft. 6 in. **49.** 28 rd. **52.** \$420. **53.** 10%. **54.** 62.5 da. **55.** 24 rd. **56.** 48 people.

**Page 164.** — **57.** 999.00199+. **58.** \$16.88. **59.** 177 rd. 4 yd. 10 in. **60.** \$8. **61.** 2 lb. 8 oz. **62.** \$1900. **63.** 1152 gal. **64.** \$36. **65.** 1$\frac{11}{16}$. **66.** John, 55 A.; James, 110 A. **67.** $\frac{61}{135}$ da. **68.** .0072. **69.** \$46.20. **70.** \$.18.

**Page 165.** — **71.** \$193.33. **72.** 50$\frac{21}{10}$ A. **73.** 1108.8 ft. **74.** \$1926. **75.** \$14.40. **76.** $\frac{7}{18}$. **77.** \$1800. **78.** \$111.30. **79.** 320 rd. **80.** 2.5. **81.** \$247.50. **82.** \$40.50. **83.** \$26.97. **84.** Latter by \$75.

**Page 169.** — **1.** \$564.35.

**Page 170.** — **2.** \$257.53. **3.** \$365.80. **4.** \$213.17. **5.** \$12,677.94. **6.** \$6.20.

Note. — In business, a half cent or more is usually counted as an additional cent.

**Page 172.** — **1.** \$159.88. **2.** \$107.21. **3.** \$17.87.

**Page 174.** — **1.** By Bal. \$7.87. **2.** By Bal. \$97.68.

**Page 178.** — **3.** 920 qt. **4.** 218 pt. **5.** 247 hr. **6.** 2640 rd. **7.** 22,160 min. **8.** 33,585 lb. **9.** 8765$\frac{1463}{1800}$ hr. **10.** 924 in. **11.** 423$\frac{1}{2}$ ft. **12.** 31,451.35 ft. **13.** 184 oz. **15.** 28 oz. **16.** 112 pt. **17.** 174,960 in. **18.** 280 rd. **19.** 16 cwt. 80 lb. **20.** 202 da. 18 hr. 40 min. **21.** 128$\frac{1}{3}$ ft. **22.** 5 da. 6 hr. **23.** 255 min. **24.** 9768 ft.

**Page 179.** — **2.** 56 gal. 1 qt. **3.** 20 bbl. 7 gal. 2 qt. **4.** 365 bu. 3 pk. **5.** 10 da. 10 hr. **6.** 16 rd. 1 ft. 4 in. **7.** 94 min. 35 sec. **8.** 1 mi. 58 rd. 4 yd. 1 ft. **9.** 2 leagues. **10.** 4 T. 2020 lb. **11.** 45 rd. 3 yd. 2 ft. 7 in. **13.** .02$\frac{25}{44}$ mi. **14.** .30025 T. **16.** $\frac{7}{96}$ ft.; $\frac{7}{288}$ yd. **17.** $\frac{3}{4}$ lb.

**Page 180.** — **18.** $\frac{3}{4}$. **19.** .87+. **20.** \$20.36. **21.** \$6. **22.** 111 boxes. **23.** 150 qt. **24.** \$1.08. **25.** 113,796 ft. **26.** \$263.01. **27.** 191 bottles. **28.** 9500 packages. **29.** \$1.65.

**Page 181.** — **30.** 12,300 ft. **31.** 429.14 lb. **32.** 13 mi. 252 rd. 2 ft. **33.** \$4.

**Page 183.** — **1.** \$182.49. **2.** \$130.27. **3.** \$111.75. **4.** \$170.33. **5.** \$136.85. **6.** \$73.34. **7.** \$151.85. **8.** \$29.20. **10.** \$50. **11.** \$131.62. **12.** \$481.64. **13.** \$23.94. **14.** \$425.89. **15.** \$148.42. **16.** \$20.22. **17.** \$143.79. **19.** £160 7*s.* 7.72*d.* **20.** 500 marks. **21.** 144 lire.

**Page 184.** — **3.** 133 bu. 2 pk. 7 qt. **4.** 97 hr. 52 min. 39 sec. **5.** 5 hr. 46 min. **6.** 11 yr. 9 mo. 3 wk. 5 da.

**Page 185.** — **8.** 57 yr. 2 mo. 8 da. **9.** 84 yr. 9 mo. 10 da. **10.** 42 yr. 11 mo. 12 da. **11.** £2006 18*s.* 9*d.* **12.** £114 10*s.* 1*d.*; £62 4*s.* 10*d.*; £192 9*s.* 4*d.* **13.** 32 hr. 40 min. **14.** 5 T. 4 cwt. 15 lb.

**Page 186.** — **3.** 52 bu. 8 baskets. **4.** 8 yd. 25$\frac{1}{2}$ in. **5.** 10,800 ft. **6.** 42 gal. 2 qt. 1 pt.

**Page 187.** — 7. Widow, \$9733.65; children, \$5840.19. **8.** \$8.40. **9.** 5940 turns. **10.** \$13.89. **11.** 96 mi. 284 rd. 4 yd. **12.** 165 T. **13.** \$9.30. **14.** 240 ft. 11 in.; 8 lb. 10 oz.; 27 bu. 2 pk. 5 qt. **15.** \$115.07. **16.** \$74.31.

**Page 188.** — **1.** \$73.88. **2.** \$305.50. **3.** $\frac{5}{32}$. **4.** 10 bbl. **5.** \$166.88. **6.** 10,000. **7.** $2\frac{1}{2}$ doz. **8.** 94.435. **9.** $54.41\frac{7}{9}$ mi. **10.** 401.44 mi. **11.** \$.0311+.

**Page 189.** — **12.** 55.97+ mi. per hour. **13.** 191.75 lb. **14.** $522.708\frac{1}{3}$. **15.** $1071\frac{3}{7}$ bu. **16.** 7¢. **17.** \$675. **18.** \$47.50. **19.** $.977\frac{4}{297}$ min. **20.** 84.5375 mi. **21.** 28.75 rd. **22.** 1876 lb. **23.** 7.

**Page 190.** — **24.** \$3.85. **25.** 100 bbl. **26.** \$1000. **27.** 3.74624625. **28.** $413\frac{1}{3}$. **29.** .138. **30.** \$8.50. **31.** 136 books. **32.** \$60,000. **33.** \$2018.75.

**Page 191.** — **34.** \$350. **35.** \$24,000. **36.** \$9.10. **37.** \$6000. **38.** 2111.101. **39.** 849.404. **40.** 6270.835. **41.** 101,260.269. **42.** 1034.25478.

**Page 193.** — **1.** 480 sq. in. **2.** 675 sq. rd. **3.** .78125 A. **4.** 60 sq. rd.

**Page 194.** — **5.** 174,240 sq. ft. **6.** 108 sq. in. **7.** 176,418 sq. in. **8.** 256,000 sq. rd. **9.** 11 A. 40 sq. rd. **10.** 11 sq. ft. **11.** 294,030 sq. ft. **12.** 94.31625 A.; \$29,473.83.

**Page 200.** — **2.** 246 sq. ft. **3.** 144.9 sq. ft. **4.** 2916 sq. ft. **5.** 218.5 sq. ft. **6.** 517 sq. ft. 72 sq. in. **7.** 945 sq. ft. **8.** 27 sq. ft. **9.** 54 sq. yd. **10.** 180 sq. yd. **11.** 2400 sq. yd. **12.** First 16 times as large as second; first $\frac{1}{4}$ as large as second. **13.** \$30.94.

**Page 201.** — **14.** $5\frac{5}{33}$ A. **15.** 30 rd. **16.** 3 lots.
**2.** \$40. **3.** \$7.89. **4.** 89 bundles.

**Page 202.** — **5.** \$54.15. **6.** \$305.21. **7.** \$231.22.
**1.** \$134.40. **2.** 12,960 tiles.

**Page 203.** — **3.** 40 sq. in. **4.** 360 slates. **5.** 240 slates; 450 lb. **6.** 5760 slates; \$88. **7.** 157 bundles. **8.** \$172.38. **9.** \$105. **10.** 30 squares.

**Page 204.** — **2.** 15 rolls. **3.** \$18.

**Page 205.** — **5.** $55\frac{7}{18}$ yd.; $73\frac{17}{18}$ yd. **6.** 5, 5, 6, 6, 6. **7.** 7, 6, 7, 8, 8. **8.** \$14.67. **9.** \$45.84.

**Page 207.** — **2.** 9 sq. ft. **3.** 140 sq. ft. **4.** 60 sq. ft. **5.** 150 sq. ft. **6.** 450 sq. ft.

**Page 208.** — **1.** $52\frac{1}{2}$ A. **2.** 28.4062 A. **3.** 75. **4.** 25 rd. **5.** 7 rd. **6.** 80 rd.
**2.** 7 sq. ft. 42 sq. in.

**Page 209.** — **3.** 10 rd. **4.** 30 rd.
**2.** 440 sq. ft. **3.** 308 sq. ft.

**Page 210.** — **1.** 47.124 ft. **2.** 78.54 ft. **3.** 188.496 ft. **4.** 33.5104 ft. **5.** 39.27 ft. **6.** 78.54 yd. **7.** 126.7112 ft. **8.** 7.854 ft. **9.** 82.7282 ft. **10.** 4 ft. **11.** 40 yd.

**Page 211.** — **1.** 78.54 sq. rd. **2.** 314.16 sq. rd. **3.** 254.4696 sq. rd. **4.** 1017.8784 sq. ft. **5.** 314.16 sq. in. **6.** 1256.64 sq. in. **7.** 5026.56 sq. rd. **8.** 490.875 sq. rd. **9.** 5026.56 sq. ft. **10.** .78 sq. ft. **11.** 3.14 sq. rd. **12.** 706.86 sq. in. **13.** 962.11 sq. yd. **14.** 314.16 sq. ft. **15.** 78.54 sq. yd. **16.** 49,087.5 sq. ft. **17.** 11,309.76 sq. yd. **18.** 1134.11 sq. yd. **19.** 21.46 sq. ft. **20.** $50.27.

**Page 213.** — **1.** 432 sq. ft. **2.** 68 sq. ft. 8 sq. in. **3.** 118 sq. ft. **4.** 412 sq. ft. **5.** 170 sq. ft. **6.** 334 sq. ft. **7.** 766 sq. ft. **8.** 96 sq. in. **9.** 6 sq. ft. **10.** 24 sq. ft. **11.** 6 sq. ft. $73\frac{1}{2}$ sq. in. **12.** 8 sq. ft. 24 sq. in. **13.** $88\frac{1}{6}$ sq. yd. **14.** 8 sq. ft. $109\frac{1}{2}$ sq. in.

**Page 214.** — **2.** 250 cu. ft. **3.** 684 cu. ft. **4.** 800 cu. ft. **5.** 9 cu. ft. **6.** 4 cu. ft. 1248 cu. in. **7.** 5 cu. yd.

**Page 215.** — **8.** $213\frac{1}{3}$ loads. **9.** 1728 cakes. **10.** 448 cu. ft. **11.** 5 C. **12.** $22\frac{1}{2}$ C. **13.** 64 boxes. **14.** $22\frac{1}{2}$ C. ; $41.63. **15.** $23\frac{1}{8}$ C. ; $42.78. **16.** $28\frac{1}{8}$ C. ; $52.03. **17.** $18\frac{3}{4}$ C. ; $34.69. **18.** $106\frac{2}{3}$ loads.

**Page 216.** — **19.** $56\frac{1}{4}$ C. **20.** $\frac{1}{27}$. **21.** $1417\frac{1}{2}$ cu. in. **22.** 111.475 cu. in. **23.** 126.6 cu. in. **24.** $1621\frac{1}{2}$ cu. in. **25.** $10,936\frac{3}{4}$ cu. in. **26.** $1595\frac{5}{8}$ cu. in. **27.** 189 cu. in. **28.** First; 1440 cu. in. **29.** $124\frac{25}{32}$ cu. in.; $451\frac{9}{16}$ cu. in.

**Page 218.** — **2.** 15 bd. ft. **3.** 24 bd. ft. **4.** 32 bd. ft. **5.** 180 bd. ft. **6.** 96 bd. ft. **7.** 36 bd. ft. **8.** 150 bd. ft. **9.** 144 bd. ft. **10.** 432 bd. ft. **11.** 360 bd. ft. **12.** 2000 bd. ft. **13.** 720 bd. ft. **14.** 1200 bd. ft. **15.** 240 bd. ft. **16.** $21.60.

**Page 219.** — **1.** $419.06. **2.** $28. **3.** $47.25. **4.** $26.25. **5.** $39.38. **6.** $10.50. **7.** 960 bd. ft. **8.** 12,800 bd. ft. **9.** 3500 bd. ft. **10.** 4800 bd. ft. **11.** 1600 bd. ft. **12.** 1500 bd. ft. **13.** 8000 bd. ft. **14.** 21,000 bd. ft. **15.** 8750 bd. ft. **16.** 1200 bd. ft. **17.** $16,666\frac{2}{3}$ bd. ft. **18.** 18,200 bd. ft. **19.** 24,000 bd. ft. **20.** 7000 bd. ft. **21.** 3600 bd. ft. **22.** 10,000 bd. ft.

**Page 220.** — **23.** $837. **24.** $1488. **25.** $118.56.
**1.** 448 cords. **2.** $805\frac{25}{27}$ cu. yd.

**Page 221.** — **3.** $100\frac{20}{27}$ loads; $503\frac{19}{27}$ loads; $201\frac{13}{27}$ loads. **4.** $6429.63. **5.** $488\frac{8}{9}$ cu. yd. **6.** 37,842 bricks. **7.** 41,328 bricks. **8.** $364.80.

**Page 222.** — **1.** 753.984 sq. in. **2.** 1413.72 sq. in. **3.** 62.832 sq. ft. **4.** 20.944 sq. ft. **5.** 8.3776 sq. ft. **6.** 282.744 sq. ft. **7.** 678.5856 sq. ft. **8.** 155.5092 sq. ft.

**Page 223.** — **10.** 3180.87 cu. ft. **11.** 14,726.25 cu. ft. **12.** 4021.248 cu. ft. **13.** 4712.4 cu. ft. **14.** 100.5312 cu. ft. **15.** 6031.872 cu. ft. **16.** 12.5664 cu. ft. **17.** 235.62 cu. ft.
**1.** 803.56+ bu. **2.** 520.71+ bu. **3.** 448.83 gal. **4.** 119.36+ bbl. **5.** 4.94+ bbl.

**Page 224.** — **1.** 7500 lb. ; 120 cu. ft. **2.** $1\frac{203}{315}$ T. hard; $1\frac{10}{27}$ T. soft. **3.** $29,687\frac{1}{2}$ lb. **4.** $5333\frac{1}{3}$ bbl. **5.** 672 bu.

**Page 225.** — **6.** $5\frac{17}{35}$ T. **7.** 471.24 gal. **8.** 84,823.2 lb. **9.** 1958.4 bu. **10.** $31\frac{23}{35}$ T. ; $38\frac{2}{7}$ T.
**1.** 50. **2.** 15 ft. **3.** 3280 tiles. **4.** 400 sq. in.; 80 sq. in. **5.** 1944 sq. in.; 5832 cu. in. **6.** 918 boxes. **7.** 216 cakes.

**Page 226.** — **8.** 264 sq. in. **9.** $57,600. **10.** $800. **11.** $7168. **12.** $398.47. **13.** 63,910 bricks. **14.** $553.96.

**Page 227.** — **15.** 1.32+ ft. **16.** $294\frac{21}{40}$ T. **17.** 72 yd. **18.** $113.47. **19.** $24.70. **20.** Grading $798; $52,800. **21.** 60°. **22.** 78°.

**Page 228.** — **23.** 75°; 30°. **24.** $76\frac{1}{4}$°; 90°. **25.** $80\frac{3}{4}$°. **26.** A, 20.75 acres; B, 26.93+ acres; R. R. 2.32− acres. **27.** 100 revolutions. **28.** 26.6455 knots per hour. **29.** $35.47. **30.** 21 double rolls. **31.** 345 sq. ft. **32.** $100.45.

**Page 229.** — **33.** $424.96. **34.** 15 ft. **35.** $96.36. **36.** $80. **37.** $666.67. **38.** $468. **39.** $826.20. **40.** 2714.34 cu. ft. **41.** 960 blocks; 55.2 T. **42.** 16.755 bbl. **43.** 3 T. **44.** $72.50.

**Page 230.** — **45.** $663.51. **46.** $67.50. **47.** $322.22. **48.** $167.68. **49.** 2025 lb. **50.** $166.59. **51.** 4 ft. **52.** $712.80. **53.** 1200 ft.

**Page 232.** — **2.** $8. **3.** 30 sheep. **4.** 60 yr. **5.** 300. **6.** 24. **7.** 100.

**Page 233.** — **11.** $11. **12.** $37.50. **13.** 16 yr. **14.** 30 books. **15.** 80¢. **16.** $100. **17.** $9000. **18.** $7.50. **19.** 60 bu. **20.** $96.

**Page 234.** — **21.** $80,000. **22.** $480. **23.** $2400. **24.** $1000. **25.** $24. **26.** $1.50. **27.** 750 girls; 450 boys. **28.** 12 yr. **29.** $24. **30.** $300. **31.** 30¢. **32.** $5000. **33.** 30 girls. **34.** $8000.

**Page 235.** — **35.** $75. **36.** $3200. **37.** 18 mi. first day. **38.** $20. **39.** 18; 54. **40.** $150. **41.** $3000. **42.** $45,000. **43.** $540. **44.** $5000; $3000. **45.** $\frac{1}{6}$, $\frac{1}{4}$.

**Page 236.** — **46.** $\frac{5}{12}$. **47.** $2\frac{2}{3}$ da. **48.** 12 da. **49.** 6 da. **50.** 4 da. **51.** $48. **52.** Harness, $20; sleigh, $40; horse, $160. **53.** $2. **54.** $2700. **55.** $\frac{4}{5}$ da. **56.** Second, $4200; third, $3000.

**Page 240.** — **2.** 13.12. **3.** 11.34. **4.** 28.93. **5.** 57. **6.** 39.72. **7.** 29.45. **8.** 236.16. **9.** 5.4. **10.** 25. **11.** $\frac{1}{2}$. **12.** 15. **13.** $\frac{3}{5}$. **14.** 147. **15.** 112. **16.** 180 girls; 220 boys. **17.** $7.50. **18.** 25.2 T. **19.** $2422.50. **20.** $16.50. **21.** The same. **22.** $73.95. **23.** $1750. **24.** $292,702.50.

**Page 241.** — **25.** $2.60; $1.73. **26.** $220, John; $198, James; $132, Henry. **27.** 240 A., wheat; 108 A., corn; 168 A., oats and grass; 84 A., rem.

**Page 242.** — **2.** 40%. **3.** $37\frac{1}{2}$%. **4.** 60%. **5.** 75%. **6.** 10%. **7.** $62\frac{1}{2}$%. **8.** 15%. **9.** 60%. **10.** $6\frac{1}{4}$%. **11.** $16\frac{2}{3}$%. **12.** $4\frac{1}{6}$%. **13.** $14\frac{2}{7}$%. **14.** 200%. **15.** 250%. **16.** 360%. **17.** 6%. **18.** 90%. **19.** 15%. **20.** 21%. **21.** 4%. **22.** 42%. **23.** $56\frac{1}{4}$%. **24.** 280%. **25.** 7%. **26.** $7\frac{1}{2}$%. **27.** 6%.

**Page 244.** — **3.** $30. **4.** $30. **5.** $728. **6.** $72. **7.** $127.50. **8.** $1200. **9.** $11.25. **10.** $8. **11.** $56.80. **12.** $32.32. **13.** $4128. **14.** $56. **15.** 920 men. **16.** $1800. **17.** $4700. **18.** $50. **19.** 600 pupils.

**Page 245.** — **2.** 300. **3.** 200. **4.** 400. **5.** 200. **6.** 46. **7.** 30. **8.** 900. **9.** 300. **10.** \$3. **11.** \$2. **12.** \$3500. **13.** \$1000. **14.** \$4700. **15.** 10 sq. mi.

**Page 246.** — **2.** 100. **3.** 300. **4.** 72. **5.** 45. **6.** 400. **7.** 400. **8.** 500. **9.** 5. **10.** \$40. **11.** \$60. **12.** \$384. **13.** \$2000; \$800; \$10,000. **14.** 549 pupils. **15.** \$20. **16.** $\frac{15}{16}$. **17.** 1000. **18.** \$160 loss. **19.** 16,000.

**Page 247.** — **1.** 100. **2.** 20. **3.** 1. **4.** \$10; $12\frac{1}{2}$%. **5.** \$854.40. **6.** 50%; 10. **7.** 300%; $33\frac{1}{3}$% **8.** 50%. **9.** $16\frac{68}{77}$%. **10.** 340. **11.** \$4000. **12.** \$500. **13.** $\frac{19}{27}$.

**Page 248.** — **14.** 85%. **15.** \$14,175. **16.** 120. **17.** \$5. **18.** $37\frac{1}{2}$%; $2\frac{1}{2}$%; $87\frac{1}{2}$%. **19.** \$200, horse; \$100, buggy. **20.** \$12,000. **21.** 2 T. **22.** 5%. **23.** \$6000; \$7200. **24.** \$144; \$150; \$180. **25.** $12\frac{1}{2}$%. **26.** \$1732.50. **27.** 60¢.

**Page 249.** — **28.** \$1620. **29.** \$9250. **30.** \$2760, increase. **31.** $74\frac{23}{24}$%.

**Page 250.** — **2.** \$15. **3.** \$32. **4.** \$17.25. **5.** \$43.13. **6.** \$74.76. **7.** \$37.10. **8.** \$24.28. **9.** \$110.10. **10.** \$260. **11.** \$2.40. **12.** \$540.

**Page 251.** — **2.** 5% gain. **3.** 20% gain. **4.** 4% loss. **5.** 35% loss. **6.** 30% gain. **7.** $33\frac{1}{3}$%. **8.** 20%. **9.** $22\frac{2}{9}$%. **10.** $\frac{2}{9}$% gain.

**Page 252.** — **2.** \$500. **3.** \$300. **4.** \$500. **5.** \$2450. **6.** \$8400. **7.** \$2500. **8.** \$35. **9.** \$225. **11.** \$200. **12.** \$600. **13.** \$880. **14.** \$960. **15.** \$1020. **16.** \$600. **17.** \$11,600. **18.** \$1250. **19.** \$1.25. **20.** \$1.25.

**Page 253.** — **22.** \$360. **23.** 60¢. **24.** $2\frac{1}{2}$%.
**1.** \$4143.75. **2.** $56\frac{1}{4}$%. **3.** 20% gain. **4.** \$88. **5.** \$3000. **6.** \$10,000. **7.** \$575. **8.** \$800.

**Page 254.** — **9.** 25%. **10.** \$30. **11.** \$7500. **12.** 4% gain. **13.** $12\frac{1}{2}$%. **14.** \$2.70. **15.** \$153. **16.** \$8000. **17.** \$150.

**Page 256.** — **2.** \$68.75. **3.** \$78.18. **4.** \$365.75. **5.** \$5484.38. **6.** 1%. **7.** 3%. **8.** 2%. **9** $2\frac{1}{2}$%.

**Page 257.** — **10.** \$69,040.80. **11.** \$152.50. **12.** \$624. **13.** \$616. **14.** \$1250. **15.** \$2268. **16.** \$3280. **17.** \$966.67. **18.** \$3795. **19.** \$19,750. **20.** \$1220. **21.** \$520 gained. **22.** 2%.
**1.** \$16,200. **2.** \$7787.50.

**Page 258.** — **3.** \$1610; \$1753. **4.** \$6085.25. **5.** \$306. **6.** \$1595.80 agent's commission; $10\frac{60}{191}$% gain. **7.** \$210.75.
**1.** Com., \$54; net pro., \$609.75. **2.** Rate, 4%; net pro., \$524.
**3.** Gross sales, \$1000; com., \$200. **4.** Rate, 10%; net pro., \$504.
**5.** Gross sales, \$600; rate, 8%. **6.** Com., \$40; net pro., \$960.
**7.** Gross sales, \$6700; expenses, none. **8.** Rate, 2%; net pro., \$2404.

**Page 259.** — **9.** \$1851.35. **10.** 360 A. **11.** \$131.60; \$6580. **12.** \$1680. **13.** \$23,294.12.

**Page 260.** — **2.** $\frac{3}{5}$%; \$24. **3.** \$58.50. **4.** $2\frac{1}{2}$%; \$2.50 per \$100. **5.** $\frac{3}{4}$%. **6.** $23\frac{1}{3}$¢. **7.** \$1900. **8.** $1\frac{3}{4}$%.

**Page 261.** — **9.** \$53.25. **10.** \$12,000. **11.** 1%. **12.** $1\frac{1}{2}$%. **13.** \$42. **14.** Farmer's loss, \$210; company's loss, \$3290. **15.** Company's loss, \$17,415; owner's loss, \$9585. **16.** \$4800. **17.** First, \$18,000; second, \$21,600; third, \$32,400; fourth, \$36,000.

**Page 263.** — **1.** \$119.80. **2.** $33\frac{103}{125}$%. **3.** $48\frac{2}{25}$%. **4.** \$821.60. **5.** \$268.45. **6.** 45 yr. **7.** \$800.40. **8.** \$450.55.

**Page 264.** — **1.** \$3; trade. **2.** Time and cash. **3.** 10%; trade. **4.** \$125.

**Page 265.** — **5.** 30%; trade. **6.** Trade. **7.** Time, \$3.75; cash, \$7.50. **9.** \$15. **10.** \$30. **12.** \$10. **13.** \$1350. **14.** 10% cash.

**Page 266.** — **4.** \$56.70. **5.** \$72. **6.** \$144. **7.** \$26.60. **8.** \$8. **9.** \$238.03. **10.** \$83.79. **11.** \$223.13. **12.** \$273.60. **13.** \$3249.

**Page 267.** — **14.** \$175. **15.** \$231.60. **16.** $24\frac{316}{441}$%. **17.** \$1. **18.** First, \$2.50. **19.** No difference. **20.** Third is \$208 better than first and \$135 better than second. **21.** \$70.13. **22.** \$1525.95. **23.** 56.8%.

**Page 268.** — **2.** \$547.20.

**Page 269.** — **3.** \$564.48. **4.** \$39.34.

**Page 271.** — **2.** Taxes, \$38; poll tax, \$3. **3.** Taxes, \$236; poll tax, \$4. **4.** $3\frac{1}{2}$ mills, rate. **5.** Est. val., \$937.50; assessed val., \$750. **6.** Est. val., \$222,900; assessed val., \$148,600. **7.** 18 mills; \$225. **8.** \$19.75. **9.** \$8125. **10.** \$22,460; \$561.50.

**Page 272.** — **11.** \$74.76. **12.** \$65.60. **13.** \$4680. **14.** \$21,173.64.

**Page 273.** — **1.** \$198. **2.** \$2500. **3.** \$6000. **4.** \$144.

**Page 274.** — **5.** \$800. **6.** \$728. **7.** \$2687.50. **8.** 3860 lb. **9.** \$2665.60. **10.** \$4.3045 per dozen. **11.** \$11,629.75. **12.** \$3289.49. **13.** \$2673. **14.** 1600 yd.

**Page 276.** — **2.** \$37.50. **3.** \$198.24. **4.** \$45.50. **5.** \$40. **6.** \$19.50. **7.** \$88. **8.** \$129. **9.** \$11. **10.** \$41.80. **11.** \$56.55.

**Page 277.** — **2.** \$1675.80. **3.** \$1226.55. **4.** \$448. **5.** \$573.80. **6.** \$3585. **7.** \$1439.36. **8.** \$215.12. **9.** \$2287.81. **10.** \$2952.50. **11.** \$225.50. **12.** \$1342.33. **13.** \$122.66. **14.** \$4056.50. **15.** \$210.43. **16.** \$342.94. **17.** \$233.14. **18.** \$1087.75. **19.** \$167.67. **20.** \$656.43. **21.** \$58.68. **22.** \$314.61. **23.** \$110.89. **24.** \$53.79. **25.** \$56.28.

**Page 278.** — **2.** \$2.75. **3.** \$1.43. **4.** \$2.58. **5.** \$2.97. **6.** \$7.13. **7.** \$4.20. **9.** \$2.50. **10.** \$3.80. **11.** \$13.02. **12.** \$2.50. **13.** \$10.06. **14.** \$5.31. **15.** \$2.50. **16.** \$2.10.

**Page 279.** — **17.** \$18. **18.** \$15.75. **19.** \$42.75. **20.** \$60.50. **21.** \$57.60. **22.** \$13.05. **23.** \$57.96. **24.** \$12.64. **25.** \$44.96. **26.** \$13.40. **27.** \$9.99. **28.** \$30. **29.** \$34.88. **30.** \$31.10. **31.** \$59.50. **32.** \$31.36. **33.** \$25.34. **34.** \$33.80. **35.** \$8.85. **36.** \$2.25. **37.** \$13. **38.** \$2.67. **39.** \$3.01. **41.** \$16.25. **42.** \$10.83. **43.** \$9.60. **44.** \$7.13. **45.** \$30.75. **46.** \$2.90. **47.** \$11. **48.** \$.51. **49.** \$3.07. **50.** \$7.05. **51.** \$13.24. **52.** \$6.71. **53.** \$11.23. **54.** \$80.03. **55.** \$171.81. **56.** \$9.90.

**Page 280.** — **57.** $1500.75. **58.** $121.60. **59.** $207.80. **61.** $27.54. **62.** $74.30. **63.** $74.80. **64.** $160.05. **65.** $305.10. **66.** $144.15. **67.** $262.60. **68.** $20.56.

**Page 281.** — **2.** $115.48. **3.** $91. **4.** $65.33. **5.** $101.28. **6.** $4.58. **7.** $9. **8.** $19.85. **9.** $114.49. **10.** $73.02. **11.** $77.25. **12.** $169.30. **13.** $337.40. **14.** $219.75. **15.** $247.50. **16.** $2.40. **17.** $4.77. **18.** $6.18. **19.** $10.50. **20.** $47.60. **21.** $60.50. **22.** $38.25. **23.** $145.13. **24.** $10.66. **25.** $19.99.

**Page 282.** — **26.** $31.93. **27.** $36.56. **28.** $1342.19. **29.** $966.60. **30.** $151.59. **31.** $895.13. **32.** $415.28. **33.** $38.48. **34.** $207.81. **35.** $2246.44. **36.** $7.15. **37.** $18.27. **38.** $12.33. **39.** $22.25. **40.** $77.19. **41.** $9.61. **42.** $1423.15. **43.** $765.53. **44.** $4044. **45.** $134.

**Page 283.** — **2.** $1000. **3.** $680. **4.** $1200. **5.** $1375.
**2.** 6%.

**Page 284.** — **3.** 5%. **4.** 5%. **5.** 6%.
**2.** 3 yr. 4 mo. **3.** 20 yr.; $16\frac{2}{3}$ yr.; $12\frac{1}{2}$ yr. **4.** 40 yr. **5.** Oct. 13, 1902.
**1.** $31.44. **2.** 2 yr. 6 mo. **3.** 3 yr. 6 mo.

**Page 285.** — **4.** $4\frac{1}{2}$%. **5.** $434. **6.** $180. **7.** $275. **8.** 6%. **9.** In 3 yr. 4 mo. **10.** $354.31. **11.** $88.20. **12.** Jan. 19, 1906. **13.** $600. **14.** July 1, 1906. **15.** $86,700. **16.** $3025.

**Page 286.** — **2.** $116.19. **3.** $8292.48.

**Page 287.** — **4.** $4954.20; $495.42.
**2.** $10.26. **3.** $35.29. **4.** $60.49. **5.** $41.34. **6.** $11.10. **7.** $20.68. **8.** $299.18. **9.** $116.99. **10.** $88.70.

**Page 288.** — **11.** $396.40. **12.** $655,912.33.

**Page 289.** — **2.** $103.81. **3.** $70.45. **4.** $27.41.

**Page 290.** — **2.** $1.52. **3.** $157.99. **4.** $80.52. **5.** $1056.07.

**Page 291.** — **6.** $206.04. **7.** $160.66. **8.** $909.93. **9.** $1233.35.

**Page 292.** — **1.** $1526.74. **2.** $14,233.12. **3.** $5642.40. **4.** $1324.90.

**Page 298.** — **1.** $103. **2.** $255.44. **3.** $530.42. **4.** $367.44. **5.** $129.13. **6.** $1219.50. **7.** $316.60.

**Page 301.** — **2.** $669.20. **3.** $295.65. **4.** $110.17.

**Page 302.** — **5.** $355.64. **6.** $9.57.
**1.** $439.70. **2.** $2380.37.

**Page 306.** — **1.** $663.15. **2.** $399.70, balance.

**Page 309.** — **2.** $203. **3.** $200. **4.** 3¢; 6¢.
**1.** Aug. 1. **2.** Aug. 22. **3.** Nov. 13. **4.** Oct. 10. **5.** April 2. **6.** May 17. **7.** June 10. **8.** Sept. 5.

**Page 310.** — **9.** April 30. **10.** July 10. **11.** Nov. 10. **12.** June 23. **13.** Oct. 14. **14.** April 2. **15.** July 23. **16.** Dec. 7. **17.** Sept. 3. **18.** Oct. 24.
**3.** Nov. 8; $ 2.25; $ 2.28. **4.** Aug. 12; $ 5.67; $ 5.76. **5.** Oct. 2; $ 11.50; $ 11.63. **6.** May 8; $ 15.63; $ 15.76. **7.** April 24; $ 4.75; $ 4.85. **8.** July 17; $ 4.53; $ 4.59. **9.** Oct. 1; $ 8.50; $ 8.78. **10.** March 11; $ 6.46; $ 6.53. **11.** May 16; $ 15.75; $ 15.97.

**Page 312.** — **1.** $ 2487.92; $ 2487.50. **2.** $ 1568.89; $ 1568.63. **3.** $ 1545.70; $ 1545.44. **4.** $ 4521.82; $ 4521.06.

**Page 313.** — **5.** $ 4455; $ 4454.25. **6.** $ 3468.50; $ 3467.92. **7.** $ 3537.87; $ 3537.27. **8.** $ 1000. **9.** $ 1207.04; $ 1206.83. **10.** $ 200; $ 209. **11.** $ 209. **12.** $ 205.62; $ 205.59. **14.** $ 224.33. **15.** $ 259.21; $ 259.10.

**Page 314.** — **16.** 9%. **17.** 12%. **18.** $ 158.76; $ 158.73. **19.** $ 1828.04; $ 1827.74. **20.** $ 1217.22; $ 1217.01. **21.** $ 2467.50; $ 2467.08. **22.** $ 6375.81; $ 6374.98. **23.** $ 1000. **24.** $ 2800.

**Page 319.** — **1.** $ 550.80. **2.** $ 1575, face; $ 2.63, exch. **3.** $ 1.30. **4.** 15¢. **5.** $ 1078.92.

**Page 320.** — **7.** Face, $ 989.01; exch., $ .99. **8.** Exch., $ 12.75; com., $ 275; face, $ 12,747.75. **9.** $ 791.98. **10.** $ 311.85.

**Page 322.** — **1.** 60 da.; 61 da. **2.** $ 1183; $ 1182.77.

**Page 323.** — **4.** $ 11,016.75; $ 11,014.59. **5.** $ 591.80; $ 591.70. **6.** $ 3809.50. **7.** $ 2456.25; $ 2455.83.

**Page 326.** — **2.** $ 6067.50. **3.** $ 8987. **4.** $ 12,142. **5.** $ 11,355.

**Page 327.** — **6.** $ 19,936. **7.** $ 8656.25. **8.** $ 20,680.63. **9.** $ 66,187.50. **10.** $ 37,250. **12.** 240 shares. **13.** 8 shares. **14.** 100 shares. **15.** 40 shares. **16.** 44 shares; surplus $ 14.50. **17.** 61+ shares; surplus $ 54.625.

**Page 328.** — **3.** $ 825. **4.** $ 1892.63. **5.** $ 2153.13. **6.** $ 22,137.50. **7.** $ 7453.13. **8.** $ 26,784. **9.** $ 13,785. **10.** $ 23,490.

**Page 329.** — **12.** 120 shares. **13.** 112 shares. **14.** 40 shares. **15.** 168 shares. **16.** 200 shares.
**1.** $ 60. **2.** $ 80. **5.** 7%. **6.** 5%. **7.** $ 2000. **8.** $ 16,000; $ 800.

**Page 330.** — **10.** 4%. **11.** 6%. **12.** $4\frac{1}{2}$%. **13.** $5\frac{1}{3}$%. **14.** $6\frac{1}{4}$%. **15.** 8%. **16.** Less. **18.** $ 1185. **19.** Former, $\frac{5}{17}$% better. **20.** $12\frac{1}{2}$%.

**Page 331.** — **21.** $ 800,000. **22.** $ 288. **24.** $ 24,025. **25.** $ 7288.75. **26.** $ 36,556.88. **27.** $ 9446.63.

**Page 333.** — **1.** $ 15,581.25. **2.** 50. **3.** $ 30,000. **4.** $ 2000. **5.** $ 35,437.50. **6.** 12%.

**Page 334.** — **7.** $6\frac{1}{4}$%. **8.** 6%. **9.** $ 1487.50. **10.** $6\frac{1}{4}$%; $5\frac{5}{9}$%; 5%; $4\frac{1}{6}$%; 4%. **11.** $ 4000. **12.** $ 1584.48. **13.** $ 400. **14.** 500%. **16.** $5\frac{5}{19}$%.

**Page 335.** — **1.** $ 60.48. **2.** $ 635.04. **3.** $ 8870. **4.** $30\frac{5}{9}$%. **5.** $ 520.52; $ 520.42. **6.** $ 36.40. **7.** $ 4000. p458

**Page 336.** — **8.** $19\frac{1}{2}\%$. **9.** 23[illegible] %. **10.** \$212.50. **11.** $5\frac{3}{5}\%$. **12.** $16\frac{984}{1001}\%$. **13.** \$585. **14.** \$1282.50; commission, \$67.50. **15.** \$480. **16.** \$7100.

**Page 338.** — **1.** 5. **2.** .25. **3.** 3. **4.** $\frac{1}{12}$. **5.** 8. **6.** .4. **7.** $\frac{3}{16}$. **8.** $\frac{1}{8}$. **9.** 100. **10.** 3. **11.** 6. **12.** 320.

**Page 339.** — **3.** $18\frac{3}{4}$. **4.** 15. **5.** 10. **6.** 5. **7.** 5. **8.** 10. **9.** $\frac{1}{5}$. **10.** .5. **11.** 2.5. **12.** 90. **14.** 30 da. **15.** \$18. **16.** 50 da.

**Page 340.** — **17.** 8. **18.** \$1921.88. **19.** \$4.17. **20.** 4 oz. **21.** $41\frac{2}{3}$ mi. **22.** \$40. **23.** \$79.55. **24.** 4 da. **25.** $37\frac{5}{7}$ sec. **26.** 3875 letters. **27.** 180 ft. **28.** 31,250 bricks.

**Page 341.** — **29.** $36\frac{36}{49}$ mi. **30.** 30 men. **31.** 24 men. **32.** \$60.30. **33.** 38 cars. **34.** \$236.25.

**Page 342.** — **2.** \$81, man; \$54, first boy; \$27, second boy. **3.** Expenses, \$5200; net savings, \$10,400. **4.** \$672,000; \$28,000; \$42,000; \$63,000. **5.** Lake, \$1000; railroad, \$1800. **6.** \$69,000; \$230,000.

**Page 343.** — **2.** A's, \$2520; B's, \$5760; C's, \$4320. **3.** M's, \$2520; N's, \$3024; R's, \$2016. **4.** E's, \$1000; F's, \$$857\frac{1}{7}$; G's, \$$2142\frac{6}{7}$. **5.** \$$1329\frac{6}{11}$; \$$920\frac{5}{11}$. **6.** Smith's, \$3225; Jones's, \$4300; Brown's, \$2150.

**Page 344.** — **8.** \$3348 to N; \$4608 to M. **9.** R's, \$1653.75; S's, \$1496.25. **10.** A's, $\frac{19}{35}$; B's, $\frac{16}{35}$.

**Page 345.** — **1.** \$600; \$400. **3.** \$3200; \$4000. **4.** \$15,000; \$12,000. **5.** 50%; 25%; 150%. **6.** Frank, \$18; Henry, \$15.

**Page 346.** — **7.** Walter's, \$1.20; Philip's, \$1.50. **8.** 40%. **9.** \$5000. **10.** Brown's, \$1050; Long's, \$1200. **11.** \$375. **12.** \$450. **13.** Moore's, \$1200; Silven's, \$1800; Rogers's, \$1500. **14.** \$45 per month. **15.** 77 da. **16.** To A, \$2700; to B, \$2400.

**Page 347.** — **17.** \$48. **18.** 6 da. **19.** 9 lots. **20.** 48 persons. **21.** 160 A. **22.** 60¢ per dozen. **23.** 24 da. **24.** \$3200; \$3600. **25.** \$1250. **26.** \$500; \$600.

**Page 348.** — **27.** F, \$1440; E, \$960. **28.** \$500; 6%. **29.** 500, 5000. **30.** 15 da. **31.** \$32. **32.** \$320. **33.** \$1000. **34.** 20,000 bu. **35.** \$5. **36.** Claim, \$1000; loss, \$400. **37.** $\frac{1}{64}$ greater.

**Page 352.** — **2.** 15°; 1 hr.; 60° W. has earlier time. **3.** 45°; 3 hr.; 120° W. **4.** 60°; 4 hr.; 15° W. **5.** 30°; 2 hr.; 30° E. **6.** 105°; 7 hr.; 75° W. **7.** 150°; 10 hr.; 120° W. **8.** 120°; 8 hr.; 90° W. **9.** 105°; 7 hr.; 30° E. **10.** 105°; 7 hr.; 45° W. **11.** 30°; 2 hr.; 15° E.

**Page 353.** — **12.** 10 hr. 8 min. 20 sec. A.M. **13.** 11 hr. 17 min. $45\frac{1}{5}$ sec. A.M. **14.** 8 hr. 58 min. $29\frac{1}{5}$ sec. A.M. **15.** 5 hr. 17 min. 33 sec. P.M. **16.** 5 hr. 58 min. $\frac{14}{15}$ sec. P.M. **17.** 6 hr. 36 min. $49\frac{3}{5}$ sec. A.M. **18.** 6 hr. 1 min. $46\frac{14}{15}$ sec. P.M. **19.** 12 min. $11\frac{4}{5}$ sec. P.M. **20.** 11 hr. 48 min. 4 sec. A.M. **21.** 5 hr. 47 min. $7\frac{1}{15}$ sec. **22.** N.Y., 3 hr. 36 min. $7\frac{4}{5}$ sec. P.M.; Cape Town, 9 hr. 46 min. 3 sec. P.M. **23.** 2 hr. 39 min. $35\frac{8}{15}$ sec.

**Page 354.** — **26**. Honolulu, 6 hr. 28 min. $37\frac{3}{5}$ sec. A.M.; Berlin, 5 hr. 53 min. $34\frac{14}{15}$ sec. P.M.; San Francisco, 8 hr. 50 min. $17\frac{1}{5}$ sec. A.M.; London, 4 hr. 59 min. $36\frac{4}{5}$ sec. P.M.
**1.** 17 da. **2.** 21 da.

**Page 355.** — **1.** 3 hr.

**Page 357.** — **5.** 160 A. **6.** 80 A. **7.** 40 A. **8.** 320 rd. **9.** 480 rd.

**Page 359.** — **8.** 225. **9.** 256. **10.** 324. **11.** 484. **12.** 625. **13.** 42.25. **14.** .5625. **15.** $2\frac{14}{25}$. **16.** $272\frac{1}{4}$. **17.** 225 sq. in. **18.** 625 sq. ft. **19.** 256 sq. yd. **20.** $72\frac{1}{4}$ sq. ft. **21.** 25 sq. in. **22.** 72.25 sq. in. **23.** 100 sq. yd. **24.** $32\frac{1}{9}$ sq. yd. **25.** 36 sq. in. **26.** 512 cu. in. **27.** 8 cu. ft. **28.** $34\frac{21}{64}$ cu. ft. **29.** $42\frac{7}{8}$ cu. ft. **30.** $56\frac{71}{216}$ cu. ft. **31.** $1876\frac{1}{27}$ cu. ft.

**Page 361.** — **2.** 15. **3.** 24. **4.** 21. **5.** 14. **6.** 40. **7.** 28. **8.** 36. **9.** 16. **10.** 48. **11.** 35. **12.** 42. **13.** 24. **14.** 56. **15.** 18. **16.** 20. **17.** 72. **18.** 25. **19.** 32. **20.** 64. **21.** 27.

**Page 364.** — **5.** 22. **6.** 24. **7.** 26. **8.** 31. **9.** 33. **10.** 43. **11.** 51. **12.** 63. **13.** $\frac{4}{5}$. **14.** $\frac{14}{15}$. **15.** $\frac{13}{18}$. **16.** .5. **17.** .15. **18.** 2.1. **19.** 2.5. **20.** .25. **21.** 17.74+. **22.** 22.91+. **23.** 34. **24.** 66. **25.** 14.5. **26.** 13.35+. **27.** 122. **28.** 163. **29.** 369. **30.** 4.56. **31.** 9.84. **32.** $13\frac{7}{8}$. **33.** 23.25. **34.** 56.5. **35.** 75.12+. **36.** .92. **37.** .114. **38.** .02+. **39.** .66+. **40.** 335.

**Page 366.** — **2.** 32.45+ in. **3.** 3 ft. **4.** 32 ft. **5.** 25 yd. **6.** 44.72+ in. **7.** 8.94+ ft. **8.** 1300 sq. rd. **9.** 22.36+ ft. **10.** 42.42 mi. **11.** 21.21+ rd. **12.** 1.41+ ft. **13.** 45 ft.

**Page 368.** — **1.** 370.8 sq. ft. **2.** 28.28 rd. **3.** 97.425 sq. in.

**Page 370.** — **1.** 250 sq. ft. **2.** $17\frac{1}{2}$ sq. ft. **3.** 108.3852 sq. ft. **4.** 613.3974 sq. in.

**Page 371.** — **1.** 169.6464 sq. ft. **2.** 768 sq. ft. **3.** 161.9 sq. ft. **4.** 700 sq. ft. **5.** \$54.17. **6.** \$58.81.

**Page 372.** — **1.** 452.3904 sq. in. **2.** 28.2744 sq. in. **3.** 530.9304 sq. in. **4.** 50.2656 sq. in. **5.** \$17.45.

**Page 373.** — **1.** 128 cu. in. **2.** 2 cu. ft. **3.** 185.4 T. **4.** 190.8522 cu. ft.

**Page 374.** — **1.** 201.0624 cu. in. **2.** 9 times. **3.** 1440 cu. in. **4.** 3456 cu. in. **5.** 256 cu. ft. **6.** 600 cu. ft. **7.** 284+ bu.

**Page 375.** — **8.** 4071.5136 cu. ft. **9.** $2\frac{26}{35}$ T. **10.** 24 in. **11.** 10 ft.
**1.** 904.7808 cu. in. **2.** 268.0832 cu. in. **3.** 288.696+ cu. in.

**Page 377.** — **1.** 75 ft. **2.** 80 rd. **3.** 12 ft. **4.** $2\frac{62}{169}$. **5.** 8 ft.

**Page 378.** — **6.** \$8.67. **7.** 21 ft.
**1.** $\frac{1}{8}$.

**Page 379.** — **2.** $\frac{1}{64}$. **3.** $\frac{1}{512}$. **4.** $\frac{1}{27}$. **6.** 20 ft.; $13\frac{1}{3}$ ft.; 8 ft. **7.** 16 ft. **9.** 200 cu. in. **10.** $179\frac{11}{16}$ bu.

**Page 380.** — **4.** 172.5 lb. **5.** 130.28+ lb. **6.** 150 lb. **7.** 1016.29+ lb. **8.** 656.25 lb. **9.** 64.375 lb. **10.** 57.5 lb. **11.** 556.25 lb. **12.** 168.75 lb. **13.** 849.375 lb. **14.** 1206.25 lb. **15.** 15 lb. **16.** 556.25 lb. **17.** 706.25 lb. **18.** 181.25 lb. **19.** 487.5 lb. **20.** 455.625 lb. **21.** 489.375 lb. **22.** 168.75 lb. **23.** 114.375 lb.

**Page 381.** — **1.** 27.98 bbl. **2.** 1000 sq. in. **3.** 10.39+ in. in diam. **4.** 3.4724 mi. **5.** $2\frac{1}{4}$ A. **6.** $9257\frac{1}{7}$ bu. **7.** 672.3024 cu. in. **8.** 4.18+ sq. ft. **9.** 18.13+ gal. **10.** 1 to $2\frac{1}{4}$.

**Page 385.** — **5.** 1.363 m.; 13.00025 m. **6.** 1 Km. 988 m. **7.** 177.192 mi.

**Page 386.** — **8.** 37.5 m. **9.** 67.5906 mi. **10.** 6 Hm.; 5 Dm.; 1 m. **11.** 6400 Km.

**Page 388.** — **2.** $72.90. **3.** $7.50. **4.** 24 sq. m. **5.** 24 steres.

**Page 390.** — **1.** 937.5 l. **2.** 188,496 l.; 188.496 M. T. **3.** $159.23. **4.** 3,000,000 l. **5.** 7.2 Hl. **6.** 9000 bottles.

**Page 391.** — **7.** $3860. **8.** 154.56 pf.; $.3678. **9.** 1000 l. **10.** $89\frac{1}{11}$ Kg. **11.** 1.3591+ cu. m. **12.** 3941.4 Kg. **13.** 169.164 m. **14.** $63.75. **15.** $11,812.50, cost of land; $1100, cost of fence. **16.** 600,000 tiles. **17.** 140,400 M. T. **18.** 4680 cu. m. **19.** $1.29.

**Page 392.** — **1.** $6.10. **2.** $9.33. **3.** $4.64. **4.** $2.31. **5.** $3.30. **6.** $11.74. **7.** $11.93. **8.** $12.06. **9.** $13.48. **10.** $28.16. **11.** $4.21. **12.** $12.01. **13.** $1.93. **14.** $3.80.

**Page 393.** — **16.** 1 to 7.9+. **17.** 1 to 3.8+. **18.** 1 to 5.27+. **19.** 1 to 5.65+. **20.** 1 to 5.8+. **21.** 1 to 8.7+. **22.** 1 to 13.7+.

**Page 394.** — **23.** 1 to 5.98+. **24.** 1 to 7.69+. **25.** 1 to 8.8+. **26.** Too wide; 1 to 7.1+. **27.** Too wide; 1 to 10.09+. **28.** Too wide; 1 to 8.2+. **29.** Too wide; 1 to 10.3+. **30.** 1 to 6.64+. **31.** 1 to 4.06+.

**Page 396.** — **1.** 234 lb.; 61.2 lb.; 52.25 lb.; 184 lb. **2.** .75 lb.; 1.14 lb.; 10.875 lb. **3.** 39 tons. **4.** 235 tons. **5.** $12.25. **6.** $23.16. **7.** $24.33. **8.** 36.4 tons.

**Page 397.** — **1.** 300 lb. of each. **2.** $2.60. **3.** $62. **4.** $274. **5.** $1140.

**Page 398.** — **6.** $131.80. **7.** $1. **8.** $1.14. **9.** $14.14. **10.** Materials: .009+ per gal.; $.115 per tree. Labor: $.01+ per gal.; $.13 per tree.

**Page 399.** — **11.** $.96. **12.** $6.25; $1.28. **13.** $30.19. **14.** 300 gal.; $7\frac{1}{2}$ lb.

**Page 400.** — **15.** $9.80. **16.** $.072−; $4.28. **17.** $58.33. **18.** $22\frac{2}{9}$%.

**Page 401.** — **1.** $1067.17. **2.** 3.1416 sq. rd. **3.** 25.42+ in.; 42.42+ in. **4.** $18. **5.** 324 cu. ft.; 10.8 min. **6.** $299.72.

**Page 402.** — **7.** 6 times. **8.** $300. **9.** $2.88. **10.** $22.62. **11.** 7.0686 cu. ft.; 21.2058 cu. ft.; 14.1372 cu. ft. **12.** 3 mills. **13.** 104.4 mi. **14.** 90 lb. **15.** $2000.

**Page 403.** — **16.** Cow, \$50; horse, \$140. **17.** 150.7968 in. **18.** 40 rd.; 160 rd. **19.** $6\frac{1}{2}$ %. **20.** $33\frac{1}{3}$ %. **21.** 17.32 in. **22.** 22.36 in. **23.** Width, 9 ft.; height, 6 ft. **24.** 1 to $3\frac{3}{8}$. **25.** \$834.40. **26.** \$13,453.

**Page 404.** — **27.** \$1080. **28.** \$2000. **29.** \$181.43; \$181.40. **30.** \$175. **31.** 20 A. **32.** A's, 320 A.; B's, 480 A.; C's, 600 A. **33.** 4.31+ %. **34.** \$630.40; \$630.29; \$639.86; \$639.75. **35.** \$1119.57; \$1118.87.

**Page 405.** — **36.** \$52.17. **37.** $85\frac{1}{3}$ ft. **38.** \$48.72. **39.** \$570. **40.** 73.24+ ft. **41.** 25. **42.** \$87.14. **43.** 5.02+ %.

**Page 406.** — **1.** 60 %. **2.** 28¢; \$2.80. **3.** \$30,000. **4.** 100 %. **5.** $6\frac{2}{3}$ hr. **6.** $6\frac{1}{4}$ %. **7.** \$1.65. **8.** $31\frac{11}{19}$ %.

**Page 407.** — **10.** M's, \$900; N's, \$800. **11.** 240 A. **12.** 20 %. **13.** 10 ft. **15.** 300 crates. **16.** \$120. **17.** 40.15 %. **18.** \$36. **19.** $108\frac{1}{3}$ %. **20.** $6\frac{1}{4}$ %.

**Page 408.** — **21.** 64 cu. in. **22.** 4 %. **23.** A's, \$450; B's, \$600; C's, \$1050. **24.** \$1400; \$1500. **25.** \$4500. **26.** \$25,000. **27.** *a.* $37\frac{1}{2}$; *b.* $\frac{4}{5}$. **28.** $3\frac{1}{2}$ %. **29.** 5456 yd.; 8342.4 yd. **30.** \$393.67. **31.** \$2510. **32.** 32,768. **33.** *a.* $4\frac{4}{9}$; *b.* $10\frac{1}{14}$.

**Page 409.** — **34.** \$3600. **35.** 288 cakes. **36.** 11,011,011.000011. **37.** $3\frac{3}{4}$. **38.** 1121.112. **39.** \$250,000. **40.** 40 A. **41.** .4, .375, .28, .5625, .75, .15625. **42.** \$419.85. **43.** 24, 15, $\frac{4}{9}$, 2. **44.** \$4800. **45.** \$4.32. **46.** $12\frac{1}{2}$ mills.

**Page 410.** — **47.** 60 da., \$986.67; 61 da., \$986.45. **48.** $20\frac{1}{4}$ sections; \$2880. **49.** \$429.84. **50.** \$505. **51.** \$6.75. **52.** July 1, 1906. **53.** \$450. **54.** \$705. **55.** A, $\frac{2}{15}$; B, $\frac{64}{225}$. **56.** \$353. **57.** \$168.48.

**Page 411.** — **58.** 60 shares. **59.** 20 yr.; $16\frac{2}{3}$ yr.; $12\frac{1}{2}$ yr. **60.** \$396. **61.** \$313.04. **62.** \$.$92\frac{1}{2}$ per yard. **63.** \$336. **64.** 1401 lb. **65.** \$19.20. **66.** \$2949.60. **67.** 21 mi.

**Page 412.** — **68.** 16.568 rd. **69.** 75 %. **70.** $83\frac{1}{3}$ %. **71.** 6283.2 sq. ft. **72.** \$39.90. **73.** 13¢. **74.** 2 yr. 4 mo. **75.** 1000 cu. ft. **76.** \$39.25. **77.** \$887.50.

**Page 413.** — **78.** Second, \$15. **79.** \$120; 20 %. **80.** 6 %. **81.** \$1000. **82.** \$38.40. **83.** First, \$13.50. **84.** \$6. **85.** A, $1\frac{1}{35}$; B, $\frac{8}{9}$. **86.** \$2125.60. **87.** $14\frac{2}{7}$ %.

**Page 414.** — **88.** \$27,984. **89.** 1020.021. **90.** 413.875 lb. **91.** \$500 loss. **92.** \$4500; \$1275. **93.** Latter by $\frac{2}{9}$ %. **94.** 48.55. **95.** \$800. **96.** 15 mills. **97.** 630 Kl.; 630,000 Kg.

**Page 415.** — **98.** 10,000 A. **99.** Nitrogen, 1890; oxygen, 7110. **100.** \$140.14 **101.** 200 men. **102.** $4\frac{43}{320}$. **103.** .0125. **104.** \$643.40. **105.** \$345.60. **106.** 50 rd. **107.** \$6000.

**Page 416**. — **108**. \$36,720. **109**. Ninety-nine hundred-thousandths. **110**. 16 ft. **111**. \$4. **112**. \$2842.50. **113**. \$144. **114**. \$601.50. **115**. 4 yr. 9 mo. 22 da. **116**. \$540. **117**. 9720 shingles.

**Page 417**. — **118**. $3\frac{1}{7}$%. **119**. \$640. **120**. \$120. **121**. \$6000. **122**. 162.5 lb. **123**. 79.5. **124**. $47\frac{16}{25}$%. **125**. \$300. **126**. \$320.25. **127**. \$3554.10; \$3555. **128**. 500 ft.

**Page 418**. — **129**. \$1979.74; \$1979.35. **130**. \$513.44. **131**. \$464.22. **132**. \$2937; \$2936.33. **133**. \$1670.14. **134**. \$267.48.

**Page 419**. — **135**. 11 hr. 33 min. 57 sec. A.M. **136**. \$384. **137**. \$1550, gain. **138**. \$558.33. **139**. 57.7269 Kl. **140**. 39 Kg. **141**. \$3213.42. **142**. Dec. 7, 9 hr. 51 min. 6 sec. A.M. **143**. 1 mi. 223 rd. 6 in.

**Page 420**. — **144**. 120. **145**. \$2371.85. **146**. 70. **147**. \$2298.75. **148**. \$1032.82. **149**. \$83,141.91. **150**. \$125,000. **151**. \$2090.

**Page 422**. — **2**. \$285. **3**. \$450. **4**. \$350. **5**. \$441.51. **6**. \$100. **7**. \$11.25. **8**. The former is \$5 better.

**Page 424**. — **4**. \$120.25. **5**. \$64.45+. **6**. \$1247.60. **7**. \$1228.96. **8**. \$148.95. **9**. \$76.43+. **10**. \$1948. **11**. 12,921 fr. 87.5 c. **12**. \$266.48. **13**. \$2697.50. **14**. \$457.90. **15**. 6489 M. 80 pf. **16**. £309 11*s*. 11*d*.+. **17**. \$1478.64; 1260 francs.

**Page 425**. — **2**. \$90. **3**. \$140. **4**. 54 rd.

**Page 431**. — **1**. 14. **2**. 16. **3**. 24. **4**. 27. **5**. 32. **6**. 45. **7**. 72. **8**. 98. **9**. 123. **10**. $\frac{3}{7}$. **11**. $\frac{12}{25}$. **12**. $\frac{5}{11}$. **13**. .9. **14**. 2.5. **15**. 3.4. **16**. 6.1. **17**. .15. **18**. $1\frac{1}{2}$.

**Page 432**. — **1**. 18 in. **2**. 441 sq. in. **3**. 13 ft. **4**. 47.55+ in. **5**. 8 ft. **6**. 6 in. **7**. 12 in. **8**. 17 ft. **9**. 5 to 7. **10**. 18 ft. high. **11**. The base of the bin is 172.8+ in. square; the height of the bin is 86.4+ in.